MATH 227: FINAL EXAM – Part II

Show your work step by step.

Problem # 1: Summarize the following data with a frequency table with five classes. For each class find the class limits, class boundaries, midpoint, and frequency. Base on the frequency table, construct the histogram. **(4 points for the frequency table and 2 points for the histogram)**

21	23	29	30	32	35	39	39	41	43
48	48	49	49	50	53	60	64	65	68

Class limits	Class boundaries	Frequency	Midpoint

Histogram:

Problem # 2: Find the population mean, median, mode, variance and standard deviation for the set of data: **13, 7, 21, 4, 15, 23, 7, 6.** <u>**Show your work step by step.**</u>

Mean =
(2 points)

Median =
(1 point)

Mode =
(1 point)

Variance =
(4 points)

Standard Deviation =
(1 point)

Problem # 3: If a die is rolled one time, find these probabilities: **(2 points each)**

a) Getting a 6 or 1

b) Getting a number less than 3 or an odd number

Problem # 4: If two dice are rolled one time, find these probability that the sum is 10. **(2 points)**

Problem # 5: A couple has three children. Find the probability that they have: **(2 points each)**

a) Exactly two boys.

b) At least two boys.

Problem # 6: A box contains 4 black balls, 7 red balls, and 3 green balls. Three balls are drawn **(without replacement).** Find the probability that the balls drawn are red, green, and black in that order. **(4 points)**

Problem # 7: A box contains 5 black balls and 15 red balls. Two balls are drawn and each one is <u>replaced</u> before the next one is drawn. Find the probability that both balls are black. **(3 points)**

Problem # 8: A student is to be selected randomly from a group of students. For each classification of freshman and sophomore, there is a music major, an English major, and a biology major. The following table summarized the number of students in each category:

	Music	English	Biology	Total
Freshman	13	41	34	
Sophomore	27	15	20	
Total				

Find the probability that:

a) A sophomore or an English major is selected **(3 points)**

a) A Biology or English major is selected **(3 points)**

Problem # 9: How many different ways a slate of officers consisting of a chair, vice-chair, and secretary be selected from a group of 7 people? **(2 points)**

Problem # 10: How many different ways committee of four can be selected from a group of 7 people? **(3 points)**

Problem # 11: The number of students using the School Bus per day is found in the distribution below. Find the mean, variance, and the standard deviation for this probability distribution.

X	15	25	10	12
P(X)	0.3	0.4	0.2	0.1

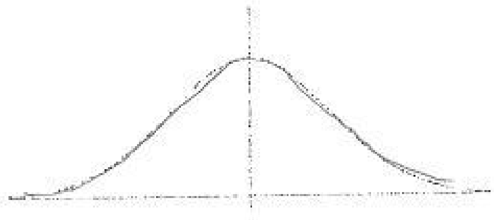
Mean =
(3 points)

Variance =
(4 points)

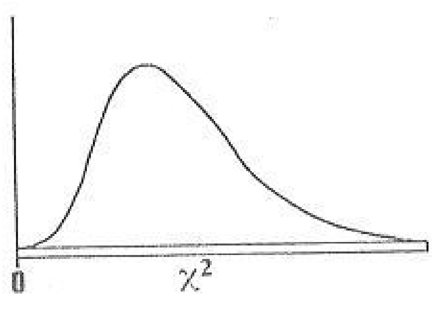
Standard deviation =
(1 point)

Problem # 12: A students takes a **5 – question**, multiple-choice exam with **four choices** for each question and guesses on each question. Find the probability of guessing exactly **3 out of 5 correctly**. **(7 points)**

Problem # 13: A salaries at the corporation are normally distributed with an average salary of \$25,000 and a standard deviation of \$2,500. What is the probability that an employee will have a salary between \$24,800 and \$26,900? (8 points)



Problem # 14: Find the 99% confidence interval **for the variance and standard deviation** for the time it takes state police inspector to check a truck for safety if a sample of 23 trucks has a standard deviation of 5.3 minutes. Assume the variable is normally distributed. **(8 points)**



(15) A sociologist found that in a sample of 50 retired men, The average number of jobs they had during their lifetime was 7.2. The population standard deviation is 2.1

(a) Find The best point estimate of The mean

(b) Find the 99% confidence interval for the mean number of Jobs.

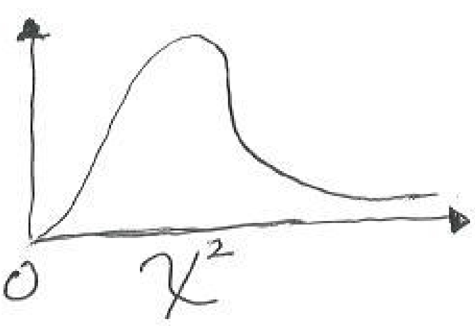
(16) A recent study of 28 employees of XYZ company showed that the mean of the distance They traveled to work was 14.3 miles with the standard deviation of 2 miles. Find the 95% confidence interval of the true mean. Assume That the population is approximately normally distributed.

(17)

In a study of 200 accidents that required treatment in an emergency room, 80 occurred at work.
Find the 90% confidence interval of the true proportion of accidents that occurred at work.

(18)

Find the 90% confidence interval for the variance and standard deviation of the ages of seniors at Oak Park college if a sample of 24 students has a standard deviation of 2.3 years. Assume the variable is normally distributed.



Formulas:

Chapter # 3:

The Sample Mean $\overline{X} = \frac{\sum X}{n}$ **The Population Mean** $\mu = \frac{\sum X}{N}$

Population Variance and Standard Deviation.

$$\sigma^2 = \frac{\sum (X-\mu)^2}{N}$$ Population Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Sample Variance and Standard Deviation.

$$s^2 = \frac{\sum (X-\overline{X})^2}{n-1}$$ Sample Standard Deviation: $s = \sqrt{s^2}$

Shortcut Formula for Sample Variance and Standard Deviation.

$$S^2 = \frac{n(\sum X^2)-(\sum X)^2}{n(n-1)}$$ $s = \sqrt{s^2}$

Chapter # 4:

Formula for Classical Probability: $P(E) = \frac{n(E)}{n(S)}$

Rule for Complimentary Events:

$P(\overline{E}) = 1 - P(E)$ or $P(E) = 1 - P(\overline{E})$ or $P(E) + P(\overline{E}) = 1$

Addition Rule 1: When two events A and B are mutually exclusive, the probability that A and B will occur is $P(A \text{ or } B) = P(A) + P(B)$

Addition Rule 2: When two events A and B are not mutually exclusive, the probability that A and B will occur is $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

Multiplication Rule 1: When two events A and B are independent, the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B)$

Multiplication Rule 2: When two events A and B are dependent, the probability of both occurring is $P(A \text{ and } B) = P(A) \cdot P(B \mid A)$

The Fundamental Counting Rule: In a sequence of n events in which the first one has k_1 possibilities and the second event has k_2 possibilities and the third event has k_2 possibilities and so forth , the total number of possibilities of the sequence will be $k_1 \cdot k_2 \cdot k_3 \cdots \cdot k_n$

Permutation Rule: The number of permutations of r objects selecting from n objects is denoted by ${}_nP_r$ and is given by the formula :

$${}_nP_r = \frac{n!}{(n-r)!}$$

Combination Rule: The number of combinations of r objects selecting from n objects is denoted by ${}_nC_r$ and is given by the formula

$${}_nC_r = \frac{n!}{(n-r)!r!}$$

Chapter # 5:

Discrete Probability Distribution:

Mean: $\mu = \sum X \cdot P(X)$ **Variance:** $\sigma^2 = \Sigma\left[X^2 \cdot P(X)\right] - \mu^2$

Standard Deviation: $\sigma = \sqrt{\sigma^2}$

Binomial Probability: $P(X) = \dfrac{n!}{(n-X)!X!} p^X q^{n-X}$ $q = 1 - p$

Mean for Binomial Distribution: $\mu = n \cdot p$

Variance for Binomial Distribution: $\sigma^2 = n \cdot p \cdot q$

Standard Deviation for Binomial Distribution: $\sigma = \sqrt{n \cdot p \cdot q}$

Chapter # 6:

z – score: $z = \dfrac{X - \mu}{\sigma}$

Mean of the sample means: $\mu_{\overline{X}} = \mu$

Standard deviation (error) of the sample means: $\sigma_{\overline{X}} = \dfrac{\sigma}{\sqrt{n}}$

z -value for the Central Limit Theorem: $z = \dfrac{\overline{X} - \mu}{\sigma / \sqrt{n}}$

Chapter # 7:

Formula for the Confidence Interval of the Mean for a Specific α When σ is Known

$$\overline{X} - z_{\alpha/2} \cdot \left(\frac{\sigma}{\sqrt{n}}\right) < \mu < \overline{X} + z_{\alpha/2} \cdot \left(\frac{\sigma}{\sqrt{n}}\right)$$

Confidence Level	$z_{\alpha/2}$
90%	1.65
95%	1.96
99%	2.58

Formula for the Minimum Sample Size Needed for an Interval Estimate of the Population Mean

$$n = \left(\frac{z_{\alpha/2} \cdot \sigma}{E}\right)^2$$ where E is the margin of error.

Formula for the Confidence Interval of the Mean for a Specific α When σ is Unknown

$$\overline{X} - t_{\alpha/2} \cdot \left(\frac{s}{\sqrt{n}}\right) < \mu < \overline{X} + t_{\alpha/2} \cdot \left(\frac{s}{\sqrt{n}}\right)$$ The degrees of freedom (d. f.) = $n-1$.

Formula for a Specific Confidence Interval for a Proportion

$$\hat{p} - z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}} < p < \hat{p} + z_{\frac{\alpha}{2}} \cdot \sqrt{\frac{\hat{p} \cdot \hat{q}}{n}}$$ when $n\hat{p} \geq 5$ and $n\hat{q} \geq 5$

Formula for Minimum Sample Size Needed for Interval Estimate of Population Proportion

$$n = \hat{p}\hat{q}\left(\frac{z_{\frac{\alpha}{2}}}{E}\right)^2$$ where E is the margin of error.

Formula for the Confidence Interval for a Variance

$$\frac{(n-1)s^2}{\chi^2_{right}} < \sigma^2 < \frac{(n-1)s^2}{\chi^2_{left}}$$

$$d.f. = n-1$$

Formula for the Confidence Interval for a Standard Deviation

$$\sqrt{\frac{(n-1)s^2}{\chi^2_{right}}} < \sigma < \sqrt{\frac{(n-1)s^2}{\chi^2_{left}}}$$

$$d.f. = n-1$$

Chapter # 8.

Z - Test for a Mean When σ is Known

Test value: $$z = \frac{\overline{X} - \mu}{\sigma / \sqrt{n}}$$

t - Test for a Mean When σ is Unknown

Test value: $$t = \frac{\overline{X} - \mu}{s / \sqrt{n}}$$

Z - Test for a Proportion

Test value:

$$z = \frac{\hat{p} - p}{\sqrt{pq/n}} \quad \text{where} \quad \hat{p} = \frac{X}{n} \qquad q = 1 - p$$

χ^2 - Test for a Variance or Standard Deviation

$$\chi^2 = \frac{(n-1)s^2}{\sigma^2} \qquad \text{d.f.} = n - 1$$