Q1)

$$\begin{vmatrix} x & 1 & 1 \\ 2 & 1 & 4 \\ -1 & 4 & 3 \end{vmatrix}$$

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x6

$$x\begin{vmatrix} 1 & 4 \\ 4 & 3 \end{vmatrix} - 1\begin{vmatrix} 2 & 4 \\ -1 & 3 \end{vmatrix} + 1\begin{vmatrix} 2 & 1 \\ -1 & 4 \end{vmatrix} = 3$$

$x(3-16) - 1\times(6-(-4)) + 1(8-(-1)\times 1) = 3$

$-13x - 10 + 9 = 3$

$-13x - 1 = 3$

$x = \frac{-4}{13}$

Don't use variable x

Q2)

$$\begin{vmatrix} 1 & 3 & -2 \\ 6 & 1 & -5 \\ 8 & 2 & 3 \end{vmatrix}$$

$$= 1\begin{vmatrix} 1 & -5 \\ 2 & 3 \end{vmatrix} - 3\begin{vmatrix} 6 & -5 \\ 8 & 3 \end{vmatrix} + (-2)\begin{vmatrix} 6 & 1 \\ 8 & 2 \end{vmatrix}$$

$= 1(3-(-10)) - 3(18-(-40)) + (2)(12-8)$

$= \cancel{13 - 174 + 8}$

$= 13 - 174 - 8$

$= -169$

x4

Q3) $x^2 - 4xy + 3y^2 \rightarrow (1)$

$x^2 + xy = 12 \rightarrow (2)$

from (2)

$$y = \frac{12 - x^2}{x}$$

put into (1)

$$x^2 - 4x\left(\frac{12 - x^2}{x}\right) \times x + 3\left(\frac{12 - x^2}{x}\right)^2 = 0$$

?

$$x^4 - 4x^2(12 - x^2) + 3(12 - x^2)^2 = 0$$

$$8x^4 - 120x^2 + 432 = 0$$

let $x^2 = u$

$$8u^2 - 120u + 432 = 0$$

$$(u - 9)(u - 6) = 0$$

$$u = 9, 6$$

$x^2 = 9 \Rightarrow x = \pm 3$

$x^2 = 6 \Rightarrow x = \pm\sqrt{6}$

Line these up vertically

for $x = 3$, $y = \frac{12 - (3)^2}{3} = 1$

for $x = -3$, $y = \frac{12 - (-3)^2}{-3} = -1$

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for $x = \sqrt{6}$, $y = \frac{12 - (\sqrt{6})^2}{\sqrt{6}} = \sqrt{6}$

for $x = -\sqrt{6}$, $y = \frac{12 - (-\sqrt{6})^2}{-\sqrt{6}} = -\sqrt{6}$

X9

So, solution are

$(3, 1), (-3, -1), (\sqrt{6}, \sqrt{6}), (-\sqrt{6}, -\sqrt{6})$

Q4)

$$A = \begin{bmatrix} 1 & -1 & -1 \\ 0 & 2 & 1 \\ 3 & 2 & 0 \end{bmatrix}$$

+10

$$= \left[\begin{array}{ccc|ccc} 1 & -1 & -1 & 1 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 3 & 2 & 0 & 0 & 0 & 1 \end{array}\right]$$

$R_1 \leftrightarrow R_3$, $R_3 \leftarrow R_3 - \frac{1}{3} R_1$

$$\left[\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 1-\frac{1}{3}\times 3 & -\frac{5}{3} & -1 & 1 & 0 & -\frac{1}{3} \end{array}\right]$$

? 0

$R_3 \rightarrow R_3 + \frac{5}{6} R_2$

Unclear

$$\left[\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 1 & 0 & 1 & 0 \\ 0 & 0 & -\frac{1}{6} & 1 & \frac{5}{6} & -\frac{1}{3} \end{array}\right]$$

This should be separate matrices

$R_3 \to -6R_3$, $R_2 \to R_2 - R_3$

$$\left[\begin{array}{ccc|ccc} 3 & 2 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 6 & 6 & -2 \\ 0 & 0 & 1 & -6 & -5 & 2 \end{array}\right]$$

$R_2 \to \frac{1}{2}R_2$, $R_1 \to R_1 - 2R_2$

This does not follow that matrix ?

$$\left[\begin{array}{ccc|ccc} 3 & 0 & 0 & -6 & -6 & 3 \\ 0 & 1 & 0 & 3 & 3 & -1 \\ 0 & 0 & 1 & -6 & -5 & 2 \end{array}\right]$$

$R_1 \to \frac{1}{3}R_1$

$$\left[\begin{array}{ccc|ccc} 1 & 0 & 0 & -2 & -2 & 1 \\ 0 & 1 & 0 & 3 & 3 & -1 \\ 0 & 0 & 1 & -6 & -5 & 2 \end{array}\right]$$

So, $A^{-1} = \begin{bmatrix} -2 & -2 & +1 \\ 3 & 3 & -1 \\ -6 & -5 & 2 \end{bmatrix}$

Work above does not support answer!

So, $A^{-1} = \begin{pmatrix} -2 & -2 & 1 \\ 3 & 3 & -1 \\ -6 & -5 & 2 \end{pmatrix}$

Q5) a) $B + AD$

$$= \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 2 & -1 & 3 \\ 0 & 4 & -2 \end{bmatrix} \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 6 - 1 * 0 + 3 & -4 + (-1)\times(-1) + 6 \\ 0 + 0 - 2 & 0 - 4 - 4 \end{bmatrix}$$

$$= \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix} + \begin{bmatrix} 9 & 3 \\ -2 & -8 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 4 \\ 0 & -3 \end{bmatrix}$$

+8

B) D.B

$$= \begin{bmatrix} 3 & -2 \\ 0 & -1 \\ 1 & 2 \end{bmatrix} \begin{bmatrix} -3 & 1 \\ 2 & 5 \end{bmatrix}$$

$$= \begin{pmatrix} 3\times(-3)+(-2)\times 2 & 3\times 1+(-2)\times 5 \\ 0\times(-3)+(-1)\times 2 & 0\times 1+(-1)\times 5 \\ 1\times(-3)+2\times 2 & 1\times 1+2\times 5 \end{pmatrix}$$

$$= \begin{pmatrix} -13 & -7 \\ -2 & -5 \\ 1 & 11 \end{pmatrix}$$

+8

Q6) $S_n = \frac{(-1)^n}{(n+6)(n+3)}$

$S_1 = \frac{(-1)^1}{(1+6)(1+3)} = \frac{-1}{7\times 4} = \frac{-1}{28}$

$S_2 = \frac{(-1)^2}{(2+6)(2+3)} = \frac{1}{8\times 5} = \frac{1}{40}$

+3

$$S_3 = \frac{(-1)^3}{(3+6)(3+3)} = \frac{-1}{9\times 6} = \frac{-1}{54}$$

? First term $a_1 = \frac{-1}{28}$

Second term $a_2 = S_2 - S_1 = \frac{1}{40} - \left(\frac{-1}{28}\right) = \frac{17}{280}$

Third term $a_3 = S_3 - S_2 = \frac{-1}{54} - \frac{1}{40} = \frac{-47}{1080}$

Q7) $\frac{1}{2\ln 2} - \frac{1}{3\ln 3} + \frac{1}{4\ln 4} - \frac{1}{5\ln 5} + \ldots \frac{1}{100\ln 100}$

$$a_n = \frac{1}{(-1)^n n\ln(n)}, \quad n \geq 2 \text{ to } n \leq 100$$

$$S_n = \sum_{n=2}^{100} \left(\frac{1}{(-1)^n}\right) \times \frac{1}{n\ln(n)}$$

+4

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Q8) $\frac{-8}{x^3-27}$

$= \frac{-8}{(x-3)(x^2+3x+9)}$

+4

$\frac{-8}{(x-3)(x^2+3x+9)} = \frac{A}{x-3} + \frac{Bx+C}{x^2+3x+9}$

$\frac{-8}{(x-3)(x^2+3x+9)} = \frac{(x-3)(Bx+C)+(x^2+3x+9)A}{(x-3)(x^2+3x+9)}$

Equate both numerator

$-8 = (x-3)(Bx+C) + (x^2+3x+9)A$

$-8 = x^2(A+B) + x(3A-3B+C) + 9A-3C$

$A+B=0, \quad 3A-3B+C=0, \quad 9A-3C=-8$

$3A - 3(-A) + \left(\frac{9A+8}{3}\right) = 0$ work?

$A = \frac{-8}{27}, \quad B = \frac{8}{27}, \quad C = \frac{16}{9}$

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$$\frac{-8}{x^2-3x+9} = \frac{(-8/27)}{x-3} + \frac{(8x/27 + 16/9)}{x^2+3x+9}$$

Q9) $\sum_{k=4}^{17} k^3$

$= 4^3 + 5^3 + 6^3 + \_\_\_\_ 17^3$

$= (1^3 + 2^3 + 3^3 + \_\_\_ 17^3) - (1^3 + 2^3 + 3^3)$

$= \frac{1}{4} \times 17^2 \times (17+1)^2 - (1 + 8 + 27)$

$= 23409 - 36$ +4

$= 23373$

Q10) Given: $9 + 12 + 16 + 64/3 + \_\_\_$ +2

a) Each term is $\frac{4}{3}$ times of its previous term

So, series is Geometric with $r = \frac{4}{3}$

Show this as done on lecture

b) For a sum of n terms

$S_n = a_1\left(\frac{r^n - 1}{r - 1}\right)$?

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$$S_n = 9\frac{\left(\left(\frac{4}{3}\right)^n - 1\right)}{\frac{4}{3} - 1}$$

$$S_n = 27 \times \left(\left(\frac{4}{3}\right)^n - 1\right)$$

?

Here, n is not given

Q11) We have

$$a_1 = 3$$

$$a_3 = \frac{4}{3}$$

We know in geometric series, nth term is

$$a_n = a_1 r^{n-1}$$

$$\frac{4}{3} = 3$$

$$\frac{4}{3} = 3 \times r^2$$

$$r^2 = \frac{4}{9}$$

$$r = \pm\frac{2}{3}$$

$$a_5 = a_1 r^4 = 3 \times \left(\frac{2}{3}\right)^4 = \frac{16}{27}$$

+4

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Q(12) we have

$$a_1 = 6, \quad d = 1/2$$

$$a_n = a_1 + (n-1)d$$

$$a_n = 6 + (n-1) \times \frac{1}{2}$$

$$a_n = \frac{n}{2} + 6 - \frac{1}{2}$$

$$a_n = \frac{n+11}{2}$$

$$a_{53} = \frac{53+11}{2} = 32$$

1/6

Q(13) $\left(1 - \frac{1}{2x}\right)^5$

use $$(a+b)^n = \sum_{k=0}^{n} {}^nC_k \times a^{n-k} b^k$$

Here $a = 1$, $b = \left(\frac{-1}{2x}\right)$, $n = 5$

Expand using binomial theorem

$$= {}^5C_0 \times (1)^{5-0} \times \left(\frac{-1}{2x}\right)^0 + {}^5C_1 \times (1)^{5-1} \times \left(\frac{-1}{2x}\right)^1 + {}^5C_2 \times (1)^{5-2} \times \left(\frac{-1}{2x}\right)^2$$

$$+ {}^5C_3 (1)^{5-3} \times \left(\frac{-1}{2x}\right)^3 + {}^5C_4 \times (1)^{5-4} \times \left(\frac{-1}{2x}\right)^4 + {}^5C_5 \times (1)^0 \times \left(\frac{-1}{2x}\right)^5$$

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$$= 1 - 5 \times \frac{1}{2x} + 10 \times \left(\frac{-1}{2x}\right)^2 + 10 \times \left(\frac{-1}{2x}\right)^3 + 5 \times \left(\frac{-1}{2x}\right)^4 + 1 \times \left(\frac{-1}{2x}\right)^5$$

+7

$$= 1 - \frac{5}{2x} + \frac{5}{2x^2} - \frac{5}{4x^3} + \frac{5}{16x^4} - \frac{1}{32x^5}$$

Direction says as done in class video instruction!

Q(14) $2^3 + 4^3 + \dots (2n)^3 = 2n^2(n+1)^2$

for S_1

RHS $= 2 \times 1^2 \times (1+1)^2 = 8$ $\times 6$

LHS $= (2 \times 1)^3 = 8$

LHS $=$ RHS

so, it is true for $n = 1$, so S_1 hold

let $S_n : 2^3 + 4^3 + \dots (2n)^3 = 2n^2 (n+1)^2$ is true

we need to prove

$S_{n+1} : 2^3 + 4^3 + \dots (2n)^3 + (2(n+1))^3 = 2(n+1)^2(n+1+1)^2$

LHS

$2^3 + 4^3 + 6^3 + \dots (2n)^3 + (2(n+1))^3$

$= 2n^2(n+1)^2 + (8 \times (n+1)^3)$ 3 only $\longrightarrow$ from S_n

$= 2(n+1)^2 \times (n^2 + 4(n+1))$

$= 2(n+1)^2 \times (n+2)^2$

Hence, S_{n+1} is also true

So, LHS $=$ RHS

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