Deception Hypotheses
What would convince you that ESP IS REAL.

A Useful Tool for Very Low/High Probabilities

- We've seen some **very unlikely data** (e.g., bag of all green candies), and very low/very high posterior probability assignments

- Convenient to express probabilities in terms of **odds (O)** and **log-odds (e)**:

  $O(A|X) = P[A|X]/P[\sim A|X] = P[A|X]/(1-P[A|X])$
  $e(A|X) = 10*\log(O(A|X))$

- $e(A|X)$ is called "evidence", in units of **decibels**

Converting from Probability to Odds/Evidence

Note that $O = P/(1-P)$ and $P = O/(1+O)$

Prob	Odds	e
1/2	1	0
2/3	2	~3
10/11	10	10
100/101	100	20
10^n/(10^n+1) ≈ 1-10^(-n)	10^n	10n
1-P	1/O	-e

Bayes' Theorem in Evidence Terms

- From Bayes' Thm we have

  $$P[H|DX] = P[H|X] * P[D|HX]/P[D|X]$$

  $$P[\sim H|DX] = P[\sim H|X] * P[D|\sim HX]/P[D|X]$$

- Dividing first equation by the second gives:

  $$P[H|DX]/P[\sim H|DX] = P[H|X]/P[\sim H|X] * P[D|HX]/P[D|\sim HX]$$

- In Odds/Evidence terms this is:

  $$O(H|DX) = O(H|X) * P[D|HX]/P[D|\sim HX]$$

  $$e(H|DX) = e(H|X) + 10*\log(P[D|HX]/P[D|\sim HX])$$

Most useful for Binary Hypothesis Test, where this is computable

What is Your Prior for An Unlikely Hypothesis?

- Imagine someone claims to be able to read your mind, specifically H: "If you write down a number from 1-10, I can tell you the number."
- What would be your prior $P[H|X]$? Here X = (everything you know).
- Imagine you have **complete control** over the experiment, and the only possibilities are H and C = "Pure chance." $\equiv \sim H$
- Assume you do n rounds of guesses, D = "He guesses n/n correct."

Hypothesis Test in Reverse

- **Claim**: $P[D|HX] = 1$
  $P[D|{\sim}HX] = 10^{(-n)}$

- So $e(H|DX) = e(H|X) + 10n$

- What value of *n* would make you **uncertain**? Meaning $e(H|DX) \approx 0$.

- It follows that $e(H|X) \approx$ **$-10n$**, that is, your $P[H|X] \approx 10^{(-n)}$

"Prior elicitation."

The Soal-Goldney Experiments

- In the 1940s, British mathematician/parapsychologist Samuel Soal claimed to experimentally verify the existence of ESP.

- Experiment involved **card-guessing**: translating sequence of numbers 1-5 to pictures of animals that the test subject would try to guess

- One subject, Gloria Stewart, was able to guess **9410/37100 ≈ 25.3%** correct

- Under "pure chance" hypothesis H_C, probability of this is $(37100 \text{ choose } 9410)\ (.2)^{9410}\ (.8)^{27690} \approx 10^{-139}$

  That is, $e(D|H_C X) \approx -1390$ db.

How Strong is This Evidence?

- Suppose we only allow a range of hypotheses {H_q; 0<q<1}
  H_q = "Subject is able to guess correctly at long-run rate *q*."
  "Pure chance" is H_0.2

- For D = "r successes out of n" we have
  P[D | H_q X] = (n choose r) q^n (1-q)^(n-r)

- If we treat them all **uniformly** at first, posterior distribution over q will be **very** sharply peaked bell curve, centered at *f* ≈ 0.253.

- P[D | H_f X] ≈ 0.005, about **136 orders of magnitude** greater

"54 sigma" deviation

Why Don’t We Believe It?

- Any hypothesis other than H_0.2 would suggest some kind of ESP!
- So why don’t we believe the evidence?
- Likely because we haven’t **completely** eliminated other possible hypotheses!
  “Results were faked/produced by some trick.”
- Even with low prior probabilities, these could be **revived** based on the data.

Comparing Deception and ESP

- Imagine we entertain the hypothesis
  H_D = "Results were produced by deception."

- Assume $P[D|H_D\ X] \approx P[D|H_f\ X]$ where H_f has highest **data-likelihood** from among possible other hypotheses

- Assuming very low data probability **kills off** all other hypotheses, we have:

$$P[D|X] \approx P[H_D\ |X] * P[D|H_D\ X] + P[H_f\ |\ X] * P[D\ |\ H_f\ X]$$

$$P[H_f\ |\ X] \approx P[H_f\ |\ X]\ /\ (P[H_f\ |\ X] + P[H_D\ |\ X])$$

The Effect of Deception

- The possibility of the deception hypothesis puts a **cap** on how willing we are to accept any other hypothesis!

- E.g., if $P[H_D \mid X] \gg P[H_f \mid X]$ we will never be convinced of H_f no matter how "unlikely" the data is to occur by chance!

- **Lesson**: In order to convince someone of something **very unlikely** (to them), you must eliminate **other possibilities** that would equally well explain the data.

  This becomes part of the background X and the prior assignment $P[H_D \mid X]$.

Famous Examples in Science

- Many scientific findings have been **hard to believe** at first because of the perceived possibility of deception/error:

  “Discovery” of cold fusion (Fleischmann and Pons, 1989)

  Non-existence of “aether” (Michelson and Morley, 1887)

  Detection of gravitational waves (LIGO, 2015)

- Epilogue: It turned out Soal had fabricated his data by changing the target numbers to match the guesses.

Summary

- Imagining a "perfect experiment" where deception is impossible can help **"draw out"** our prior probability assignments for unlikely hypotheses.

- In practice, no experiment is perfect! Any residual probability of deception may **block** us from believing extraordinary claims.

- Probabilistic thinking can give us a **language** to describe this, even without exact numbers.
  (Standard statistics is mostly useless, other than telling us to reject the "chance" hypothesis.)