2. Let $A = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$.

- Verify that $A^2 = \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$.
- In general, for any integer $k \geq 2$, explain that $A^k = \begin{pmatrix} 1 & k \\ 0 & 1 \end{pmatrix}$.

3. Consider the following map from $\mathbf{R}^3$ to $\mathbf{P}_2$:

$$\mathbf{T}\begin{pmatrix} \alpha \\ \beta \\ \gamma \end{pmatrix} = (\alpha+\beta) + (\beta+\gamma)\,x + (\gamma+\alpha)\,x^2.$$

Is $\mathbf{T}$ a linear transformation? If so, is it an isomorphism?

8. Let matrices $B \in \mathbf{R}^{n\times k}$ and $C \in \mathbf{R}^{k\times n}$ for $0 < k < n$. Define $A = BC$. Explain that $\mathbf{rank}(A) \leq k$.