

4.5 Assess Your Understanding

'Are You Prepared?' Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.

- Find $f(-1)$ if $f(x) = 2x^2 - x$. (pp. 49–52)
- Factor the expression $6x^2 + x - 2$. (pp. A28–A29)
- Find the quotient and remainder if $3x^4 - 5x^3 + 7x - 4$ is divided by $x - 3$. (pp. A25–A28 or A32–A35)
- Find the intercepts of $f(x) = x^2 + x - 3$. (pp. 138–139)

SECTION 4.5 The Real Zeros of a Polynomial Function 231

Concepts and Vocabulary

- In the process of polynomial division, (Divisor)(Quotient) + _____ = _____.
- When a polynomial function f is divided by $x - c$, the remainder is _____.
- If a function f , whose domain is all real numbers, is even and if 4 is a zero of f , then _____ is also a zero.
- True or False** Every polynomial function of degree 3 with real coefficients has exactly three real zeros.
- If f is a polynomial function and $x - 4$ is a factor of f , then $f(4) = \underline{\hspace{2cm}}$.
- True or False** If f is a polynomial function of degree 4 and if $f(2) = 5$, then

$$\frac{f(x)}{x - 2} = p(x) + \frac{5}{x - 2}$$
 where $p(x)$ is a polynomial of degree 3.

Skill Building

In Problems 11–20, use the Remainder Theorem to find the remainder when $f(x)$ is divided by $x - c$. Then use the Factor Theorem to determine whether $x - c$ is a factor of $f(x)$.

- $f(x) = 4x^3 - 3x^2 - 8x + 4; x - 2$
- $f(x) = 3x^4 - 6x^3 - 5x + 10; x - 2$
- $f(x) = 3x^6 + 82x^3 + 27; x + 3$
- $f(x) = 4x^6 - 64x^4 + x^2 - 15; x + 4$
- $f(x) = 2x^4 - x^3 + 2x - 1; x - \frac{1}{2}$
- $f(x) = -4x^3 + 5x^2 + 8; x + 3$
- $f(x) = 4x^4 - 15x^2 - 4; x - 2$
- $f(x) = 2x^6 - 18x^4 + x^2 - 9; x + 3$
- $f(x) = x^6 - 16x^4 + x^2 - 16; x + 4$
- $f(x) = 3x^4 + x^3 - 3x + 1; x + \frac{1}{3}$

In Problems 21–32, tell the maximum number of real zeros that each polynomial function may have. Do not attempt to find the zeros.

- $f(x) = -4x^7 + x^3 - x^2 + 2$
- $f(x) = 5x^4 + 2x^2 - 6x - 5$
- $f(x) = 2x^6 - 3x^2 - x + 1$
- $f(x) = -3x^5 + 4x^4 + 2$
- $f(x) = 3x^3 - 2x^2 + x + 2$
- $f(x) = -x^3 - x^2 + x + 1$
- $f(x) = -x^4 + x^2 - 1$
- $f(x) = x^4 + 5x^3 - 2$
- $f(x) = x^5 + x^4 + x^2 + x + 1$
- $f(x) = x^5 - x^4 + x^3 - x^2 + x - 1$
- $f(x) = x^6 - 1$
- $f(x) = x^6 + 1$

In Problems 33–44, list the potential rational zeros of each polynomial function. Do not attempt to find the zeros.

- $f(x) = 3x^4 - 3x^3 + x^2 - x + 1$
- $f(x) = x^5 - x^4 + 2x^2 + 3$
- $f(x) = x^5 - 6x^2 + 9x - 3$
- $f(x) = 2x^5 - x^4 - x^2 + 1$
- $f(x) = -4x^3 - x^2 + x + 2$
- $f(x) = 6x^4 - x^2 + 2$
- $f(x) = 6x^4 - x^2 + 9$
- $f(x) = -4x^3 + x^2 + x + 6$
- $f(x) = 2x^5 - x^3 + 2x^2 + 12$
- $f(x) = 3x^5 - x^2 + 2x + 18$
- $f(x) = 6x^4 + 2x^3 - x^2 + 20$
- $f(x) = -6x^3 - x^2 + x + 10$

In Problems 45–56, use the Rational Zeros Theorem to find all the real zeros of each polynomial function. Use the zeros to factor f over the real numbers.

- $f(x) = x^3 + 2x^2 - 5x - 6$
- $f(x) = x^3 + 8x^2 + 11x - 20$
- $f(x) = 2x^3 - x^2 + 2x - 1$
- $f(x) = 2x^3 + x^2 + 2x + 1$
- $f(x) = 2x^3 - 4x^2 - 10x + 20$
- $f(x) = 3x^3 + 6x^2 - 15x - 30$
- $f(x) = 2x^4 + x^3 - 7x^2 - 3x + 3$
- $f(x) = 2x^4 - x^3 - 5x^2 + 2x + 2$
- $f(x) = x^4 + x^3 - 3x^2 - x + 2$