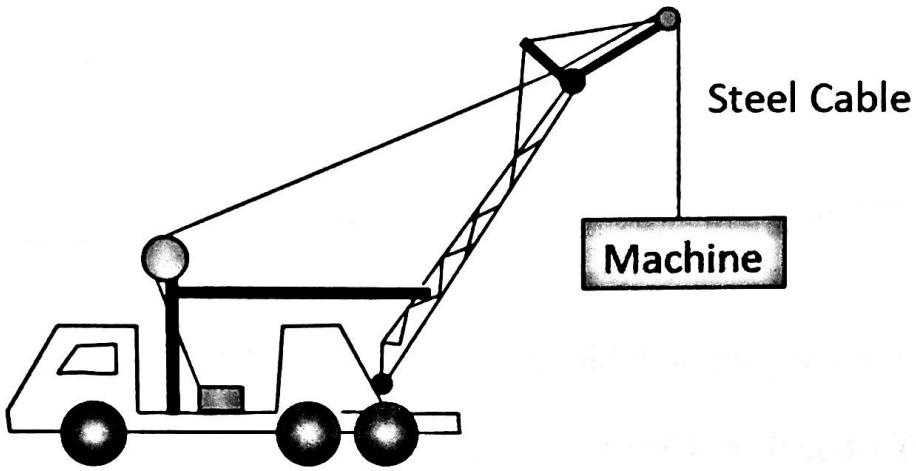
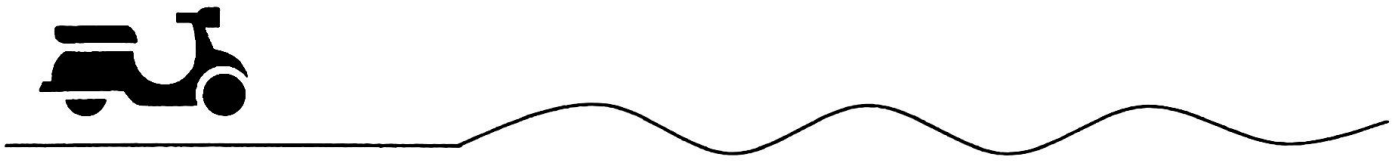
1. You are going to go tandem skydiving with a friend and your combined weight is 300 lb. You jump out of the plane with an initial zero velocity. Assuming that your parachute is immediately deployed, you encounter air resistance equal to 10 times the square of the descent velocity. (10 points)
    a. Derive the differential equation for your descent velocity.
    b. Solve the equation for your descent velocity.
    c. How much time will it take for you to reach the ground from a height of 10,000 ft?
    d. What will be your impact velocity?

2. A crane is lowering a 450 kg machine to the ground as illustrated below. The stranded steel cable that holds the machine has a stiffness of 550 N/m. Each strand has a diameter of 5 mm. The machine is being lowered at 0.5 m/s when the crane suddenly seizes and jams, stopping the cable from lowering the machine any further. (10 points)



a. What is the frequency of motion of the machine when the crane seizes?
b. What is the maximum tension in the cable after the crane seizes?
c. Will the cable break as a result of this motion if the maximum tensile stress is 170 MPa?

3. You are driving a motorcyle supported by a suspension system. You have a water pack with 50 kg of water. The combined mass of you and the motorcycle is 150 kg. The suspension system has a spring constant of k = 20,000 N/m and a damping coefficient of c = 2,000 N/m. You are driving on a wavy road surface that exerts a cyclic vertical force on the motorcycle that can be approximated by the function 2000 sin(10t) N. (10 points)

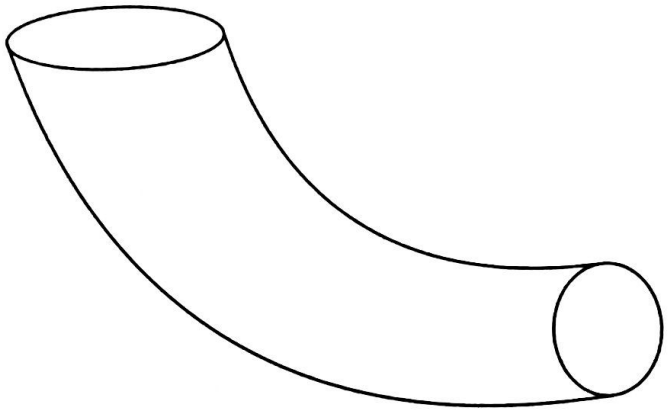


a) Determine the differential equation that would describe your movement (y) in the motorcyle as a function of time.
b) Find the solution to the differential equation if $y(0) = 0$ and $y'(0) = 5\ m/s$.
c) What is your position 2 s after you hit the first bump in the road?
d) What would the differential equation be if your water pack springs a leak when you hit the bump and you lose 2 kg of water every second?

4. If possible, find a series solution for the following equations about $x_0 = 0$. (10 points)

$$x^2y'' + xy' + (x - 5)\, y = 0$$

5. For fluid flow through this pipe (10 points):



a) Derive the governing differential equations in three dimensions for fluid velocity if the fluid is an incompressible Newtonian fluid with the following constitutive relations:

$$\sigma_{xx} = -p + 2\mu\frac{\partial u}{\partial x} \qquad \tau_{xy} = \tau_{yx} = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)$$

$$\sigma_{yy} = -p + 2\mu\frac{\partial v}{\partial y} \qquad \tau_{yz} = \tau_{zy} = \mu\left(\frac{\partial v}{\partial z} + \frac{\partial w}{\partial y}\right)$$

$$\sigma_{zz} = -p + 2\mu\frac{\partial w}{\partial z} \qquad \tau_{xz} = \tau_{zx} = \mu\left(\frac{\partial u}{\partial z} + \frac{\partial w}{\partial x}\right)$$

- where σ is normal stress
- τ is shear stress
- p is the pressure
- μ is viscosity
- and u, v, w are the x-, y-, and z- components of velocity respectively.

b) How would the differential equations change if the fluid is an inviscid fluid?

c) Select a non-Newtonian fluid and find its constitutive equation. How would the governing differential equations change in this case?