The Ballistic Pendulum. Projectile Motion 5

The principle of conservation of momentum follows directly from Newton's laws of motion. According to this principle, if there are no external forces acting on a system containing several bodies, then the momentum of the system remains constant. In this experiment, the principle is applied to the case of a collision, using a ballistic pendulum. A ball is fired by a gun into the pendulum's bob. The initial velocity of the ball is determined in terms of the masses of the ball and the bob and the height to which the bob rises after impact. This velocity can also be obtained by firing the ball horizontally and allowing it to fall freely toward the earth. The velocity is then determined in terms of the range and the vertical distance of fall. It is the object of this experiment to study the law of conservation of momentum and the elements of projectile motion; more precisely, the purpose is to determine the initial velocity of a projectile (1) by measurements of its range and vertical distance of fall and (2) by means of a ballistic pendulum. If the theory is correct, the two results should agree.

THEORY

The momentum of a body is defined as the product of the mass of the body and its velocity. Newton's second law of motion states that the net force acting on a body is proportional to the time rate of change of momentum. Hence, if the sum of the external forces acting on a body is zero, the linear momentum of the body is constant. This is essentially a statement of the principle of conservation of momentum. Applied to a system of bodies, the principle states that if no external forces act on a system containing two or more bodies, then the momentum of the system *as a whole* does not change.

In a collision between two bodies, each one exerts a force on the other. These forces are equal and opposite, and if no other forces are brought into play, the total momentum of the two bodies is not changed by the impact. Hence, the total momentum of the system after collision is equal to the total momentum of the system before collision. During the collision the bodies become deformed and a certain amount of energy is used to change their shape. If the bodies are perfectly elastic, they will recover completely from the distortion and will return all of the energy that was expended in distorting them. In this case, the total kinetic energy of the system also remains constant. If the bodies are not perfectly elastic, they will remain permanently distorted, and the energy used up in producing the distortion is not recovered.

Inelastic impact can be illustrated by a device known as the ballistic pendulum, which is sometimes employed to determine the speed of a bullet. If a bullet is fired into a pendulum bob and remains imbedded in it, the momentum of the bob and bullet just after the collision is equal to the momentum of the bullet just before the collision. This follows from the law of conservation of momentum. The velocity of the pendulum before collision is zero, so that the total momentum of the system is then just that of the bullet alone. After the collision, the pendulum and the bullet move with the same velocity. Thus, conservation of the system's momentum requires

$$mv = (M + m)V \qquad (5.1)$$

where m is the mass of the bullet in kilograms, v is the velocity in meters per second of the bullet just before the collision, M is the mass of the pendulum bob, and V is the common velocity of the bob and bullet just after the collision.

As a result of the collision, the pendulum with the imbedded bullet swings about its point of support, and the center of gravity of the system rises through a vertical distance h. From a measurement of this distance, it is possible to calculate the velocity V. The kinetic energy of the system just after the collision must be equal to the increase in potential energy of the system as the pendulum reaches its highest point. This follows from the law of conservation of energy; here we assume that the loss of energy due to friction at the point of support is negligible. The energy equation gives

$$\frac{1}{2}(M + m)V^2 = (M + m)gh \qquad (5.2)$$

where M is the mass of the pendulum bob, m is the mass of the bullet, V is their common velocity just after the collision, g is the known value of the acceleration of gravity, and h is the vertical distance through which the center of gravity of the system rises. The left-hand side of Equation 5.2 represents the kinetic energy of the system just after the impact; the righthand side represents the change in the potential energy of the system. Solving Equation 5.2 for V yields

$$V = \sqrt{2gh} \tag{5.3}$$

By substituting this value of V and the values of the masses M and m in Equation 5.1, the velocity of the bullet before the collision can be calculated.

The velocity of the bullet can also be determined from measurements of the range and vertical distance of fall when the bullet is fired horizontally and allowed to fall to the floor without striking the pendulum bob.

The motion of a projectile is a special case of a freely falling body in which the initial velocity may be in any direction with respect to the vertical. The projectile's path is a curve produced by a combination of the uniform velocity of projection and the velocity due to the acceleration of gravity. This type of motion may be studied very effectively by considering it as made up of two independent motions, one of constant speed in the horizontal direction, and the other of constant acceleration in the vertical direction. Here the problem is simplified by neglecting air resistance and also by firing the projectile horizontally so that there is no vertical component of the initial velocity v.

Because the initial velocity is in the horizontal or x direction, in which there is no acceleration, the horizontal distance covered is given by

$$x = vt \tag{5.4}$$

where t is the time of flight in seconds. In the y direction, however, the initial velocity is zero, so that the motion in this direction is that of a freely falling body starting from rest. Therefore, the distance traveled vertically is given by

$$y = \frac{1}{2}gt^2 \tag{5.5}$$

where y is the vertical distance moved and g is the known acceleration of gravity. Combining Equations 5.4 and 5.5 to eliminate the time, in which we have no interest, and solving for v yields

$$v = x\sqrt{\frac{g}{2y}} \tag{5.6}$$

The apparatus used in this experiment (see Fig. 5.1) is a combination of a ballistic pendulum and a spring gun. The pendulum consists of a massive cylindrical bob, hollowed out to receive the projectile and suspended by a strong, light rod pivoted at its upper end. The projectile is a brass ball that is fired into the pendulum bob and is held there by a spring in such a position that its center of gravity lies on the axis of the suspension rod. A brass index is attached to the pendulum bob to indicate the height of the center of gravity of the loaded pendulum. When the projectile is fired into the bob, the pendulum swings upward and is caught at its highest point by a pawl that engages a tooth on a curved rack.

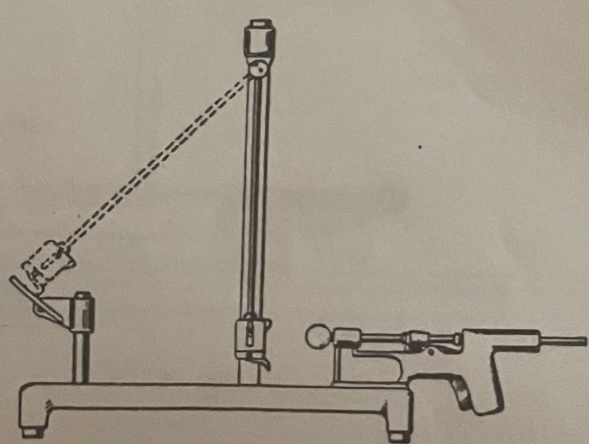

Figure 5.1 *The Blackwood ballistic pendulum*

APPARATUS

1. Blackwood ballistic pendulum
2. Triple-beam balance
3. Metric steel scale
4. $11\frac{1}{2} \times 1\frac{11}{16} \times \frac{1}{2}$-in. plate
5. 2-m stick
6. 1-m stick
7. Plumb bob
8. Carbon paper
9. White paper and masking tape
10. Level

PROCEDURE

1. In the first part of the experiment, the initial velocity of the projectile is obtained from measurements of the range and fall. The apparatus should be set near one edge of a level table. The pendulum is not used in this part and should be swung up onto the rack so that it will not interfere with the free flight of the ball. In some cases, the rack itself will interfere and must be removed. Your instructor will supply the proper tool for doing this. The pendulum must then be held up out of the way when the gun is fired. Check that the apparatus is level; if it is not, place sheets of paper under the feet as required.

2. Get the gun ready for firing by placing the ball on the end of the firing rod and pushing it back, compressing the spring until the trigger is engaged. The ball is fired horizontally so that it strikes a target placed on the floor. Fire the ball several times and determine approximately where it strikes the floor. Place a sheet of white paper on the floor so that the ball will hit it near its center; secure the paper with masking tape. Cover this sheet with a sheet of carbon paper, carbon side down, so that a record will be made of the exact spot where the ball strikes. Fire the ball ten more times. *Note:* It is imperative that the apparatus not move on the bench between shots. If you have trouble keeping it in position, ask your instructor to provide a clamp to fix it to the bench. *Do not overtighten the clamp.*

3. Measure the range for each shot; this is the *horizontal* distance from the point of projection to the point of contact with the floor. To do this, place the metal plate, long edge down, on the apparatus base so that it extends forward from the gun's firing pin. Then place the meter stick edge down on the top edge of the plate and hold it there with one end butted against the end of the firing pin. With the gun fired (that is, not cocked) this is very closely the position of the center of the ball at the instant it leaves the gun. Hang the plumb bob from the meter stick at some convenient position from which the plumb line can hang down beyond the table edge (see Fig. 5.2). Place a second sheet of white paper on the floor under the plumb bob, secure it in position with masking tape, and mark the spot indicated by the plumb bob's point. Make sure the bob hangs freely and has stopped swinging when you make the mark. Note and record the reading on the meter stick at the point where you have hung the plumb line; this is the *horizontal* distance from the tip of the firing pin to the mark on the floor. You can now use the 2-m stick to

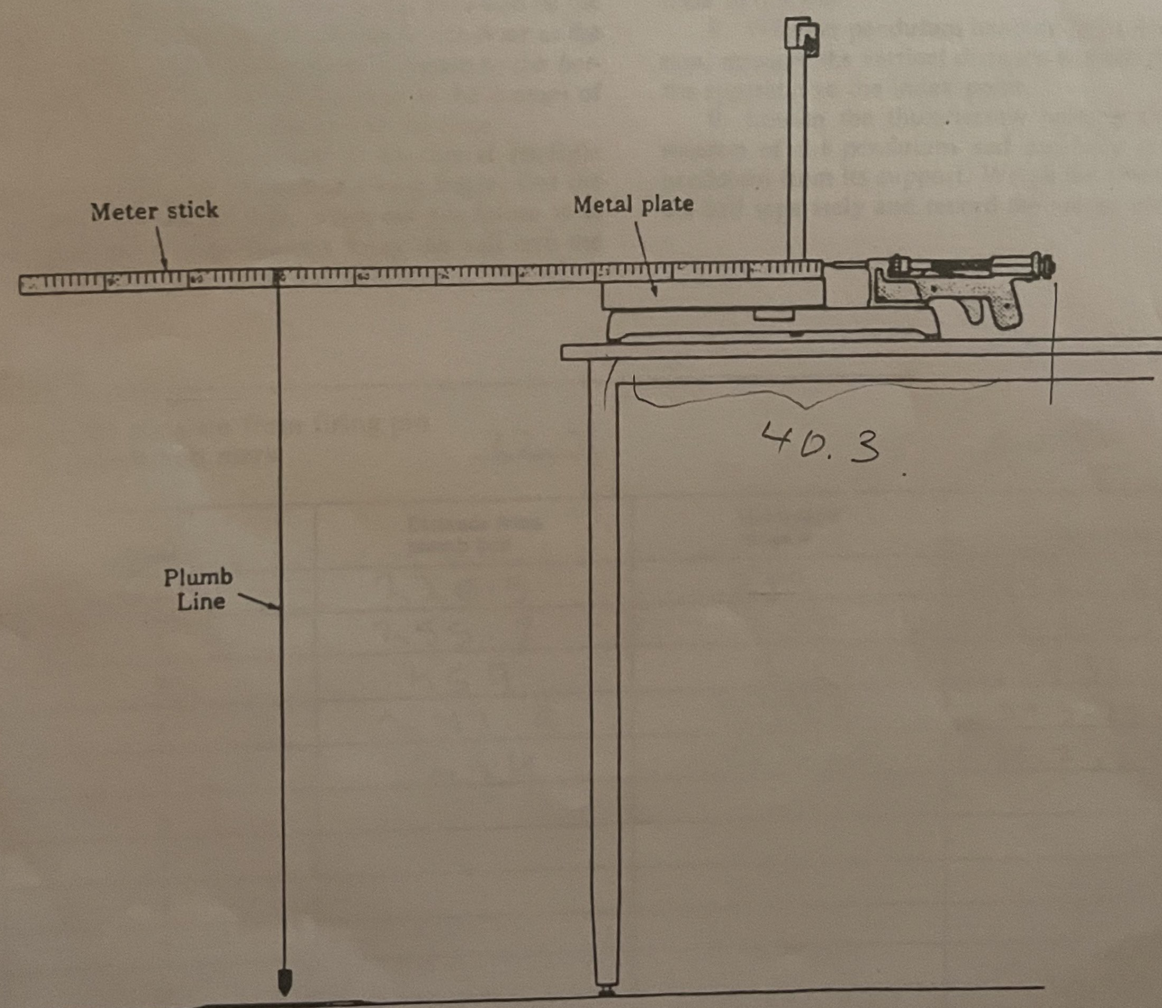

Figure 5.2 *Setup for range measurements*

measure the distance from your mark to each of the ten carbon spots made by the ball when you fired it in Procedure 2. Record all your data as called for in the data table, noting that the range of each shot is the pertinent distance along the floor measured with the 2-m stick *plus* the distance from the plumb-bob mark to the firing pin read from the 1-m stick as described above.

4. Measure the ball's vertical fall, that is, the vertical distance of the point of projection above the floor. For this measurement, use the meter stick as a straightedge. Lay it on top of the metal plate as shown in Fig. 5.2 but broadside rather than edge down. In this position its upper surface should just slide under the ball when the latter is on the firing pin so that this surface is at the level of the bottom of the ball at the instant of firing. Because the meter stick projects beyond the edge of the table, you can use the 2-m stick to measure the vertical distance from the floor to its upper surface. This should then also be the height of the bottom of the ball above the floor at the time of firing. Note that it is the distance to the *bottom* of the ball that you want, as it is the *bottom* of the ball (not its center) that strikes the floor.

5. Return the pendulum to its central (straight down) position and allow it to swing freely. Get the gun ready for firing, and, when the pendulum is at rest, pull the trigger, thereby firing the ball into the pendulum bob. This will cause the pendulum with the ball inside it to swing up along the rack where it will be caught at its highest point. Record the notch on the curved scale reached by the pawl when it catches the pendulum. To remove the ball from the pendulum, push it out with your finger or a rubber-tipped pencil while holding up the spring catch.

6. Repeat Procedure 5 four more times, recording the position of the pendulum on the rack each time.

7. From the data of Procedures 5 and 6, compute the average value of the position of the pendulum on the rack. Set the pendulum with the pawl engaged in the notch that corresponds most closely to the average reading. Measure the vertical distance h_1 from the base of the apparatus to the index point attached to the pendulum. This index point is at the vertical position of the center of gravity of the pendulum and ball. Use the metric steel scale and read the measurement to 0.1 mm.

8. With the pendulum hanging in its lowest position, measure the vertical distance h_2 from the base of the apparatus to the index point.

9. Loosen the thumbscrew holding the axis of rotation of the pendulum and carefully remove the pendulum from its support. Weigh the pendulum and the ball separately and record the values obtained.

DATA

Horizontal distance from firing pin to plumb bob mark 32.3

Avg -

Shots	Distance from plumb bob	Horizontal range x	Deviations
1	220.5	232.	232-233.64 = 1.64
2	255.7	233	233-233.64= 0.64
3	259	233	233-233.64=0.64
4	243.5	234.5	234.5-233.64= 0.86
5	254	235.7	235.7-233.64 =2.06
6			
7			
8			
9			
10			
Averages		233.64	1.168

Name ________ Section ________ Date ________

~~Vertical distance of fall y~~ 83.6

Highest position of ballistic pendulum:

Reading of curved scale

Average reading ________

Vertical distance h_1 or the average height of the center of gravity of the pendulum in its highest position 13.8 cm

Vertical distance h_2 or the height of the center of gravity of the pendulum in its lowest position 5.5 cm

Vertical distance h through which the center of gravity of the pendulum was raised ________

h = h2 - h1

Velocity of projection from range and fall measurements ________

Trials

1. 14 2. 21 3. 20 4. 18 5. 19

Mass of the pendulum bob 265.5

Mass of the ball 62.5 g

Velocity of the pendulum and ball just after the collision ________

Velocity of the ball before the collision ________

Difference between this value of v and that found from the range measurement ________

Percent difference ________

CALCULATIONS

1. From the data of Procedure 3, compute the average range of the projectile.

2. Calculate the deviation of each of your range measurements from your average and then calculate the average deviation (a.d.).

~~3. Compute the velocity of projection from Equation 5.6. Attach the proper error to your result by~~ using the a.d. found in Calculation 2 as the error in x and attaching no errors to your measurement of y or the known value of g.

4. From the data of Procedures 7 and 8, calculate the vertical distance h through which the center of gravity of the loaded ballistic pendulum was raised as a result of the collision.

5. Compute the value of V, the common velocity of the pendulum bob and ball just after the collision, by using Equation 5.3.

6. Calculate the velocity of the ball before the collision by substituting the value of V found in Calculation 5 and the measured values of the masses of the pendulum bob and of the ball in Equation 5.1.

and expressing it as a percent. Note also

7. Compare the values of v obtained by your two methods by finding the difference between them and expressing it as a percent. Note also whether your value of v from Calculation 6 falls within the range of error found in Calculation 3.

QUESTIONS

1. Using the data of your experiment, calculate the kinetic energy of the ball just before impact from the value of the velocity of the ball obtained with the ballistic pendulum and the mass of the ball.

2. Calculate the kinetic energy of the pendulum bob and ball just after impact from the value of their common velocity and of their masses.

3. (a) Using the results of Questions 1 and 2, calculate the fractional loss of energy during this inelastic impact. Express it in percent. (b) What became of the energy lost?

4. (a) Compute the ratio of the mass of the pendulum bob to the total mass of the bob and the ball. Express it in percent. (b) How does this ratio compare with the fraction of energy lost during the impact?

5. Compare the momentum and the kinetic energy of an automobile weighing 2000 lb and moving with a velocity of 60 mi/h with the momentum and the kinetic energy of a projectile weighing 70 lb and moving with a velocity of 2500 ft/s.

6. A bullet of mass 10 g is fired horizontally into a block of wood of mass 2 kg and suspended like a ballistic pendulum. The bullet sticks in the block and the impact causes the block to swing so that its center of gravity rises 0.1 m. Find the velocity of the bullet just before the impact.

initial energy = final energy

$0.5 \times (2 + 10 \times 10^{-3}) \times v^2 = (2 + 10 \times 10^{-3}) \times g \times 0.1$

$0.5 \times v^2 = g \times 0.1 \quad v \Rightarrow 1.4\ m/s$

initial momentum = final momentum

$10 \times 10^{-3} \times V = (2 + 10 \times 10^{-3}) \times 1.4 \quad V = 281.4\ m/s$

$\Rightarrow$ 281.4 m/s

7. Using your range data from Procedures 1–3, calculate the average deviation of the mean (A.D.) and the standard deviation (σ). Refer to the section titled "Theory of Errors" in the Introduction for definitions and explanations of these quantities. Then find the error to be attached to your value of the spring gun's muzzle velocity v when the A.D. is used as the error in the range and also when σ is so used. In each case, see if your value of v from the ballistic pendulum measurement falls within the error range obtained.

8. In your calculations of v from your range measurements, no attempt was made to attach errors to the values you used for the acceleration of gravity g or the distance of vertical fall y. Justify your ignoring the errors that must exist for these quantities. Also justify the fact that you did not calculate an error for the value of v you obtained from your measurements with the ballistic pendulum.

9. (a) Derive Equation 5.6 from Equations 5.4 and 5.5. (b) Derive an equation giving the gun's muzzle velocity v in terms of h and the masses of the ball and pendulum by combining Equations 5.1 and 5.3.