KEY

# Intermediate Macroeconomic Theory
## Economics 317: Section B
## Pre-Midterm Exam: Spring 2015

**Directions**: ***Please answer all questions. Your answers should be as thorough and as precise as possible. If necessary you may continue your answer to any question at the back of the page.***

This exam is based on the following description of 317land. 317land is a monetary economy in which the typical household has production opportunities summarized by the production function $y_t = f(l_t)$. In addition, households have access to a competitive commodity or goods market and to a perfect credit or bond market. The bonds in 317land are one period bonds that pay real interest at rate R. Assume that the typical household in 317land has an infinite planning horizon and seeks to maximize horizon long utility given by $U = u(c_1, l_1) + 1/(1+\rho)\, u(c_2, l_2) + 1/(1+\rho)^2\, u(c_3, l_3) + \ldots$, where c and l represent consumption and work effort and $\rho$ is the subjective rate of time preference. Finally, assume that households in 317land behave in accordance with the permanent income hypothesis.

1. In class we noted that in order to maximize horizon-long utility households in an economy like 317land will have to allocate resources optimally along three dimensions. These are: intratemporal allocation (i.e. the choice of (c, l) in any period t, intertemporal allocation of consumption, and intertemporal allocation of work effort. Fundamental economic principles instruct that optimizing households' allocation of resources along these dimensions will be guided by the fundamental principle that ***the optimal level of a good service or activity is the level at which marginal benefit is equal to marginal costs.***

(a) Write an essay that thoroughly proves that the representative optimizing household's intertemporal allocation of work effort will be consistent with this principle. (20 points)

The first step in constructing a convincing response to this question is to rigorously identify MC and MB. To do so assume that a hh is considering $\uparrow l_1$ and $\downarrow l_2$. Under the assumption that the proposed $\Delta$ in $l_1 = 1$, this will reduce the hh's period 1 utility by $-MU_{l_1}$. This is the marginal utility loss or marginal costs in terms of utility. Since the 1 unit increase in $l_1$ yields $MPL_1$ additional units of output in pd 1 the hh will use bonds to transfer these resources into the future (i.e. pd 2). As such the hh will have $MPL_1(1+R)$ units of the comm. to buy leisure in pd 2 (i.e. $\downarrow l_2$).

Cont'd Since $MPL_2$ is the best available estimate of the price of pd 2 leisure, the hh will be able to buy $MPL_1(1+R)/MPL_2$ add'l units of leisure in pd 2. The effect on utility in pd 2 will be given by.

$$\left[\begin{array}{c}\text{Appropriately}\\ \text{discounted}\\ \text{Marginal Disutility}\end{array}\right] \times \left[\begin{array}{c}\#\text{ of add'l units}\\ \text{of leisure or } \uparrow\\ \text{in } l_2.\end{array}\right] = \frac{1}{1+\rho} MU_{l_2}\left[\frac{MPL_1(1+R)}{MPL_2}\right]$$

This is the marginal utility gain or MB in terms of utility. If the hh's intertemporal alloc of w.e. is consistent with stated principle, it should choose the $(l_1, l_2)$ combination for which

$$-MU_{l_1} = \frac{1}{(1+\rho)} MU_{l_2}\left[\frac{MPL_1(1+R)}{MPL_2}\right].$$ If $MB \neq MC$, it will be possible for the hh to intertemporally reallocate work effort and achieve an $\uparrow$ in overall ~~up~~ utility. For example, if $MC > MB$ the hh will be able to $\uparrow$ its utility by $\downarrow l_1$ and $\uparrow l_2$.

(b) Thoroughly evaluate the accuracy of the following statement.

***"Holding all else constant, a reduction in the intertemporal relative price of current consumption will induce optimizing households to intertemporally reallocate work effort from the future (or future periods) to the current period." (10 points)***

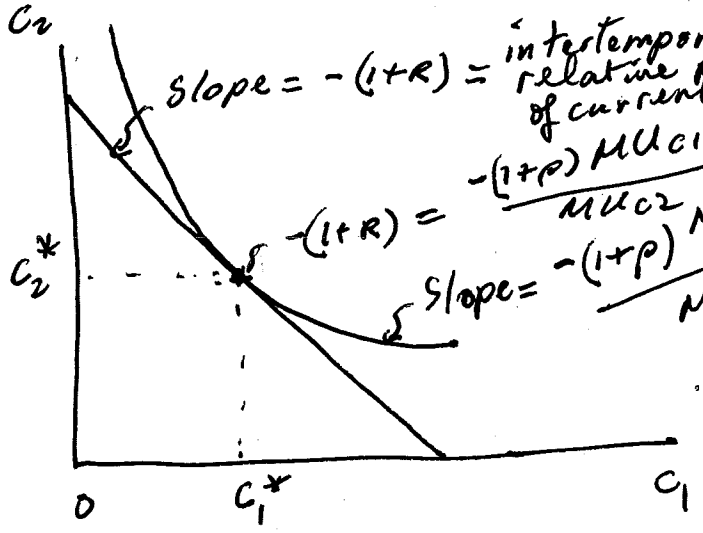
The intertemporal relative price of ~~the~~ current consumption is $(1+R)$. We know that the intertemporal relative price of leisure is $\left[\frac{MPL_1(1+R)}{MPL_2}\right]$ and that changes in $[\cdot]$ will induce hh's to engage in intertemporal reallocation of work effort. Consequently, if $(1+R)\downarrow$ current leisure will become relatively ~~more expensive~~ cheaper and hhs will $\uparrow$ current leisure (i.e. $\downarrow l_1$) and decrease future leisure (i.e. $\uparrow l_2$). Consequently the statement is false.

(c) Suppose 317land is hit by a natural disaster. Use whatever combination of written and graphical exposition you deem appropriate to explain how this change in the economic environment is likely to affect the representative household's intertemporal allocation of work effort. (10 points)

Recall that the relative price that is pertinent for the intertemporal alloc of work effort is $\left[\frac{MPL_1}{MPL_2}(1+R)\right]$. A natural disaster will temporarily reduce the MPL so that $\frac{MPL_1}{MPL_2} \downarrow$. This will make current leisure relatively ~~more expensive~~ cheaper and induce hhs to ↑ $l_1$ and ↑ $l_2$. The temp downward shift in the prodn fnc will reduce wealth (as measured by x) and lead to upward pressure on work effort in both pds. However, note that this effect will be small since temp.

2. Prove that the $(c_1, c_2)$ combination that is consistent with maximization of the representative optimizing household's lifetime utility is the $(c_1, c_2)$ combination at which the marginal benefit is equal to marginal costs by doing the following.

(a) Use whatever combination of written and graphical exposition you deem necessary to demonstrate that the representative optimizing household in 317land will pick the $(c_1, c_2)$ combination for which the marginal rate of substitution of consumption in period 2 for consumption in period one is equal to the intertemporal relative price of period one consumption. (15 points)




Slope = $-(1+R)$ = intertemporal relative price of current cons.

$-(1+R) = \frac{-(1+\rho)MUc_1}{MUc_2}$

Slope = $\frac{-(1+\rho)MUc_1}{MUc_2} = MRS$

To be fully convincing, I expected you to derive the slope of the IBC and the slope of the indiff curve.

⇒ Utility maximized by picking the comb $(c_1^* c_2^*)$ @ the pt of tangency btwn the IBC and the indiff curve.

(b) Thoroughly demonstrate/explain the exact sense in which the behavior you described in your answer to question 2(a) is consistent with the fundamental principle that ***the optimal level of a good service or activity is the level at which marginal benefit is equal to marginal costs.*** (15 points)

Your analysis in 2(a) should have yielded the following optimality condition:

$-(1+R) = -\frac{(1+\rho)\,MU_{c1}}{MU_{c2}}$. To demonstrate that the implied behavior is consistent with the stated principle, you must first identify MC and MB. To do so, suppose the hh in question decides to ↓ $c_1$, ~~and~~ by 1 unit and to transfer the freed-up resources into pd 2 using bonds. The reduction in $c_1$ will reduce utility in pd 1 by $MU_{c1}$. This is the marginal utility loss or marginal cost in terms of utility. Given that bonds pay interest @ rate R, the hh will have (1+R) additional units to consume in pd 2. Each additional unit of $c_2$ will yield $\frac{1}{1+\rho} MU_{c2}$ units of utility (appropriately discounted). Consequently (1+R) units will ↑ lifetime utility by $\frac{1}{1+\rho} MU_{c2}(1+R)$. This is the marginal utility gain or MB in terms of utility. Setting MC = MB yields.

(*) $MU_{c1} = \frac{1}{1+\rho} MU_{c2}(1+R)$. If you multiply the optimality condition in part 2(a) by $MU_{c2}$ and divide both sides by $-(1+\rho)$ you will get eqn (*). As such, the behavior in 2(a) is consistent with equating MB to MC!

3. The president of 317land believes that it is important to stimulate current economic activity in 317land and is considering a stimulus package that includes tax cuts. His advisors have provided him with two alternative policies. Under policy A, the representative household will receive a permanent tax cut that will reduce their tax liability in the current period and in every subsequent period by \$100. Under policy B, the representative household will receive a one-time tax rebate of \$1000.00 dollars. Under that assumption that the government's primary goal is to encourage current consumption and that the real interest rate is 3 percent, evaluate which of these policies is most consistent with the government's objective by doing the following.

(a) **Thoroughly derive** the marginal propensity to consume that is applicable for policy A and use it to predict the effect of policy A on per capita consumption. (state your final result as a dollar figure. (15 points)

Policy A permanently ↑ the rep hh's disposable income so we need to derive the MPC|perm.

Under the PIH $C_t = y_t^P \iff \Delta C_t = \Delta y_t^P$. Since $y_t^P = \frac{xR}{(1+R)}$, $\Delta y_t^P = \frac{(\Delta x)R}{(1+R)}$. Therefore,

$$\Delta C_t = \left[\frac{R}{(1+R)}\right]\left\{\Delta y_1 + \frac{\Delta y_2}{(1+R)} + \frac{\Delta y_3}{(1+R)^2} + \cdots + \Delta\left[\frac{b_0(1+R)}{P}\right]\right\}$$

Since perm let $\Delta y_1 = \Delta y_2 = \cdots = 1$ (we can multiply by 100 later).

$$\Delta C_t = \left[\frac{R}{1+R}\right]\left\{1 + \frac{1}{(1+R)} + \frac{1}{(1+R)^2} + \frac{1}{(1+R)^3} + \cdots\right\}.$$

Note that $\Delta\left[\frac{b_0(1+R)}{P}\right] = 0$

This is a geometric series that collapses to $\frac{1}{1-\frac{1}{1+R}} = \frac{(1+R)}{R}$.

$$\therefore \Delta C_t = \left[\frac{R}{1+R}\right]\left\{\frac{(1+R)}{R}\right\} = 1. \quad MPC|_{perm} = \frac{\Delta C_t}{\Delta y_t} = \frac{1}{1} = 1$$

The effect on per capita consumption is given by $\left[\Delta \text{ in disposable income}\right] \times MPC|_{perm} = \$100.$

(b) **Thoroughly derive** the marginal propensity to consume that is applicable for policy B and use it to predict the effect of policy B on per capita consumption. (state your final result as a dollar figure). Identify the policy you would recommend and explain why. (15 points)

In this case (i.e Policy B), the applicable marginal propensity to consume is $MPC/_{temp}$.

$MPC/_{temp} = \frac{\Delta C_t}{\Delta y_t}$. Under the PIH $\Delta C_t = \Delta y^{\Delta}_t$

so $\Delta C_t = \left[\frac{R}{1+R}\right]\left\{\Delta y_1 + \frac{\Delta y_2}{(1+R)} + \frac{\Delta y_3}{(1+R)^2} + \cdots + \Delta\left[\frac{b_0(1+R)}{P}\right]\right\}$

Let $\Delta y_1 = 1$; $\Delta y_2 = \Delta y_3 = \cdots = 0$ & $\Delta\left[\frac{b_0(1+R)}{P}\right] = 0$.

$\therefore \Delta C_t = \left[\frac{R}{1+R}\right]\{1\} = \frac{R}{1+R}$. $MPC_{temp} = \frac{\Delta C_t}{\Delta y_t} = \frac{\frac{R}{1+R}}{1}$

With $R = .03$ $MPC/_{temp} = \frac{.03}{1.03} = .02913$

Effect of \$1000 rebate on current cons is given by $MPC/_{temp} \times \$1000 = .02913(1000) = \$29.13$. Since the gov'ts primary goal is to encourage current cons. Policy A has greater impact

(c) Bonus Question: Will these policies affect work effort? Explain briefly. (10 points)

Both policies increase hhs' disposable income this means that $x$ or the real present value of resources available to hhs to dispose of as they deem appropriate ↑. This is an ↑ in wealth. Consequently, this should lead to an increase in leisure under both policies. However, the impact of policy A on wealth will be much larger so work-effort will ↓ by more than it will under policy B.

3. The president of 317land believes that it is important to encourage the citizens of 317land to increase their ***current*** savings and is considering doing so by implementing tax cuts. His advisors have provided him with two alternative policies. Under policy A, the representative household will receive a permanent tax cut that will reduce their tax liability in the current period and in every subsequent period by $200. Under policy B, the representative household will receive a one-time tax rebate of $2000.00 dollars. Under the assumption that the government's primary goal is to encourage current savings and that the real interest rate is 2 percent, determine which of these policies is most consistent with the government's objective by doing the following.

(a) **Thoroughly derive** the marginal propensity to save that is applicable for policy A and use it to predict the effect of policy A on current per capita savings. (state your final result as a dollar figure) (15 points)

Policy A is a permanent tax cut. To derive the MPS, recall that MPC + MPS = 1.

∴ derive the $MPC|_{perm}$ as shown earlier and subtract it from 1 to get the $MPS|_{perm} = 0$. This means that policy A will ~~increase~~ fail to induce an increase in savings. In particular the $\Delta S = (200)(MPS|_{perm}) = \$0$.

(b) **Thoroughly derive** the marginal propensity to save that is applicable for policy B and use it to predict the effect of policy B on current per capita savings. (state your final result as a dollar figure) Identify the policy you would recommend and explain why. (15 points)

Policy B is a temporary tax cut. To derive the $MPS_{temp}$, recall that MPS + MPC = 1. Therefore derive the $MPC_{temp}$ as done earlier and subtract it from 1 to get $MPS_{temp}$.

$MPS_{temp} = 1 - \frac{R}{1+R}$. Since $R = .02$ $MPS_{temp} = 1 - \frac{.02}{1.02}$

$MPS_{temp} = 1 - .0196 = .9804$. $\Delta S = .9804(2000) = \$1960.80$

So current savings would ↑ by $1,960.80. Since the objective is to encourage savings Policy B preferred.

(c) Bonus Question: Will these policies affect current period leisure? Explain briefly. (10 points)

See earlier ans.