4.1 Assess Your Understanding

'Are You Prepared?' *Answers are given at the end of these exercises. If you get a wrong answer, read the pages listed in red.*

1. The intercepts of the equation $9x^2 + 4y = 36$ are _____. (pp. 11–12)
2. Is the expression $4x^3 - 3.6x^2 - \sqrt{2}$ a polynomial? If so, what is its degree? (pp. A22–A29)
3. To graph $y = x^2 - 4$, you would shift the graph of $y = x^2$ _____ a distance of _____ units. (pp. 90–99)
4. Use a graphing utility to approximate (rounded to two decimal places) the local maximum value and local minimum value of $f(x) = x^3 - 2x^2 - 4x + 5$, for $-3 < x < 3$. (p. 74)
5. ***True or False*** The x-intercepts of the graph of a function $y = f(x)$ are the real solutions of the equation $f(x) = 0$. (pp. 61–63)
6. If $g(5) = 0$, what point is on the graph of g? What is the corresponding x-intercept of the graph of g? (pp. 61–63)

Concepts and Vocabulary

7. The graph of every polynomial function is both _____ and _____.
8. If r is a real zero of even multiplicity of a function f, then the graph of f _____ (crosses/touches) the x-axis at r.
9. The graphs of power functions of the form $f(x) = x^n$, where n is an even integer, always contain the points _____, _____, and _____.
10. If r is a solution to the equation $f(x) = 0$, name three additional statements that can be made about f and r assuming f is a polynomial function.
11. The points at which a graph changes direction (from increasing to decreasing or decreasing to increasing) are called _____.
12. The graph of the function $f(x) = 3x^4 - x^3 + 5x^2 - 2x - 7$ will behave like the graph of _____ for large values of $|x|$.
13. If $f(x) = -2x^5 + x^3 - 5x^2 + 7$, then $\lim_{x \to -\infty} f(x) =$ _____ and $\lim_{x \to \infty} f(x) =$ _____.
14. Explain what the notation $\lim_{x \to \infty} f(x) = -\infty$ means.

Skill Building

In Problems 15–26, determine which functions are polynomial functions. For those that are, state the degree. For those that are not, tell why not.

15. $f(x) = 4x + x^3$
16. $f(x) = 5x^2 + 4x^4$
17. $g(x) = \dfrac{1 - x^2}{2}$
18. $h(x) = 3 - \dfrac{1}{2}x$
19. $f(x) = 1 - \dfrac{1}{x}$
20. $f(x) = x(x - 1)$
21. $g(x) = x^{3/2} - x^2 + 2$
22. $h(x) = \sqrt{x}(\sqrt{x} - 1)$
23. $F(x) = 5x^4 - \pi x^3 + \dfrac{1}{2}$
24. $F(x) = \dfrac{x^2 - 5}{x^3}$
25. $G(x) = 2(x - 1)^2(x^2 + 1)$
26. $G(x) = -3x^2(x + 2)^3$

In Problems 27–40, use transformations of the graph of $y = x^4$ or $y = x^5$ to graph each function.

27. $f(x) = (x + 1)^4$
28. $f(x) = (x - 2)^5$
29. $f(x) = x^5 - 3$
30. $f(x) = x^4 + 2$
31. $f(x) = \dfrac{1}{2}x^4$
32. $f(x) = 3x^5$
33. $f(x) = -x^5$
34. $f(x) = -x^4$
35. $f(x) = (x - 1)^5 + 2$
36. $f(x) = (x + 2)^4 - 3$
37. $f(x) = 2(x + 1)^4 + 1$
38. $f(x) = \dfrac{1}{2}(x - 1)^5 - 2$
39. $f(x) = 4 - (x - 2)^5$
40. $f(x) = 3 - (x + 2)^4$

In Problems 41–48, form a polynomial function whose real zeros and degree are given. Answers will vary depending on the choice of a leading coefficient.

41. Zeros: $-1, 1, 3$; degree 3
42. Zeros: $-2, 2, 3$; degree 3
43. Zeros: $-3, 0, 4$; degree 3
44. Zeros: $-4, 0, 2$; degree 3
45. Zeros: $-4, -1, 2, 3$; degree 4
46. Zeros: $-3, -1, 2, 5$; degree 4