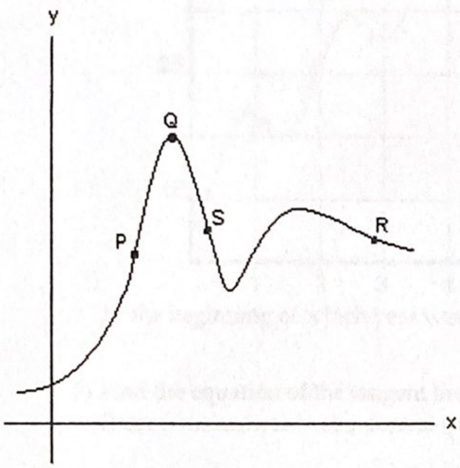
SOLUTIONS

MATH 211 - Test #2 Review

Referring to the graph below, assign one of the following descriptors to the point: large positive slope, small positive slope, zero slope, small negative slope, large negative slope.



1) P — 1) Large positive slope

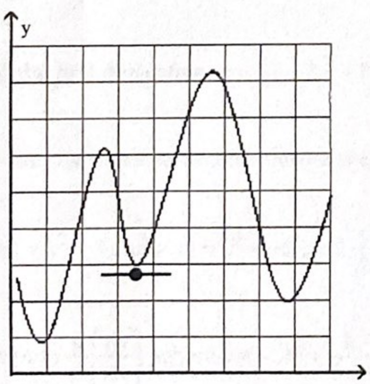
2) Q — 2) Zero slope

3) S — 3) Large negative slope

4) R — 4) small negative slope

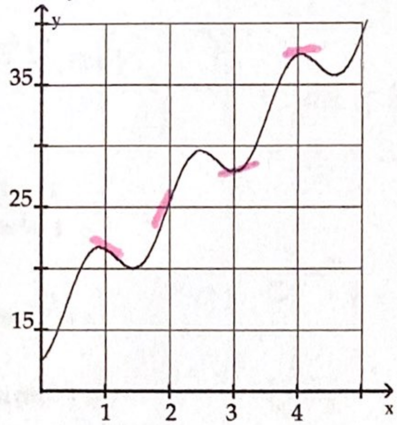
Estimate the slope of the curve at the designated point.

5) — 5) 0



Solve the problem.

6) The graph below shows the sales (in thousands of dollars) at a certain company over the course of 4 years. 6) 2



At the beginning of which year were the sales rising the most?

7) Find the equation of the tangent line to the curve $y = x^3 + 4x^2 + 4$ at $(1, 9)$. Enter your answer in standard slope–intercept form. 7) $y = 11x - 2$

8) Find the equation of the tangent line to the graph of $y = -\frac{3}{5x + 2}$ at $x = 3$. 8) $y = \frac{15}{289}x - \frac{96}{289}$

Differentiate.

9) $f(x) = 3x^4 + 5x^3 + 1$ $\quad f'(x) = 12x^3 + 15x^2$ 9) $12x^3 + 15x^2$

10) $y = (x^2 + 4)^3$ $\quad \frac{dy}{dx} = 3(x^2+4)^2 \cdot 2x = 6x(x^2+4)^2$ 10) $6x(x^2+4)^2$

11) $y = \frac{3}{x^5} - \frac{2}{x}$ 11) $-15/x^6 + 2/x^2$

12) Find the first derivative of $w(t) = 7t^2 - 19\sqrt{t} + 23$ 12) $14t - \frac{19}{2\sqrt{t}}$

13) Use the chain rule to find the derivative of $\sqrt{6x^3 - 2x}$ at $x = 1$. 13) 4

14) If $f(x) = x^2 - 9$ and $g(x) = x^2 - 16$, find $\frac{d}{dx} g(f(x))$. 14) $4x(x^2-9)$

15) Compute $\frac{d}{dt}\left(\frac{dv}{dt}\right)$, where $v = -5t^3 + \frac{2}{1 - t}$ at $t = -1$. 15) $61/2$

16) Compute $f''(2)$ when $f(t) = \frac{3}{(3t - 1)^2}$. 16) $162/625$

17) $f(x) = (x^2 + 1)^7$, find $f''(x)$.

17) $14(x^2+1)^5(13x^2+1)$

18) $y = \frac{x^4 + 1}{x^2}$, find y''.

$y = \frac{x^4}{x^2} + \frac{1}{x^2} = x^2 + x^{-2}$

$y' = 2x - 2x^{-3}$

$y'' = 2 - 2(-3)x^{-4} = 2 + 6/x^4$

18) $2 + \frac{6}{x^4}$

Differentiate.

19) $y = \frac{9x - 4}{6x^2 + 1}$

19) $\frac{-54x^2+48x+9}{(6x^2+1)^2}$

20) $f(x) = e^{4x^2} + x$

$f'(x) = e^{4x^2} \cdot 8x + 1$

20) $8xe^{4x^2} + 1$

21) $y = \ln(6x^3 - x^2)$

$y' = \frac{1}{6x^3-x^2} \cdot 18x^2-2x = \frac{\cancel{x}(18x-2)}{\cancel{x}(6x^2-x)}$

21) $\frac{18x-2}{6x^2-x}$

22) $x^3 \ln x$

$x^3 \cdot \frac{1}{x} + \ln x \cdot 3x^2$

$x^2 + 3x^2 \ln x$

22) $x^2 + 3x^2 \ln x$

Find all antiderivatives of the function.

23) $f(x) = e^{-x/2}$

23) $-2e^{-1/2x} + C$

24) Find: $\int \left(\frac{x^2}{4} - 4\right) dx$

24) $\frac{1}{12}x^3 - 4x + C$

25) Find: $\int (x^5 + e^{5x})\, dx$

25) $\frac{1}{6}x^6 + \frac{1}{5}e^{5x} + C$

Calculate.

26) $\int_{-2}^{-1} (x^2 - 2x^{-3} + 3)\, dx$

26) $73/12$

27) $\int_{1}^{4} \sqrt{x}\, dx$

27) $14/3$

Compute the net change of the function.

28) Given $f'(x) = 3x^2 + 7$, compute $f(2) - f(-2)$.

28) 44

Find the partial derivative.

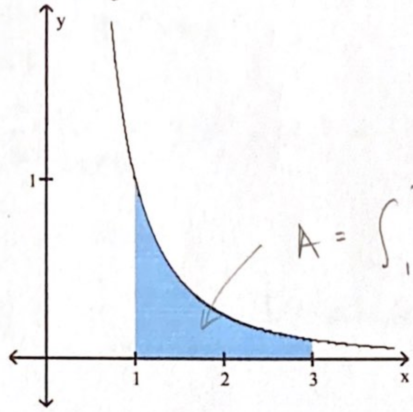
29) $f(x,y) = 3x^2 - 15xy + 7y^3$. Find $\frac{\partial f}{\partial x}$.

29) $6x - 15y$

30) $f(x, y) = e^{-2x + 3y}$. Find $\frac{\partial f}{\partial x}$.

30) $-2e^{-2x+3y}$

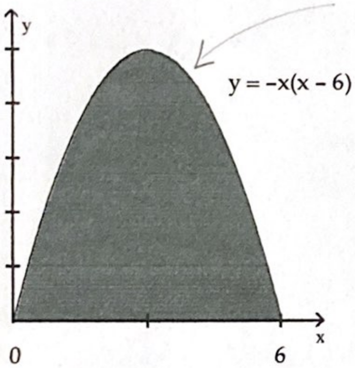
31) Given the graph of the function $y = \frac{1}{x^2}$, set up the definite integral that gives the area of the shaded region. 31) $\int_1^3 \frac{1}{x^2}\,dx$



$A = \int_1^3 \left(\frac{1}{x^2}\right) dx$

Compute the area of the shaded region.

32) 32) 36



$A = \int_0^6 (-x(x-6))\,dx$

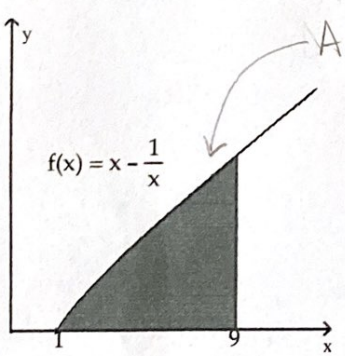
$= \int_0^6 (-x^2 + 6x)\,dx$

$\left(-\frac{1}{3}x^3 + 3x^2\right)\Big|_0^6$

$-\frac{1}{3}(6)^3 + 3(6)^2 - 0$

$= 36$

33) 33) $40 - \ln 9$



$A = \int_1^9 \left(x - \frac{1}{x}\right) dx$

$\left(\frac{1}{2}x^2 - \ln x\right)\Big|_1^9$

$\frac{1}{2}(9)^2 - \ln 9 - \left(\frac{1}{2}(1)^2 - \ln(1)\right)$

$\frac{81}{2} - \ln 9 - \left(\frac{1}{2}\right)$

$\frac{80}{2} - \ln 9$

34) Find the area of the region bounded by $y = 4x - x^2$ and the x-axis. 34) $32/3$

35) Find the area of the region bounded by the curve $y = -x^2 + 3$ and the line $y = 2x$. 35) $32/2$

SOLUTIONS

#7 $y = x^3 + 4x^2 + 4$, $(1,9)$ Equation of tangent line

$$\frac{dy}{dx} = 3x^2 + 8x$$

$$\left.\frac{dy}{dx}\right|_{x=1} = 3(1)^2 + 8(1) = 11$$

$$\boxed{y - 9 = 11(x-1)}$$

$$y - 9 = 11x - 11$$
$$+9 \qquad\qquad +9$$

$$\boxed{y = 11x - 2}$$

#8 $y = -\frac{3}{5x+2}$ Equation of tangent line @ $x = 3$

$$\left(3, \frac{-3}{5(3)+2}\right) \Rightarrow \left(3, -\tfrac{3}{17}\right)$$

$$y = -3(5x+2)^{-1}$$

$$\frac{dy}{dx} = -3(-1)(5x+2)^{-2} \cdot 5$$

$$\frac{dy}{dx} = \frac{15}{(5x+2)^2} \qquad \left.\frac{dy}{dx}\right|_{x=3} = \frac{15}{(5(3)+2)^2} = \frac{15}{289}$$

$$\boxed{\left(y + \frac{3}{17}\right) = \frac{15}{289}(x-3)}$$

or $y = \frac{15}{289}x - \frac{45}{289} - \frac{3}{17}\left(\frac{17}{17}\right)$

$$\boxed{y = \frac{15}{289}x - \frac{96}{289}}$$

#11 $y = \frac{3}{x^5} - \frac{2}{x} = 3x^{-5} - 2x^{-1}$

$$\frac{dy}{dx} = 3(-5)x^{-6} - 2(-1)x^{-2}$$

$$\frac{dy}{dx} = -15x^{-6} + 2x^{-2} = \frac{-15}{x^6} + \frac{2}{x^2}$$

#12 $w(t) = 7t^2 - 19\sqrt{t} + 23$

$w(t) = 7t^2 - 19t^{1/2} + 23$

$w'(t) = 14t - 19 \cdot \frac{1}{2} t^{1/2 - 1}$

$w'(t) = 14t - \frac{19}{2} t^{-1/2} = 14t - \frac{19}{2\sqrt{t}}$

#13 $\frac{d}{dx}\left(\sqrt{6x^3 - 2x}\right)\Big|_{x=1}$

$\frac{d}{dx}\left((6x^3 - 2x)^{1/2}\right)\Big|_{x=1}$

$\left(\frac{1}{2}(6x^3 - 2x)^{-1/2}(18x^2 - 2)\right)\Big|_{x=1}$

$\left(\frac{18x^2 - 2}{2\sqrt{6x^2 - 2x}}\right)\Big|_{x=1}$

$\frac{18(1)^2 - 2}{2\sqrt{6(1)^2 - 2(1)}} = \frac{18 - 2}{2\sqrt{6 - 2}} = \frac{16}{2\sqrt{4}} = \frac{16}{2 \cdot 2} = 4$

#14 $f(x) = x^2 - 9$, $g(x) = x^2 - 16$

$g(f(x)) = (x^2 - 9)^2 - 16$

$\frac{d}{dx}(g(f(x))) = \frac{d}{dx}\left((x^2 - 9)^2 - 16\right)$

$= 2(x^2 - 9) \cdot 2x$

$= 4x(x^2 - 9)$

#15 $\frac{d}{dt}\left(\frac{dV}{dt}\right)\Big|_{t=-1}$

$V(t) = -5t^3 + \frac{2}{1-t}$

$V(t) = -5t^3 + 2(1-t)^{-1}$

$V'(t) = -15t^2 + 2(-1)(1-t)^{-2} \cdot -1$

$V'(t) = -15t^2 + 2(1-t)^{-2}$

$V''(t) = -30t + 2(-2)(1-t)^{-3}(-1)$

$V''(t) = -30t + 4(1-t)^{-3}$

$V''(t) = -30t + \frac{4}{(1-t)^3}$

$V''(-1) = -30(-1) + \frac{4}{(1-(-1))^3}$

$= 30 + \frac{4}{8}$

$= 30.5 = 61/2$

#16 $f(t) = \frac{3}{(3t-1)^2} = 3(3t-1)^{-2}$

$f'(t) = 3(-2)(3t-1)^{-3} \cdot 3 = -18(3t-1)^{-3}$

$f''(t) = -18(-3)(3t-1)^{-4} \cdot 3$

$f''(t) = 162(3t-1)^{-4} = \frac{162}{(3t-1)^4}$

$f''(2) = \frac{162}{(3(2)-1)^4} = \frac{162}{625}$

#17 $f(x) = (x^2+1)^7$

$f'(x) = 7(x^2+1)^6 \cdot 2x = 14x(x^2+1)^6$

$f''(x) = 14x \cdot 6(x^2+1)^5 \cdot 2x + (x^2+1)^6\, 14$

$f''(x) = 14(x^2+1)^5\,[12x^2 + (x^2+1)]$

$f''(x) = 14(x^2+1)^5\,(13x^2+1)$

#19 $y = \dfrac{9x-4}{(6x^2+1)}$

$\dfrac{dy}{dx} = \dfrac{(6x^2+1)9 - (9x-4)(12x)}{(6x^2+1)^2}$

$\dfrac{dy}{dx} = \dfrac{54x^2+9 - 108x^2 + 48x}{(6x^2+1)^2}$

$\dfrac{dy}{dx} = \dfrac{-54x^2+48x+9}{(6x^2+1)^2}$

#23 $f(x) = e^{-x/2} = e^{-\frac{1}{2}x}$

$\int e^{-\frac{1}{2}x}\,dx = \dfrac{1}{-\frac{1}{2}} e^{-\frac{1}{2}x} + C$

$= -2e^{-\frac{1}{2}x} + C$

#24 $\int \left(\frac{x^2}{4} - 4\right)dx$

$\frac{1}{4} \cdot \frac{1}{2+1} x^{2+1} - 4x + C$

$\frac{1}{12} x^3 - 4x + C$

#25 $\int (x^5 + e^{5x})\,dx$

$\frac{1}{5+1} x^{5+1} + \frac{1}{5} e^{5x} + C$

$\frac{1}{6} x^6 + \frac{1}{5} e^{5x} + C$

#26 $\int_{-2}^{-1} (x^2 - 2x^{-3} + 3)\,dx$

$\left(\frac{1}{3} x^3 - 2\frac{1}{-3+1} x^{-3+1} + 3x\right)\Big|_{-2}^{-1}$

$\left(\frac{1}{3} x^3 + x^{-2} + 3x\right)\Big|_{-2}^{-1}$

$\frac{1}{3}(-1)^3 + (-1)^{-2} + 3(-1) - \left(\frac{1}{3}(-2)^3 + (-2)^{-2} + 3(-2)\right)$

$-\frac{1}{3} + 1 - 3 - \left(\frac{1}{3}(-8) + \frac{1}{4} - 6\right)$

$-\frac{7}{3} - \left(\frac{-101}{12}\right) = 73/12$

#27 $\int_1^4 \sqrt{x}\,dx = \int_1^4 x^{1/2}\,dx$

$\left(\frac{1}{\frac{1}{2}+1} x^{\frac{1}{2}+1}\right)\Big|_1^4$

$\frac{1}{\frac{3}{2}} x^{3/2}\Big|_1^4$

$\frac{2}{3} x^{3/2}\Big|_1^4 = \frac{2}{3}(4)^{3/2} - \frac{2}{3}(1)^{3/2}$

$\frac{2}{3} \cdot 8 - \frac{2}{3}(1)$

$\frac{14}{3}$

#28 $\int_{-2}^{2}(3x^2+7)\,dx$

$\left.(x^3+7x)\right|_{-2}^{2}$

$(2)^3+7(2)-\left((-2)^3+7(-2)\right)$

$8+14-(-8-14)$

$8+14-(-22)$

$22+22=44$

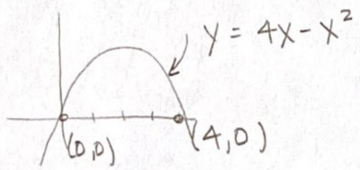
#29 $f(x,y)=3x^2-15xy+7y^3$

$\frac{\partial f}{\partial x}=6x-15y$

#30 $f(x,y)=e^{-2x+3y}$

$\frac{\partial f}{\partial x}=e^{-2x+3y}\cdot(-2)=-2e^{-2x+3y}$

#34



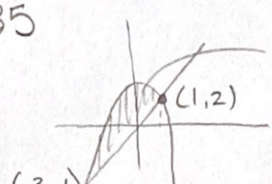
$A=\int_0^4(4x-x^2)\,dx$

$=\left.\left(2x^2-\frac{1}{3}x^3\right)\right|_0^4$

$=2(4)^2-\frac{1}{3}(4)^3-0$

$=32/3$

#35



$A=\int_{-3}^{1}\left((-x^2+3)-2x\right)dx$

$=\int_{-3}^{1}(-x^2-2x+3)\,dx$

$=\left.\left(-\frac{1}{3}x^3-x^2+3x\right)\right|_{-3}^{1}$

$=-\frac{1}{3}(1)^3-(1)^2+3(1)-\left(-\frac{1}{3}(-3)^3-(-3)^2+3(-3)\right)$

$=5/3-(9-9-9)=32/2$