Problem 1 (20 pts)

Consider the following matrices:

$$A = \begin{pmatrix} 4 & 3 & 2 \\ 5 & 0 & 1 \end{pmatrix}, \quad B = \begin{pmatrix} 2 & -4 & 7 \\ 1 & 3 & -3 \\ -1 & 2 & 0 \end{pmatrix} \quad C = \begin{pmatrix} -4 & 0 \\ -1 & 5 \\ -1 & -2 \end{pmatrix}, \quad \text{and} \quad D = \begin{pmatrix} 1 & -2 \\ -3 & 3 \end{pmatrix}.$$

If possible, compute the following:

(i) $BC - A$

(ii) $AC + 3B$

(iii) $(A^T + C)D$

(iv) $AB^2 + C$

Problem 2 (20 pts)

If it exists, find the inverse of

$$\begin{pmatrix} 1 & 3 & 4 \\ 1 & -1 & 2 \\ -2 & -2 & 0 \end{pmatrix}.$$

Problem 3 (20 pts)

Suppose

$$\det \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix} = 3.$$

Find the determinants of the following matrices:

(a) $\begin{pmatrix} g & h & i \\ d-6g & e-6h & f-6i \\ a & b & c \end{pmatrix}$.

(b) $\begin{pmatrix} 2g & 2h & 2i \\ a & b & c \\ -d & -e & -f \end{pmatrix}$.

(c) $\begin{pmatrix} b & 4e & h \\ a & 4d & g \\ c & 4f & i \end{pmatrix}$.

Problem 4 (20 pts)

Let $A = \begin{pmatrix} k^2 & 12 & -k \\ k & k & 0 \\ 0 & k & k \end{pmatrix}$.

- Find all values of k for which A is not invertible (Hint: Although the polynomial to be solved has degree 4, there are two repeated roots).
- Let $T(\mathbf{x}) = A\mathbf{x}$. For what values of k is T 1-1? Onto?

Problem 5 (10 pts)

Compute the determinant of the following matrix:

$$\begin{pmatrix} -2 & 4 & -1 & -5 \\ -1 & -2 & -1 & 0 \\ 1 & 2 & -1 & 4 \\ 3 & 0 & 2 & 0 \end{pmatrix}.$$

Problem 6 (10 pts)

Use inverse matrices to find a matrix A which is the matrix of the linear transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ with

$$T\begin{pmatrix} 3 \\ 5 \end{pmatrix} = \begin{pmatrix} 3 \\ 2 \end{pmatrix} \quad \text{and} \quad T\begin{pmatrix} 2 \\ 3 \end{pmatrix} = \begin{pmatrix} 5 \\ -7 \end{pmatrix}.$$

Bonus Problem (10 pts)

Let A be an $m \times m$ matrix. Prove that if $A^T = -A$ and if m is odd, then A is singular (i.e., A is NOT invertible).