48 F

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Q#1)

SOLUTION :-

$$\because (f,g) = \int_0^1 f(x)g(x)\,dx$$

wrong function

$$= \int_0^1 (5t-3)(t^3-t^2)\,dt$$

$$= \int_0^1 (5t^4 - 8t^3 + 3t^2)\,dt$$

$$= 5\int_0^1 t^4\,dt - 8\int_0^1 t^3\,dt + 3\int_0^1 t^2\,dt$$

$$= \left[t^5 - 2t^4 + t^3\right]_0^1$$

$$= \left[t^3(t-1)^2\right]_0^1 + C$$

incorrect work

−8

$$= 0$$

$$\|g\| = \sqrt{(1)^3 + (-1)^2 + 0 + 0} = \sqrt{1+1} = \sqrt{2}$$

Not $\sqrt{2}$.

Q # 2

ANSWER

$y = \begin{bmatrix} 2 \\ 6 \end{bmatrix}$, $u = \begin{bmatrix} 7 \\ 1 \end{bmatrix}$

Vector notation missing. (-1)

$\hat{y} = \frac{y \cdot u}{u \cdot u} u$

$\Rightarrow y \cdot u = (2,6).(7,1) = 14 + 6 = 20$

$\Rightarrow u \cdot u = (7,1) \cdot (7,1) = 49 + 1 = 50$

$\hat{y} = \frac{20}{50} u = \frac{20}{50} \begin{bmatrix} 7 \\ 1 \end{bmatrix} = \begin{bmatrix} 14/5 \\ 2/5 \end{bmatrix}$

as, $\hat{y}$ is the part that is in span u. Calculating other part.

$y - \hat{y} = \begin{bmatrix} 2 \\ 6 \end{bmatrix} - \begin{bmatrix} 14/5 \\ 2/5 \end{bmatrix} = \begin{bmatrix} -4/5 \\ 28/5 \end{bmatrix}$

Result:

$y = \begin{bmatrix} 14/5 \\ 2/5 \end{bmatrix} + \begin{bmatrix} -4/5 \\ 28/5 \end{bmatrix}$

Q #3

ANSWER:

$T(p) = (t^2+1)\,p(t)$

Vector notation missing. (-1)

$$\text{Matrix} = \Big[\, [T(1)]_c \quad [T(t)]_c \quad [T(t^2)]_c \,\Big]$$

$$= \Big[\, [t^2+1]_c \quad [t^3+t]_c \quad [t^4+t^2]_c \,\Big]$$

$$= \begin{array}{c} \begin{array}{ccc} 1 & t & t^2 \end{array} \\ \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \end{array} = \begin{bmatrix} 1 & 0 & 0 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}$$

-9

wrong entries.

Q#4

SOLUTION:

Many valid steps missing.

$f(t) = t-1$

$[-\pi, \pi]$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi}(t-1)\,dt$$

steps missing here.

$$a_0 = \frac{1}{2\pi}(-2\pi) = -1$$

Not -1.

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t-1)\cos nt\,dt$$

steps missing here.

?

$$= \frac{1}{\pi}\left[-\frac{2\sin(\pi n)}{n}\right]$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi}(t-1)\sin nt\,dt$$

steps missing here.

?

$$= \frac{1}{\pi}\left[\frac{2\sin(\pi n) - \pi n\cos(\pi n))}{n^2}\right]$$

so,

$$\boxed{-1 + \sum_{n=1}^{\infty} = 1 - \frac{2(-1)^n \sin(nx)}{nx}}$$

?

-1/2

Q # 5

SOLUTION

Starting with

$$\begin{bmatrix} -1 & 6 & 6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix}$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 3 & -8 & 3 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix} \quad (-1)R_1$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 10 & 21 \\ 1 & -2 & 6 \\ 1 & -4 & -3 \end{bmatrix} \quad R_2 - 3R_1$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 10 & 21 \\ 0 & 4 & 12 \\ 1 & -4 & -3 \end{bmatrix} \quad R_3 - (1)R_1$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 21/10 \\ 0 & 4 & 12 \\ 0 & 2 & 3 \end{bmatrix} \quad \tfrac{1}{10}R_2$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 21/10 \\ 0 & 0 & 15/5 \\ 0 & 2 & 3 \end{bmatrix} \quad R_3 - 4R_2$$

invalid
incorrect work

5

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 21/10 \\ 0 & 0 & 18/5 \\ 0 & 0 & -6/5 \end{bmatrix} \quad R_4 - 2R_2$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 21/10 \\ 0 & 0 & 1 \\ 0 & 0 & -6/5 \end{bmatrix} \quad \frac{5}{18} R_3$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 21/10 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_4 + 6/5 \, R_3$$

$$\sim \begin{bmatrix} 1 & -6 & -6 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_2 - 21/10 \, R_3$$

$$\sim \begin{bmatrix} 1 & -6 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 + 6R_3$$

$$\sim \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix} \quad R_1 + 6R_2$$

incorrect work

Many valid steps missing.

−12

Recalling that a leading 1 is the first non-zero entry in a row. Colums conteining leading ones are the pivot columns. To optain basis for column space, we use pivot colums from original matrix.

(6)

$$\left\{ \begin{bmatrix} -1 \\ 3 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 6 \\ -8 \\ -2 \\ -4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 6 \\ -3 \end{bmatrix} \right\}$$

wrong vectors.

Q#6

SOLUTION:

let, $A = \begin{bmatrix} 0 & -4 & -6 \\ -1 & 0 & -3 \\ 1 & 2 & 5 \end{bmatrix}$, a matrix to be diagonalized using $\lambda = 1, 2$.

for $\lambda = 1$,

eigenvector: $\begin{bmatrix} -2 \\ -1 \\ 1 \end{bmatrix}$

for $\lambda = 2$

eigenvector: $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

for $\lambda = 2$

eigenvector: $\begin{bmatrix} -3 \\ 0 \\ 1 \end{bmatrix}$

work not shown for eigenvectors.

−9

For a matrix P, whose i-th colum is i-th eigenvector.

$$P = \begin{bmatrix} -2 & -2 & -3 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

From diagonal matrix D, $D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$

It has property $A = PDP^{-1}$!

$$P = \begin{bmatrix} -2 & -2 & -3 \\ -1 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$