For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch13.

EXAMPLE 13.2: Process Control Chart Design

An insurance company wants to design a control chart to monitor whether insurance claim forms are being completed correctly. The company intends to use the chart to see if improvements in the design of the form are effective. To start the process, the company collected data on the number of incorrectly completed claim forms over the past 10 days. The insurance company processes thousands of these forms each day, and due to the high cost of inspecting each form, only a small representative sample was collected each day. The data and analysis are shown in Exhibit 13.6.

Analytics

excel For the Excel template, visit www.mhhe.com/jacobs14e.

SOLUTION

To construct the control chart, first calculate the overall fraction defective from all samples. This sets the centerline for the control chart.

$$\bar{p} = \frac{\text{Total number of defective units from all samples}}{\text{Number of samples} \times \text{Sample size}} = \frac{91}{3{,}000} = .03033$$

Next calculate the sample standard deviation:

$$s_p = \sqrt{\frac{\bar{p}(1-\bar{p})}{n}} = \sqrt{\frac{.03033(1-.03033)}{300}} = .00990$$

Finally, calculate the upper and lower process control limits. A z-value of 3 gives 99.7 percent confidence that the process is within these limits.

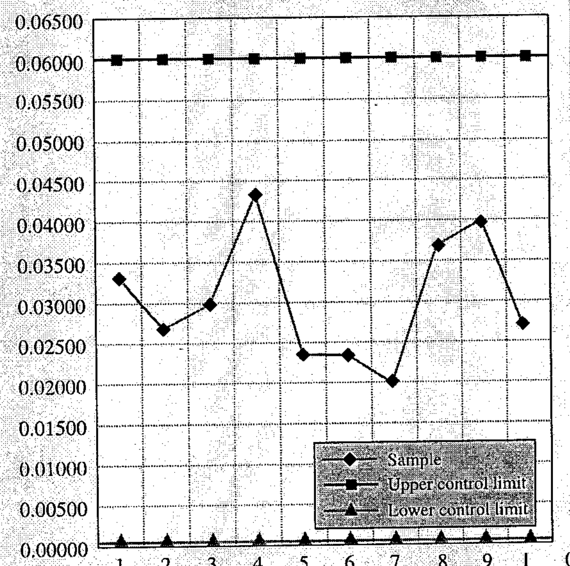
$$\text{UCL} = \bar{p} + 3s_p = .03033 + 3(.00990) = .06003$$

$$\text{LCL} = \bar{p} - 3s_p = .03033 - 3(.00990) = .00063$$

The calculations in Exhibit 13.6, including the control chart, are included in the spreadsheet "SPC.xls."

exhibit 13.6 Insurance Company Claim Form

Sample	Number Inspected	Number of Forms Completed Incorrectly	Fraction Defective
1	300	10	0.03333
2	300	8	0.02667
3	300	9	0.03000
4	300	13	0.04333
5	300	7	0.02333
6	300	7	0.02333
7	300	6	0.02000
8	300	11	0.03667
9	300	12	0.04000
10	300	8	0.02667
Totals	3,000	91	0.03033
Sample standard deviation			0.00990



Process Control with Attribute Measurements: Using *c*-Charts

In the case of the *p*-chart, the item was either good or bad. There are times when the product or service can have more than one defect. For example, a board sold at a lumberyard may have multiple knotholes and, depending on the quality grade, may or may not be defective. When it is desired to monitor the number of defects per unit, the *c*-chart is appropriate.

Analytics

The underlying distribution for the *c*-chart is the Poisson, which is based on the assumption that defects occur randomly on each unit. If c is the number of defects for a particular unit, then $\bar{c}$ is the average number of defects per unit, and the standard deviation is $\sqrt{\bar{c}}$. For the purposes of our control chart we use the normal approximation to the Poisson distribution and construct the chart using the following control limits.

$$\bar{c} = \text{Average number of defects per unit} \qquad [13.8]$$

$$s_p = \sqrt{\bar{c}} \qquad [13.9]$$

$$\text{UCL} = \bar{c} + z\sqrt{\bar{c}} \qquad [13.10]$$

$$\text{LCL} = \bar{c} - z\sqrt{\bar{c}} \text{ or 0 if less than 0} \qquad [13.11]$$

Just as with the *p*-chart, typically $z = 3$ (99.7 percent confidence) or $z = 2.58$ (99 percent confidence) is used.

EXAMPLE 13.3

The owners of a lumberyard want to design a control chart to monitor the quality of 2 × 4 boards that come from their supplier. For their medium-quality boards they expect an average of four knotholes per 8-foot board. Design a control chart for use by the person receiving the boards using three-sigma (standard deviation) limits.

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch13.

SOLUTION

For this problem, $\bar{c} = 4$, $s_p = \sqrt{\bar{c}} = 2$

$$\text{UCL} = \bar{c} + z\sqrt{\bar{c}} = 4 + 3(2) = 10$$

$$\text{LCL} = \bar{c} - z\sqrt{\bar{c}} = 4 - 3(2) = -2 \rightarrow 0$$ (Zero is used since it is not possible to have a negative number of defects.)

Process Control with Variable Measurements: Using $\bar{X}$- and *R*-Charts

$\bar{X}$- and *R*- (range) charts are widely used in statistical process control.

In attribute sampling, we determine whether something is good or bad, fits or doesn't fit—it is a go/no-go situation. In **variables** sampling, however, we measure the actual weight, volume, number of inches, or other variable measurements, and we develop control charts to determine the acceptability or rejection of the process based on those measurements. For example, in attribute sampling, we might decide that if something is over 10 pounds we will reject it and under 10 pounds we will accept it. In variables sampling, we measure a sample and may record weights of 9.8 pounds or 10.2 pounds. These values are used to create or modify control charts and to see whether they fall within the acceptable limits.

Variables
Quality characteristics that are measured in actual weight, volume, inches, centimeters, or other measure.

There are four main issues to address in creating a control chart: the size of the samples, number of samples, frequency of samples, and control limits.

Size of Samples For industrial applications in process control involving the measurement of variables, it is preferable to keep the sample size small. There are two main reasons. First, the sample needs to be taken within a reasonable length of time; otherwise, the process might change while the samples are taken. Second, the larger the sample, the more it costs to take.

SAMPLE No.	OBSERVATIONS				MEAN	RANGE
1	27.34667	27.50085	29.94412	28.21249	28.25103	2.59745
2	27.79695	26.15006	31.21295	31.33272	29.12317	5.18266
3	33.53255	29.32971	29.70460	31.05300	30.90497	4.20284
4	37.98409	32.26942	31.91741	29.44279	32.90343	8.54130
5	33.82722	30.32543	28.38117	33.70124	31.55877	5.44605
6	29.68356	29.56677	27.23077	34.00417	30.12132	6.77340
7	32.62640	26.32030	32.07892	36.17198	31.79940	9.85168
8	30.29575	30.52868	24.43315	26.85241	28.02750	6.09553
9	28.43856	30.48251	32.43083	30.76162	30.52838	3.99227
10	28.27790	33.94916	30.47406	28.87447	30.39390	5.67126
11	26.91885	27.66133	31.46936	29.66928	28.92971	4.55051
12	28.46547	28.29937	28.99441	31.14511	29.22609	2.84574
13	32.42677	26.10410	29.47718	37.20079	31.30221	11.09669
14	28.84273	30.51801	32.23614	30.47104	30.51698	3.39341
15	30.75136	32.99922	28.08452	26.19981	29.50873	6.79941
16	31.25754	24.29473	35.46477	28.41126	29.85708	11.17004
17	31.24921	28.57954	35.00865	31.23591	31.51833	6.42911
18	31.41554	35.80049	33.60909	27.82131	32.16161	7.97918
19	32.20230	32.02005	32.71018	29.37620	31.57718	3.33398
20	26.91603	29.77775	33.92696	33.78366	31.10110	7.01093
21	35.05322	32.93284	31.51641	27.73615	31.80966	7.31707
22	32.12483	29.32853	30.99709	31.39641	30.96172	2.79630
23	30.09172	32.43938	27.84725	30.70726	30.27140	4.59213
24	30.04835	27.23709	22.01801	28.69624	26.99992	8.03034
25	29.30273	30.83735	30.82735	31.90733	30.71869	2.60460
				Means	30.40289	5.932155

Questions

1 Prepare $\bar{X}$- and R-charts using these data with the method described in the chapter.

2 Analyze the charts and comment on whether the process appears to be in control and stable.

3 Twelve additional samples of curetime data from the molding process were collected from an actual production run. The data from these new samples are shown below. Update your control charts and compare the results with the previous data. The $\bar{X}$- and R-charts are drawn with the new data using the same control limits established before. Comment on what the new charts show.

SAMPLE No.	OBSERVATIONS				MEAN	RANGE
1	31.65830	29.78330	31.87910	33.91250	31.80830	4.12920
2	34.46430	25.18480	37.76689	39.21143	34.15686	14.02663
3	41.34268	39.54590	29.55710	32.57350	35.75480	11.78558
4	29.47310	25.37840	25.04380	24.00350	25.97470	5.46960
5	25.46710	34.85160	30.19150	31.62220	30.53310	9.38450
6	46.25184	34.71356	41.41277	44.63319	41.75284	11.53828
7	35.44750	38.83289	33.08860	31.63490	34.75097	7.19799
8	34.55143	33.86330	35.18869	42.31515	36.47964	8.45185
9	43.43549	37.36371	38.85718	39.25132	39.72693	6.07178
10	37.05298	42.47056	35.90282	38.21905	38.41135	6.56774
11	38.57292	39.06772	32.22090	33.20200	35.76589	6.84682
12	27.03050	33.63970	26.63060	42.79176	32.52314	16.16116

Case: Quality Management—Toyota

Quality Control Analytics at Toyota

As part of the process for improving the quality of their cars, Toyota engineers have identified a potential improvement to the process that makes a washer that is used in the accelerator assembly. The tolerances on the thickness of the washer are fairly large since the fit can be loose, but if it does happen to get too large, it can cause the accelerator to bind and create a potential problem for the driver. (*Note:* This part of the case has been fabricated for teaching purposes, and none of these data were obtained from Toyota.)

Let's assume that, as a first step to improving the process, a sample of 40 washers coming from the machine that produces the washers was taken and the thickness measured in millimeters. The following table has the measurements from the sample:

1.9	2.0	1.9	1.8	2.2	1.7	2.0	1.9	1.7	1.8
1.8	2.2	2.1	2.2	1.9	1.8	2.1	1.6	1.8	1.6
2.1	2.4	2.2	2.1	2.1	2.0	1.8	1.7	1.9	1.9
2.1	2.0	2.4	1.7	2.2	2.0	1.6	2.0	2.1	2.2

Questions

1 If the specification is such that no washer should be greater than 2.4 millimeters, assuming that the thicknesses are distributed normally, what fraction of the output is expected to be greater than this thickness?

2 If there are an upper and lower specification, where the upper thickness limit is 2.4 and the lower thickness limit is 1.4, what fraction of the output is expected to be out of tolerance?

3 What is the C_{pk} for the process?

4 What would be the C_{pk} for the process if it were centered between the specification limits (assume the process standard deviation is the same)?

5 What percentage of output would be expected to be out of tolerance if the process were centered?

6 Set up $\bar{X}$- and range control charts for the current process. Assume the operators will take samples of 10 washers at a time.

7 Plot the data on your control charts. Does the current process appear to be in control?

8 If the process could be improved so that the standard deviation were only about .10 millimeter, what would be the best that could be expected with the processes relative to fraction defective?

Practice Exam

1. A Six Sigma process that is running at the center of its control limits would expect this defect rate.
2. Variation that can be clearly identified and possibly managed.
3. Variation inherent in the process itself.
4. If a process has a capability index of 1 and is running normally (centered on the mean), what percentage of the units would one expect to be defective?
5. An alternative to viewing an item as simply good or bad due to it falling in or out of the tolerance range.
6. Quality characteristics that are classified as either conforming or not conforming to specification.
7. A quality characteristic that is actually measured, such as the weight of an item.
8. A quality chart suitable for when an item is either good or bad.
9. A quality chart suitable for when a number of blemishes are expected on each unit, such as a spool of yarn.
10. Useful for checking quality when we periodically purchase large quantities of an item and it would be very costly to check each unit individually.
11. A chart that depicts the manufacturer's and consumer's risks associated with a sampling plan.

1. Two parts per billion units 2. Assignable variation 3. Common variation 4. Design limits are at $\pm 3\sigma$ or 2.7 defects per thousand 5. Taguchi loss function 6. Attributes 7. Variable 8. *p*-chart 9. *c*-chart 10. Acceptance sampling 11. Operating characteristic curve

Selected Bibliography

Evans, James R., and William M. Lindsay. *The Management and Control of Quality*, 8th ed. Cincinnati: South-Western College Publications, 2010.

Rath & Strong. *Rath & Strong's Six Sigma Pocket Guide.* Rath & Strong, Inc., 2003.

Ryan, Thomas P., *Statistical Methods for Quality Improvement.* New York: Wiley Series in Probability and Statistics, 2011.

Small, B. B. (with committee). *Statistical Quality Control Handbook.* Western Electric Co., Inc., 1956.

Footnotes

1. E. L. Grant and R. S. Leavenworth, *Statistical Quality Control* (New York: McGraw-Hill, 1996). Copyright © 1996 McGraw-Hill Companies, Inc. Used with permission.
2. There is some controversy surrounding AQLs. This is based on the argument that specifying some acceptable percentage of defectives is inconsistent with the philosophical goal of zero defects. In practice, even in the best QC companies, there is an acceptable quality level. The difference is that it may be stated in parts per million rather than in parts per hundred. This is the case in Motorola's Six Sigma quality standard, which holds that no more than 3.4 defects per million parts are acceptable.
3. See, for example, H. F. Dodge and H. G. Romig, *Sampling Inspection Tables—Single and Double Sampling* (New York: John Wiley & Sons, 1959); and *Military Standard Sampling Procedures and Tables for Inspection by Attributes* (MIL-STD-105D) (Washington, DC: U.S. Government Printing Office, 1983).

Using these factors, if we expected demand for next year to be 1,100 units, we would forecast the demand to occur as

	EXPECTED DEMAND FOR NEXT YEAR	AVERAGE SALES FOR EACH SEASON (1,100/4)		SEASONAL FACTOR		NEXT YEAR'S SEASONAL FORECAST
Spring		275	×	0.8	=	220
Summer		275	×	1.4	=	385
Fall		275	×	1.2	=	330
Winter		275	×	0.6	=	165
Total	1,100					

The seasonal factor may be periodically updated as new data are available. The following example shows the seasonal factor and multiplicative seasonal variation.

For a step-by-step walkthrough of this example, visit www.mhhe.com/jacobs14e_sbs_ch18.

EXAMPLE 18.4: Computing Trend and Seasonal Factor from a Linear Regression Line Obtained with Excel

Forecast the demand for each quarter of the next year using trend and seasonal factors. Demand for the past two years is in the following table:

QUARTER	AMOUNT	QUARTER	AMOUNT
1	300	5	520
2	200	6	420
3	220	7	400
4	530	8	700

SOLUTION

First, we plot as in Exhibit 18.10 and then calculate the slope and intercept using Excel. For Excel the quarters are numbered 1 through 8. The "known *y*s" are the amounts (300, 200, 220, etc.), and the "known *x*s" are the quarter numbers (1, 2, 3, etc.). We obtain a slope = 52.3 (rounded), and intercept = 176.1 (rounded). The equation for the line is

$$\text{Forecast Including Trend (FIT)} = 176.1 + 52.3t$$

Next we can derive a seasonal index by comparing the actual data with the trend line, as in Exhibit 18.11. The seasonal factor was developed by averaging the same quarters in each year.

We can compute the 2013 forecast including trend and seasonal factors (FITS) as follows:

$$\text{FITS}_t = \text{FIT} \times \text{Seasonal}$$

$$\text{I—2013 FITS}_9 = [176.1 + 52.3(9)]1.25 = 808$$

$$\text{II—2013 FITS}_{10} = [176.1 + 52.3(10)]0.79 = 552$$

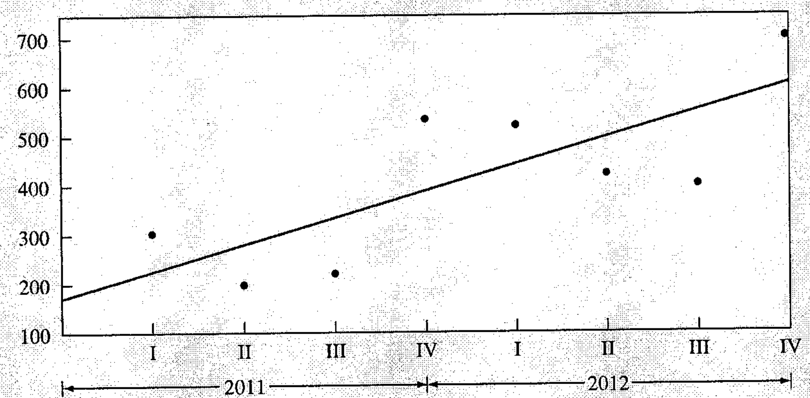
$$\text{III—2013 FITS}_{11} = [176.1 + 52.3(11)]0.70 = 526$$

$$\text{IV—2013 FITS}_{12} = [176.1 + 52.3(12)]1.28 = 1{,}029$$

Note, these numbers were calculated using Excel, so your numbers may differ slightly due to rounding.

exhibit 18.10

Computing a Seasonal Factor from the Actual Data and Trend Line



eXcel

For the Excel template, visit www.mhhe.com/jacobs14e.

Quarter	Actual Amount	From Trend Equation $FIT_t = 176.1 + 52.3t$	Ratio of Actual ÷ Trend
2011			
I	300	228.3	1.31
II	200	280.6	0.71
III	220	332.9	0.66
IV	530	385.1	1.38
2012		—	—
I	520	437.4	1.19
II	420	489.6	0.86
III	400	541.9	0.74
IV	700	594.2	1.18

Seasonal Factor (Average of Same Quarters in Both Years)
I 1.25
II 0.79
III 0.70
IV 1.28

exhibit 18.11

Deseasonalized Demand

(1) Period (t)	(2) Quarter	(3) Actual Demand (y)	(4) Average of the Same Quarters of Each Year	(5) Seasonal Factor	(6) Deseasonalized Demand (y_d) Col. (3) ÷ Col. (5)	(7) t^2 (Col. 1)^2	(8) $t \times y_d$ Col. (1) × Col. (6)
1	I	600	(600 + 2,400 + 3,800)/3 = 2,266.7	0.82	735.7	1	735.7
2	II	1,550	(1,550 + 3,100 + 4,500)/3 = 3,050	1.10	1,412.4	4	2,824.7
3	III	1,500	(1,500 + 2,600 + 4,000)/3 = 2,700	0.97	1,544.0	9	4,631.9
4	IV	1,500	(1,500 + 2,900 + 4,900)/3 = 3,100	1.12	1,344.8	16	5,379.0
5	I	2,400		0.82	2,942.6	25	14,713.2
6	II	3,100		1.10	2,824.7	36	16,948.4
7	III	2,600		0.97	2,676.2	49	18,733.6
8	IV	2,900		1.12	2,599.9	64	20,798.9
9	I	3,800		0.82	4,659.2	81	41,932.7
10	II	4,500		1.10	4,100.4	100	41,004.1
11	III	4,000		0.97	4,117.3	121	45,290.1
12	IV	4,900		1.12	4,392.9	144	52,714.5
78		33,350*		12.03	33,350.1*	650	265,706.9

$\bar{t} = \frac{78}{12} = 6.5 \qquad b = \frac{\sum t y_d - n\bar{t}\bar{y}_d}{\sum t^2 - n\bar{t}^2} = \frac{265{,}706.9 - 12(6.5)2{,}779.2}{650 - 12(6.5)^2} = 342.2$

$\bar{y}_d = 33{,}350/12 = 2{,}779.2 \qquad a = \bar{y}_d - b\bar{t} = 2{,}779.2 - 342.2(6.5) = 554.9$

Therefore, $Y = a + bt = 554.9 + 342.2t$

*Column 3 and column 6 totals should be equal at 33,350. Differences are due to rounding. Column 5 was rounded to two decimal places.