

Math 5A - Final Exam - Fall 2021

100 points . Each problem is worth 9 points. There is an additional 1 "free" point. Please show your work.

#1.) Where is  $f(x) = \frac{\sqrt{x^2 - 9}}{x^2 - 2}$  continuous?

#2.) Find the limit.  $\lim_{x \rightarrow 1} \frac{\sin(x-1)}{x^2 + x - 2}$

#3.) If  $y = \tan(x^2 \sin x)$  find  $\frac{dy}{dx}$

#4.)  $y = \sqrt{x + \sqrt{x + \sqrt{x}}}$  find  $\frac{dy}{dx}$

#5.) Use implicit differentiation to find an equation of the tangent line to the curve

$\tan(x + y) + \sec(x - y) = 2$  at the point  $\left(\frac{\pi}{8}, \frac{\pi}{8}\right)$ .

#6.) Two men stand 10m apart on level ground near the edge of a cliff. One man drops a stone and one second later the other man drops a stone. One second after that, how fast is the distance between the two stones changing? [Use the fact that the distance (in meters) a dropped stone falls after  $t$  seconds is  $d = 4.9t^2$  ].

#7.) Find the dimensions of a rectangle with area  $1000m^2$  whose perimeter is as small as possible.

#8.) Evaluate  $\int \frac{\sec^2 x}{\tan^2 x} dx$

#9.) Evaluate  $\int_0^4 \frac{x}{\sqrt{1+2x}} dx$

#10.) Find the area enclosed by the curves  $y = \sqrt{x-1}$  and  $x - y = 1$  .

#11.) Find the volume of the solid obtained by rotating the region bounded by  $y = 6 - x^2$  and  $y = 2$  about the x-axis.