Review Exercises

See CalcChat.com for tutorial help and worked-out solutions to odd-numbered exercises.

4.1 Using Radian or Degree Measure In Exercises 1–4, (a) sketch the angle in standard position, (b) determine the quadrant in which the angle lies, and (c) determine two coterminal angles (one positive and one negative).

1. $\frac{15\pi}{4}$
2. $-\frac{4\pi}{3}$
3. $-110°$
4. $280°$

Converting from Degrees to Radians In Exercises 5–8, convert the degree measure to radian measure. Round to three decimal places.

5. $450°$
6. $190°$
7. $-16°$
8. $-112°$

Converting from Radians to Degrees In Exercises 9–12, convert the radian measure to degree measure. Round to three decimal places, if necessary.

9. $\frac{3\pi}{10}$
10. $-\frac{11\pi}{6}$
11. -3.5
12. 5.7

Converting to D° M′ S″ Form In Exercises 13 and 14, convert the angle measure to D° M′ S″ form.

13. $198.4°$
14. $-5.96°$

15. **Arc Length** Find the length of the arc on a circle of radius 20 inches intercepted by a central angle of 138°.
16. **Phonograph** Phonograph records are vinyl discs that rotate on a turntable. A typical record album is 12 inches in diameter and plays at $33\frac{1}{3}$ revolutions per minute.
    (a) Find the angular speed of a record album.
    (b) Find the linear speed (in inches per minute) of the outer edge of a record album.

Area of a Sector of a Circle In Exercises 17 and 18, find the area of the sector of a circle of radius r and central angle θ.

	Radius r	Central Angle θ
17.	20 inches	150°
18.	7.5 millimeters	$2\pi/3$ radians

4.2 Finding a Point on the Unit Circle In Exercises 19–22, find the point (x, y) on the unit circle that corresponds to the real number t.

19. $t = 2\pi/3$
20. $t = 7\pi/4$
21. $t = 7\pi/6$
22. $t = -4\pi/3$

Evaluating Trigonometric Functions In Exercises 23 and 24, evaluate (if possible) the six trigonometric functions at the real number.

23. $t = \frac{3\pi}{4}$
24. $t = -\frac{2\pi}{3}$

Using Period to Evaluate Sine and Cosine In Exercises 25–28, evaluate the trigonometric function using its period as an aid.

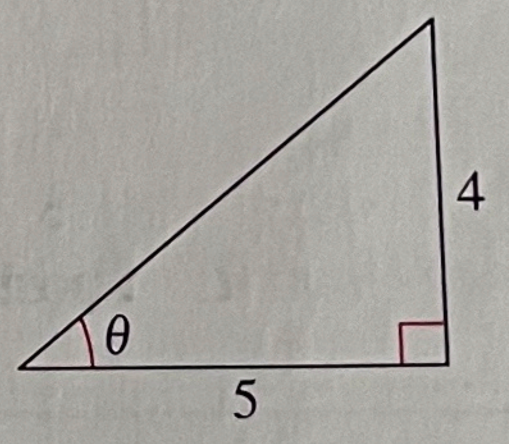
25. $\sin \frac{11\pi}{4}$
26. $\cos 4\pi$
27. $\cos\left(-\frac{17\pi}{6}\right)$
28. $\sin\left(-\frac{13\pi}{3}\right)$

Using a Calculator In Exercises 29–32, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

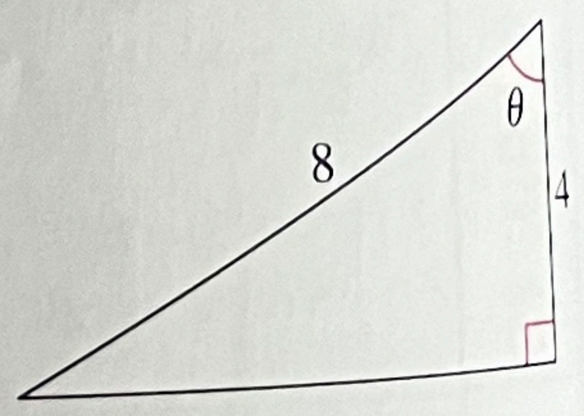
29. $\sec \frac{12\pi}{5}$
30. $\sin\left(-\frac{\pi}{9}\right)$
31. $\tan 33$
32. $\csc 10.5$

4.3 Evaluating Trigonometric Functions In Exercises 33 and 34, find the exact values of the six trigonometric functions of the angle θ.

33.



34.



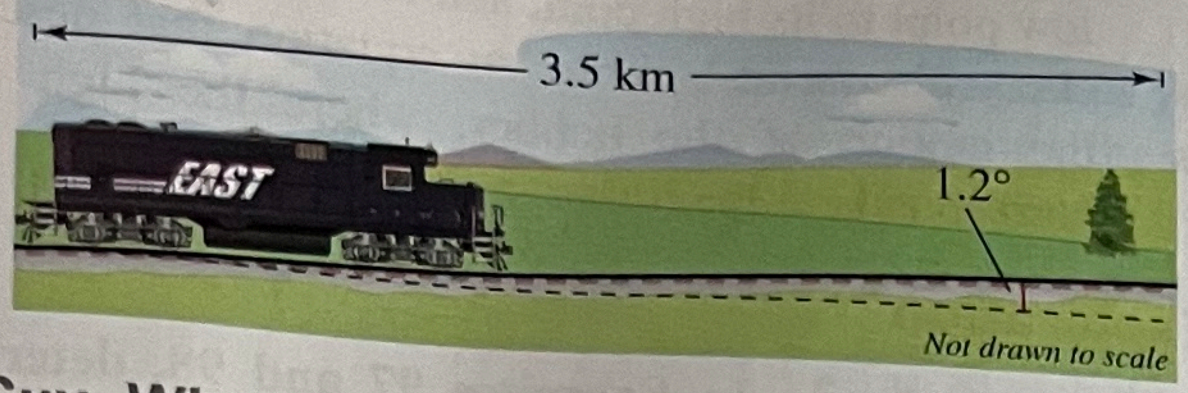
Using a Calculator In Exercises 35–38, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)

35. $\tan 33°$
36. $\sec 79.3°$
37. $\cot 15°\ 14'$
38. $\cos 78°\ 11'\ 58''$

Applying Trigonometric Identities In Exercises 39 and 40, use the given function value and the trigonometric identities to find the exact value of each indicated trigonometric function.

39. $\sin\theta = \frac{1}{3}$
    (a) $\csc\theta$ (b) $\cos\theta$
    (c) $\sec\theta$ (d) $\tan\theta$
40. $\csc\theta = 5$
    (a) $\sin\theta$ (b) $\cot\theta$
    (c) $\tan\theta$ (d) $\sec(90° - \theta)$

41. Railroad Grade A train travels 3.5 kilometers on a straight track with a grade of 1.2° (see figure). What is the vertical rise of the train in that distance?



42. Guy Wire A guy wire runs from the ground to the top of a 25-foot telephone pole. The angle formed between the wire and the ground is 52°. How far from the base of the pole is the guy wire anchored to the ground? Assume the pole is perpendicular to the ground.

4.4 Evaluating Trigonometric Functions **In Exercises 43–46, the point is on the terminal side of an angle in standard position. Find the exact values of the six trigonometric functions of the angle.**

43. $(12, 16)$ **44.** $(3, -4)$

45. $(0.3, 0.4)$ **46.** $\left(-\frac{10}{3}, -\frac{2}{3}\right)$

Evaluating Trigonometric Functions **In Exercises 47–50, find the exact values of the remaining five trigonometric functions of θ satisfying the given conditions.**

47. $\sec \theta = \frac{6}{5}$, $\tan \theta < 0$

48. $\csc \theta = \frac{3}{2}$, $\cos \theta < 0$

49. $\cos \theta = -\frac{2}{5}$, $\sin \theta > 0$

50. $\sin \theta = -\frac{1}{2}$, $\cos \theta > 0$

Finding a Reference Angle **In Exercises 51–54, find the reference angle θ'. Sketch θ in standard position and label θ'.**

51. $\theta = 264°$ **52.** $\theta = 635°$

53. $\theta = -6\pi/5$ **54.** $\theta = 17\pi/3$

Using a Reference Angle **In Exercises 55–58, evaluate the sine, cosine, and tangent of the angle without using a calculator.**

55. $-150°$ **56.** $495°$

57. $\pi/3$ **58.** $-5\pi/4$

Using a Calculator **In Exercises 59–62, use a calculator to evaluate the trigonometric function. Round your answer to four decimal places. (Be sure the calculator is in the correct mode.)**

59. $\sin 106°$

60. $\tan 37°$

61. $\tan(-17\pi/15)$

62. $\cos(-25\pi/7)$

4.5 Sketching the Graph of a Sine or Cosine Function **In Exercises 63–68, sketch the graph of the function. (Include two full periods.)**

63. $y = \sin 6x$

64. $f(x) = -\cos 3x$

65. $y = 5 + \sin \pi x$

66. $y = -4 - \cos \pi x$

67. $g(t) = \frac{5}{2} \sin(t - \pi)$

68. $g(t) = 3 \cos(t + \pi)$

69. Sound Waves Sound waves can be modeled using sine functions of the form $y = a \sin bx$, where x is measured in seconds.

(a) Write an equation of a sound wave whose amplitude is 2 and whose period is $\frac{1}{264}$ second.

(b) What is the frequency of the sound wave described in part (a)?

70. Meteorology The times S of sunset (Greenwich Mean Time) at 40° north latitude on the 15th of each month starting with January are: 16:59, 17:35, 18:06, 18:38, 19:08, 19:30, 19:28, 18:57, 18:10, 17:21, 16:44, and 16:36. A model (in which minutes have been converted to the decimal parts of an hour) for the data is

$$S(t) = 18.10 - 1.41 \sin\left(\frac{\pi t}{6} + 1.55\right)$$

where t represents the month, with $t = 1$ corresponding to January. *(Source: NOAA)*

(a) Use a graphing utility to graph the data and the model in the same viewing window.

(b) What is the period of the model? Is it what you expected? Explain.

(c) What is the amplitude of the model? What does it represent in the context of the problem?

4.6 Sketching the Graph of a Trigonometric Function **In Exercises 71–74, sketch the graph of the function. (Include two full periods.)**

71. $f(t) = \tan\left(t + \frac{\pi}{2}\right)$ **72.** $f(x) = \frac{1}{2} \cot x$

73. $f(x) = \frac{1}{2} \csc \frac{x}{2}$ **74.** $h(t) = \sec\left(t - \frac{\pi}{4}\right)$

Analyzing a Damped Trigonometric Graph **In Exercises 75 and 76, use a graphing utility to graph the function and the damping factor of the function in the same viewing window. Describe the behavior of the function as x increases without bound.**

75. $f(x) = x \cos x$

76. $g(x) = e^x \cos x$

4.7 Evaluating an Inverse Trigonometric Function In Exercises 77–80, find the exact value of the expression.

77. $\arcsin(-1)$
78. $\cos^{-1} 1$
79. $\operatorname{arccot} \sqrt{3}$
80. $\operatorname{arcsec}(-\sqrt{2})$

Calculators and Inverse Trigonometric Functions In Exercises 81–84, use a calculator to approximate the value of the expression, if possible. Round your result to two decimal places.

81. $\tan^{-1}(-1.3)$
82. $\arccos 0.372$
83. $\operatorname{arccot} 15.5$
84. $\operatorname{arccsc}(-4.03)$

Graphing an Inverse Trigonometric Function In Exercises 85 and 86, use a graphing utility to graph the function.

85. $f(x) = \arctan(x/2)$
86. $f(x) = -\arcsin 2x$

Evaluating a Composition of Functions In Exercises 87–90, find the exact value of the expression.

87. $\cos\left(\arctan \frac{3}{4}\right)$
88. $\tan\left(\arccos \frac{3}{5}\right)$
89. $\sec\left(\arctan \frac{12}{5}\right)$
90. $\cot\left[\arcsin\left(-\frac{12}{13}\right)\right]$

Writing an Expression In Exercises 91 and 92, write an algebraic expression that is equivalent to the given expression.

91. $\tan[\arccos(x/2)]$
92. $\sec[\arcsin(x-1)]$

4.8

93. **Angle of Elevation** The height of a radio transmission tower is 70 meters, and it casts a shadow of length 30 meters. Draw a right triangle that gives a visual representation of the problem. Label the known and unknown quantities. Then find the angle of elevation.

94. **Height** A football lands at the edge of the roof of your school building. When you are 25 feet from the base of the building, the angle of elevation to the football is 21°. How high off the ground is the football?

95. **Air Navigation** From city A to city B, a plane flies 650 miles at a bearing of 48°. From city B to city C, the plane flies 810 miles at a bearing of 115°. Find the distance from city A to city C and the bearing from city A to city C.

96. **Wave Motion** A fishing bobber oscillates in simple harmonic motion because of the waves in a lake. The bobber moves a total of 1.5 inches from its low point to its high point and returns to its high point every 3 seconds. Write an equation that describes the motion of the bobber, where the high point corresponds to the time $t = 0$.

Exploration

True or False? In Exercises 97 and 98, determine whether the statement is true or false. Justify your answer.

97. $y = \sin\theta$ is not a function because $\sin 30° = \sin 150°$.
98. Because $\tan(3\pi/4) = -1$, $\arctan(-1) = 3\pi/4$.

99. **Writing** Describe the behavior of $f(\theta) = \sec\theta$ at the zeros of $g(\theta) = \cos\theta$. Explain.

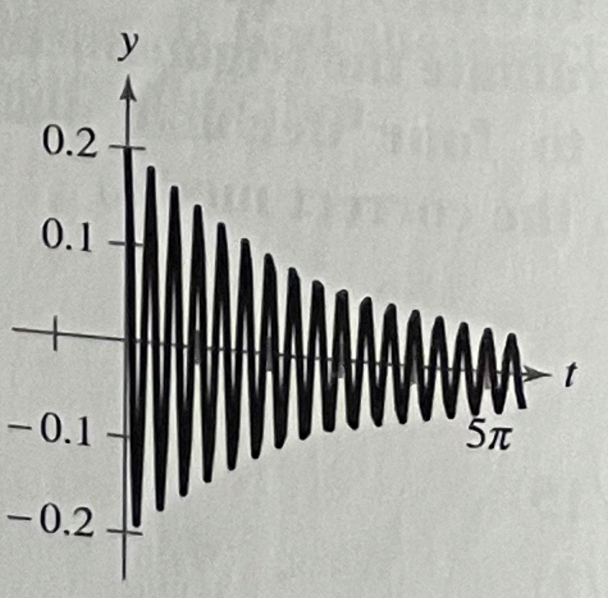
100. **Conjecture**
(a) Use a graphing utility to complete the table.

θ	0.1	0.4	0.7	1.0	1.3
$\tan\left(\theta - \frac{\pi}{2}\right)$					
$-\cot\theta$					

(b) Make a conjecture about the relationship between $\tan[\theta - (\pi/2)]$ and $-\cot\theta$.

101. **Writing** When graphing the sine and cosine functions, determining the amplitude is part of the analysis. Explain why this is not true for the other four trigonometric functions.

102. **Oscillation of a Spring** A weight is suspended from a ceiling by a steel spring. The weight is lifted (positive direction) from the equilibrium position and released. The resulting motion of the weight is modeled by $y = Ae^{-kt}\cos bt = \frac{1}{5}e^{-t/10}\cos 6t$, where y is the distance (in feet) from equilibrium and t is the time (in seconds). The figure shows the graph of the function. For each of the following, describe the change in the graph without graphing the resulting function.
(a) A is changed from $\frac{1}{5}$ to $\frac{1}{3}$.
(b) k is changed from $\frac{1}{10}$ to $\frac{1}{3}$.
(c) b is changed from 6 to 9.



Review Exercises

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5.1 Recognizing a Fundamental Identity In Exercises 1–4, name the trigonometric function that is equivalent to the expression.

1. $\dfrac{\cos x}{\sin x}$
2. $\dfrac{1}{\cos x}$
3. $\sin\left(\dfrac{\pi}{2} - x\right)$
4. $\sqrt{\cot^2 x + 1}$

Using Identities to Evaluate a Function In Exercises 5 and 6, use the given conditions and fundamental trigonometric identities to find the values of all six trigonometric functions.

5. $\cos\theta = -\frac{2}{5}$, $\tan\theta > 0$
6. $\cot x = -\frac{2}{3}$, $\cos x < 0$

Simplifying a Trigonometric Expression In Exercises 7–16, use the fundamental trigonometric identities to simplify the expression. (There is more than one correct form of each answer.)

7. $\dfrac{1}{\cot^2 x + 1}$
8. $\dfrac{\tan\theta}{1 - \cos^2\theta}$
9. $\tan^2 x(\csc^2 x - 1)$
10. $\cot^2 x(\sin^2 x)$
11. $\dfrac{\cot\left(\dfrac{\pi}{2} - u\right)}{\cos u}$
12. $\dfrac{\sec^2(-\theta)}{\csc^2\theta}$
13. $\cos^2 x + \cos^2 x \cot^2 x$
14. $(\tan x + 1)^2 \cos x$
15. $\dfrac{1}{\csc\theta + 1} - \dfrac{1}{\csc\theta - 1}$
16. $\dfrac{\tan^2 x}{1 + \sec x}$

Trigonometric Substitution In Exercises 17 and 18, use the trigonometric substitution to write the algebraic expression as a trigonometric function of θ, where $0 < \theta < \pi/2$.

17. $\sqrt{25 - x^2}$, $x = 5\sin\theta$
18. $\sqrt{x^2 - 16}$, $x = 4\sec\theta$

5.2 Verifying a Trigonometric Identity In Exercises 19–26, verify the identity.

19. $\cos x(\tan^2 x + 1) = \sec x$
20. $\sec^2 x \cot x - \cot x = \tan x$
21. $\sin\left(\dfrac{\pi}{2} - \theta\right)\tan\theta = \sin\theta$
22. $\cot\left(\dfrac{\pi}{2} - x\right)\csc x = \sec x$
23. $\dfrac{1}{\tan\theta\csc\theta} = \cos\theta$
24. $\dfrac{1}{\tan x \csc x \sin x} = \cot x$
25. $\sin^5 x \cos^2 x = (\cos^2 x - 2\cos^4 x + \cos^6 x)\sin x$
26. $\cos^3 x \sin^2 x = (\sin^2 x - \sin^4 x)\cos x$

5.3 Solving a Trigonometric Equation In Exercises 27–32, solve the equation.

27. $\sin x = \sqrt{3} - \sin x$
28. $4\cos\theta = 1 + 2\cos\theta$
29. $3\sqrt{3}\tan u = 3$
30. $\frac{1}{2}\sec x - 1 = 0$
31. $3\csc^2 x = 4$
32. $4\tan^2 u - 1 = \tan^2 u$

Solving a Trigonometric Equation In Exercises 33–42, find all solutions of the equation in the interval $[0, 2\pi)$.

33. $\sin^3 x = \sin x$
34. $2\cos^2 x + 3\cos x = 0$
35. $\cos^2 x + \sin x = 1$
36. $\sin^2 x + 2\cos x = 2$
37. $2\sin 2x - \sqrt{2} = 0$
38. $2\cos\dfrac{x}{2} + 1 = 0$
39. $3\tan^2\left(\dfrac{x}{3}\right) - 1 = 0$
40. $\sqrt{3}\tan 3x = 0$
41. $\cos 4x(\cos x - 1) = 0$
42. $3\csc^2 5x = -4$

Using Inverse Functions In Exercises 43–46, solve the equation.

43. $\tan^2 x - 2\tan x = 0$
44. $2\tan^2 x - 3\tan x = -1$
45. $\tan^2\theta + \tan\theta - 6 = 0$
46. $\sec^2 x + 6\tan x + 4 = 0$

5.4 Evaluating Trigonometric Functions In Exercises 47–50, find the exact values of the sine, cosine, and tangent of the angle.

47. $75° = 120° - 45°$
48. $375° = 135° + 240°$
49. $\dfrac{25\pi}{12} = \dfrac{11\pi}{6} + \dfrac{\pi}{4}$
50. $\dfrac{19\pi}{12} = \dfrac{11\pi}{6} - \dfrac{\pi}{4}$

Rewriting a Trigonometric Expression In Exercises 51 and 52, write the expression as the sine, cosine, or tangent of an angle.

51. $\sin 60° \cos 45° - \cos 60° \sin 45°$
52. $\dfrac{\tan 68° - \tan 115°}{1 + \tan 68° \tan 115°}$

Evaluating a Trigonometric Expression In Exercises 53–56, find the exact value of the trigonometric expression given that $\tan u = \frac{3}{4}$ and $\cos v = -\frac{4}{5}$. (u is in Quadrant I and v is in Quadrant III.)

53. $\sin(u + v)$
54. $\tan(u + v)$
55. $\cos(u - v)$
56. $\sin(u - v)$

Verifying a Trigonometric Identity In Exercises 57–60, verify the identity.

57. $\cos\left(x + \dfrac{\pi}{2}\right) = -\sin x$

58. $\tan\left(x - \dfrac{\pi}{2}\right) = -\cot x$

59. $\tan(\pi - x) = -\tan x$

60. $\sin(x - \pi) = -\sin x$

Solving a Trigonometric Equation In Exercises 61 and 62, find all solutions of the equation in the interval $[0, 2\pi)$.

61. $\sin\left(x + \dfrac{\pi}{4}\right) - \sin\left(x - \dfrac{\pi}{4}\right) = 1$

62. $\cos\left(x + \dfrac{\pi}{6}\right) - \cos\left(x - \dfrac{\pi}{6}\right) = 1$

5.5 Evaluating Functions Involving Double Angles In Exercises 63 and 64, use the given conditions to find the exact values of $\sin 2u$, $\cos 2u$, and $\tan 2u$ using the double-angle formulas.

63. $\sin u = \frac{4}{5}$, $0 < u < \pi/2$

64. $\cos u = -2/\sqrt{5}$, $\pi/2 < u < \pi$

Verifying a Trigonometric Identity In Exercises 65 and 66, use the double-angle formulas to verify the identity algebraically and use a graphing utility to confirm your result graphically.

65. $\sin 4x = 8\cos^3 x \sin x - 4\cos x \sin x$

66. $\tan^2 x = \dfrac{1 - \cos 2x}{1 + \cos 2x}$

Reducing Powers In Exercises 67 and 68, use the power-reducing formulas to rewrite the expression in terms of first powers of the cosines of multiple angles.

67. $\tan^2 3x$

68. $\sin^2 x \cos^2 x$

Using Half-Angle Formulas In Exercises 69 and 70, use the half-angle formulas to determine the exact values of the sine, cosine, and tangent of the angle.

69. $-75°$

70. $5\pi/12$

Using Half-Angle Formulas In Exercises 71–74, use the given conditions to (a) determine the quadrant in which $u/2$ lies, and (b) find the exact values of $\sin(u/2)$, $\cos(u/2)$, and $\tan(u/2)$ using the half-angle formulas.

71. $\tan u = \dfrac{4}{3}$, $\pi < u < \dfrac{3\pi}{2}$

72. $\sin u = \dfrac{3}{5}$, $0 < u < \dfrac{\pi}{2}$

73. $\cos u = -\dfrac{2}{7}$, $\dfrac{\pi}{2} < u < \pi$

74. $\tan u = -\dfrac{\sqrt{21}}{2}$, $\dfrac{3\pi}{2} < u < 2\pi$

Using Product-to-Sum Formulas In Exercises 75 and 76, use the product-to-sum formulas to rewrite the product as a sum or difference.

75. $\cos 4\theta \sin 6\theta$

76. $2\sin 7\theta \cos 3\theta$

Using Sum-to-Product Formulas In Exercises 77 and 78, use the sum-to-product formulas to rewrite the sum or difference as a product.

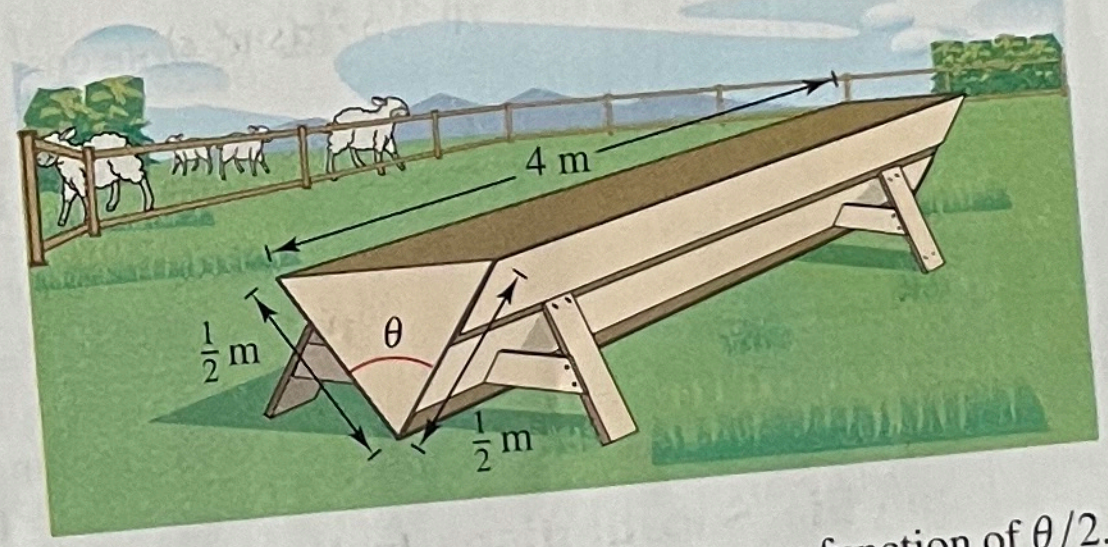
77. $\cos 6\theta + \cos 5\theta$

78. $\sin 3x - \sin x$

79. **Projectile Motion** A baseball leaves the hand of a player at first base at an angle of θ with the horizontal and at an initial velocity of $v_0 = 80$ feet per second. A player at second base 100 feet away catches the ball. Find θ when the range r of a projectile is

$$r = \frac{1}{32}v_0^2 \sin 2\theta.$$

80. **Geometry** A trough for feeding cattle is 4 meters long and its cross sections are isosceles triangles with the two equal sides being $\frac{1}{2}$ meter (see figure). The angle between the two sides is θ.



(a) Write the volume of the trough as a function of $\theta/2$.

(b) Write the volume of the trough as a function of θ and determine the value of θ such that the volume is maximized.

Exploration

True or False? In Exercises 81–84, determine whether the statement is true or false. Justify your answer.

81. If $\dfrac{\pi}{2} < \theta < \pi$, then $\cos\dfrac{\theta}{2} < 0$.

82. $\cot x \sin^2 x = \cos x \sin x$

83. $4\sin(-x)\cos(-x) = -2\sin 2x$

84. $4\sin 45° \cos 15° = 1 + \sqrt{3}$

85. **Think About It** Is it possible for a trigonometric equation that is not an identity to have an infinite number of solutions? Explain.