Lab# 3 – Theorems & Bridges

Mechatronics systems use signal conditioning circuitry on both input and output. Design and analysis of such circuits could get very complicated. To simplify the process, we use a number of circuit theorems. In first section of this lab, students will build circuits to verify some of these theorems.

Thevenin's theorem aims to simplify circuit analysis by modeling the circuit as a voltage source V_{TH} in series with a resistance R_{TH} across the pair of terminals of interest. A typical use case for this theorem is to analyze the effects of various load resistances on a circuit.

Superposition theorem helps to simplify circuit analysis by finding response of each voltage or current source individually and them summing them up to find total response of the circuit.

Maximum Power Transfer theorem establishes that maximum power is transferred to load when output resistance of a circuit matches the load resistance. You may have applied this theorem when matching a speaker with amplifier for your music system.

In Mechatronics, many sensors work on the principal of change in resistance due to external factors such as stress, strain, deflection, light, temperature or pressure. This change in resistance results in a change in voltage that could be read by the processor. However these changes could be very, very small. One mechanism used in such situations is to use a Wheatstone bridge with a voltage amplifier. In second section of this lab, students will build a Wheatstone bridge to find an unknown resistance. Students will also find resistance variation of a photocell (light dependent resistor) under different light conditions using bridge circuit.

Finally, students will use Strain Gauge with Balanced Bridge and an amplifier to measure voltage changes under different load conditions. The will also interface circuit with Arduino and display results in the Serial Monitor.

More information on theorems and Wheatstone bridge is given in the appendix of this manual.

Section 1

Objective: Analysis and verification of basic electrical theorems

Equipment:

1. Breadboard
2. Multi-meter
3. DC BK Power Supply
4. Decade Resistance Box

Components:

1. Resistors (3) 560Ω, 1kΩ, 4.7kΩ
2. Jumper wires

Note(s):

1. Connect Decade Resistance Box in your circuit using the two **GREEN** terminals
2. To short circuit voltage source, **REMOVE Source** and connect circuit with jumper wires
3. To measure current, you need to **BREAK** the circuit i.e. disconnect a wire and place the multi-meter in series

Procedure(s):

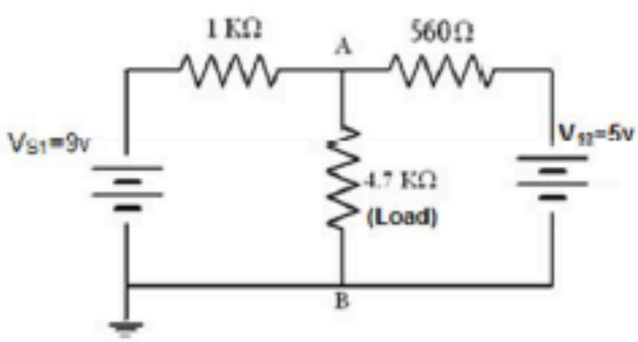


Figure 1

Thevenin Equivalent Circuit

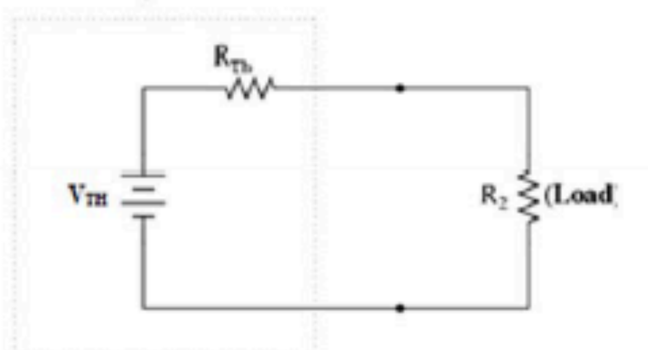


Figure 2

I. Verify Thevenin's theorem
   a. Connect the circuit as shown in figure 1 on the breadboard
   b. Measure the voltage across AB (i.e. across load resistor 4.7 KΩ)
      V_L = 6.04
   c. Remove load resistor.

d. Measure the open circuit voltage across AB using multi-meter

   V_{TH} = 6.51 ______________

e. Measure the Thevenin's resistance with the help of multi-meter (with short circuiting voltage sources

   R_{TH} = 352 ______________

f. Connect the circuit as shown in figure 2; using only one voltage source for V_{TH} and R_{TH} as measured in previous steps.

g. Connect the load resistor (4.7kΩ) and measure the voltage across load with the help of voltmeter.

   V_L = 6.56 ______________

h. Is V_L in step b equal to step g? yes __________

II. Verify Superposition theorem

a. Connect the circuit as shown in figure 1 on the breadboard.

b. Set the value of V_{S1} to 0V and V_{S2} to 5V and measure the voltage across load resistor.

   V_{L1} = 4.54 __________

c. Also measure the current through load of 4.7KΩ.

   I_{L1} = 5.08 __________

d. Set the value of V_{S1} to 9V and V_{S2} to 0V and measure the voltage across load resistor.

   V_{L2} = 7.44 __________

e. Also measure the current through load of 4.7KΩ.

   I_{L2} = 1.61 __________

f. Calculate the total voltage as below

   $V_L = V_{L1} + V_{L2}$ = 12 ______________

g. Calculate total current as below

   $I_L = I_{L1} + I_{L2}$ = 6.7 ______________

h. Now set the value of V_{S1} to 9V and V_{S2} to 5V and measure the voltage across load resistor.

   V_L = 6.03 ______________

i. Measure current through load of 4.7KΩ.

   I_L = 1.30 ______________

j. Is voltage V_L in step f equal to V_L in step h No ______________

k. Is voltage I_L in step g equal to I_L in step i No ______________

III. Verify Maximum Power Transfer theorem

a. Connect Decade Resistance Box between the terminals A-B of Figure 1 circuit.
b. Set the resistance values as given in the table below
c. For each value of the resistance, measure and record the voltage V, across it
d. Calculate the power for each reading and plot against resistor values in table below
e. At what value of resistor, does maximum power occur 352

R	50	100	250	500	1k	2k	5k	R_{TH} 352
V	0.81	1.43	2.70	3.80	4.80	5.52	6.07	3.23
$P = V^2/R$	0.013	0.020	0.029	0.028	0.023	0.015	0.007	0.03

Section 2

Objective: Wheatstone bridge and resistance measurement

Equipment:

1. Breadboard
2. Multi-meter
3. DC BK Power Supply
4. Decade Resistance Box
5. Arduino Development Board

Components:

1. Resistors (5) 1kΩ, 10kΩ (2), 1MΩ, Unknown
2. Operational Amplifier LM324
3. Photocell
4. Jumper wires

Note(s):

1. Connect Decade Resistance Box in your circuit using the two **GREEN** terminals

Procedure(s):

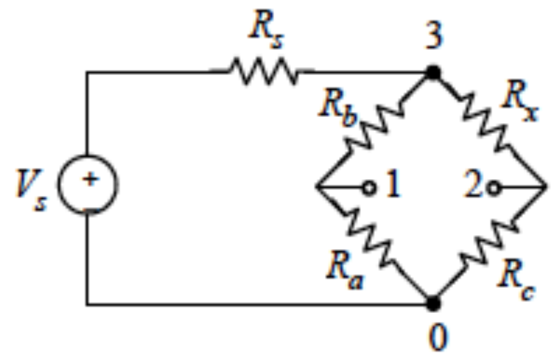


Figure 3 (Wheatstone bridge)

I. Measure unknown Resistance

a. Construct the circuit shown in Figure 3. using Ra =1kΩ, Rc = 10kΩ, and Rs =10kΩ

b. Connect Decade Resistance Box for Rb.

c. Adjust the DC power supply Vs to 5 volts

d. Obtain unknown resistance from your instructor and use as Rx
Adjust Rb (decade box), monitor V_{12} with the multi-meter until it is 0V.
What is value of Rb? 40 ________________
Calculate resistance Rx using equation in Appendix 400 __________________
Using multi-meter measure resistance of unknown resistor 330 __________________

e. Obtain a photo cell and connect in circuit as Rx
Adjust Rb (decade box), monitor V_{12} with the multi-meter until it is 0V
What is value of Rb? ________________
Calculate resistance Rx using equation in Appendix __________________

f. Cover photo cell, adjust Rb (decade box) and monitor V_{12} with the meter until it is 0V
What is value of Rb? ________________
Calculate resistance Rx using equation in Appendix __________________

II. Strain Gauge

a. Measure resistance between P+ (Green wire) and S+ (Red wire). What value do you read? 262.3

b. Measure resistance between P+ (Green wire) and S- (Blue wire). This is the resistance of the strain gauge. What value do you read? 262.8

c. Put load on the strain gauge so that it is fully pressed. What resistance value do you read on the meter now? 263

d. For strain gauges, change in resistance is very, very small. Is this your observation as well? yes

e. Connect the P+ (Green wire) to 5V and P- (White wire) to ground of power supply

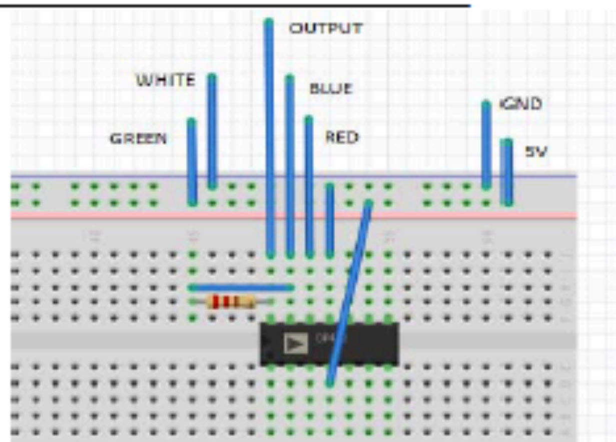
f. Measure voltage between S+ (Red wire) and S- (Blue wire)

g. Since the Wheatstone bridge is balanced on the strain gauge, you should 0V. What value do you read? 0

h. Put load on the strain gauge so that it is fully pressed. What voltage do you read on the meter now? 0.3

i. Change in voltage from the output of Wheatstone bridge with a strain gauge is very, very small. Is this your observation as well? yes

j. Setup circuit shown using LM324 and 1MΩ resistor



k. Measure voltage between OUTPUT and GND under various load conditions.

Value at no load: 3.89

Value 1: 3.9

Value 2: 3.9

Value at maximum load: 3.9

l. Connect OUTPUT to A0 of the Arduino and connect GND on breadboard to GND on Arduino

m. Start a new Sketch in Arduino IDE and initialize Serial Monitor

n. In *loop()* function, write following statements

```
float v = analogRead(0);
Serial.println(v);
delay(1000);
```

o. Upload Sketch to Arduino and open Serial Monitor. Record values displayed under various load conditions:

Value at no load: 805

Value 1: 804

Value 2: ______

Value at maximum load: ______

Note: Arduino has a 10bit Analog to Digital converter (ADC) and it maps 0-5V analog values to 0-1023. Your recorded values should be within this range.

Appendix

Thevenin's Theorem

In circuit theory, Thevenin's theorem for linear electrical networks states that any combination of voltage sources, current sources and resistors with two terminals is electrically equivalent to a single voltage source V_{TH} and a single series resistor R_{TH}.

The Thevenin voltage V_{TH} used in thevenin's theorem is an ideal voltage source equal to the open circuit voltage at the terminals whereas the thevenin resistance R_{TH} used in Thevenin's Theorem is the resistance measured at terminals AB with all voltage sources replaced by short circuits and all current sources replaced by open circuits.

Superposition Theorem

The superposition theorem for electrical circuits states that the response (Voltage or Current) in any branch of a bilateral linear circuit having more than one independent source equals the algebraic sum of the responses caused by each independent source acting alone, while all other independent sources are replaced by their internal impedances.

To ascertain the contribution of each individual source, all of the other sources first must be "turned off" (set to zero) by:

1. Replacing all other independent voltage sources with a short circuit (thereby eliminating difference of potential. i.e. V=0, internal impedance of ideal voltage source is ZERO (short circuit)).
2. Replacing all other independent current sources with an open circuit (thereby eliminating current. i.e. I=0, internal impedance of ideal current source is infinite (open circuit).
3. This procedure is followed for each source in turn, and then the resultant responses are added to determine the true operation of the circuit. The resultant circuit operation is the superposition of the various voltage and current sources.

Electrical Power

Power is understood as the rate at which work is done or energy is transferred from one form to other. In an electrical component power is given by

$$P = V * I$$

Where, P is the power, measured in watts, V is the potential difference across the component, measured in volts and I is the current measured in amperes.

The above equation could also be expressed as

$$P = I^2 * R = V^2 / R$$

Maximum Power Transfer Theorem

The Maximum Power Transfer Theorem states that the maximum amount of power will be dissipated by a load resistance if it is equal to the Thevenin or Norton resistance of the network supplying power. If the load resistance is lower or higher than the Thevenin/Norton resistance of the source network, its dissipated power will be less than maximum.

The Maximum Power Transfer Theorem does not satisfy the goal of maximum efficiency.

Wheatstone bridge

A bridge is a special class of circuits that can be used for measuring resistance, capacitance, or inductance. A resistance bridge is especially useful when a very accurate measurement of a resistance is required. The Wheatstone bridge or four arm bridge, invented by C. Wheatstone in 1843, is the most widely used resistance bridge for measuring resistance values above 1 Ω. Commercial Wheatstone bridges are accurate to about 0.1 percent, making the values of resistance obtained far more accurate than values obtained from many types of meters.

A Wheatstone bridge consists of a voltage source and two parallel voltage dividers, as shown in figure 1. The bridge is said to be balanced when $V_{12} = 0$. For the balanced condition, the voltage V_3 is divided in the path containing resistors Ra and Rb in the same ratio as in the path containing resistors Rc and Rx, which allows the unknown resistance Rx to be determined in terms of Ra, Rb and Rc.

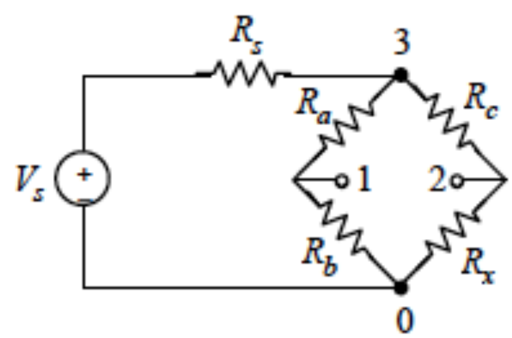


Figure 4

We can find Rx in terms of Ra, Rb and Rc as follows. Using the voltage divider relation,

$$V1 = \frac{Rb}{Ra+Rb}V3 \quad and \quad V2 = \frac{Rx}{Rx+Rc}V3$$

For the balanced condition, $V_{12} = 0$, or $V_1 = V_2$. Equating the above expressions for V_1 and V_2 gives

$$\frac{Rb}{Ra+Rb} = \frac{Rx}{Rx+Rc}$$

Solving for Rx, gives

$$Rx = \frac{Rb * Rc}{Ra}$$

In order to achieve balance for a specific unknown resistance Rx, let Ra and Rc have fixed, known values, and let Rb be a calibrated (adjustable) resistor. The procedure is to adjust Rb until $V_{12} = 0$, and then use the expression derived above to determine Rx.

1. What was the objective of this lab (0.5 pts)?

2. What components and equipment was used in this lab (0.5 pts)?

3. List the new concepts you learned in this lab (3 pts)?

4. Suggest one improvement that would make this lab better (1 pts)?