HW from Section 2.2



**49.** $f(x) = x^3 - x$

**50.** $f(x) = 2x^3 - x^2 + 1$

**51.** $f(x) = \dfrac{1}{x}$

**52.** $f(x) = \sqrt{x}$

**53. Restaurant Revenue** Nicole owns and operates two restaurants. The revenue of the first restaurant at time $t$ is $f(t)$ dollars, and the revenue of the second restaurant at time $t$ is $g(t)$ dollars. What does the function $F(t) = f(t) + g(t)$ represent?

**54. Birthrate of Endangered Species** The birthrate of an endangered species of whales in year $t$ is $f(t)$ whales/year. This species of whales is dying at the rate of $g(t)$ whales/year in year $t$. What does the function $F(t) = f(t) - g(t)$ represent?

**55. Value of an Investment** The number of IBM shares that Nancy owns is given by $f(t)$. The price per share of the stock of IBM at time $t$ is $g(t)$ dollars. What does the function $f(t)g(t)$ represent?

**56. Production Costs** The total cost incurred by time $t$ in the production of a certain commodity is $f(t)$ dollars. The number of products produced by time $t$ is $g(t)$ units. What does the function $f(t)/g(t)$ represent?

**57. Carbon Monoxide Pollution** The number of cars running in the business district of a town at time $t$ is given by $f(t)$. Carbon monoxide pollution coming from these cars is given by $g(x)$ parts per million, where $x$ is the number of cars being operated in the district. What does the function $g \circ f$ represent?

**58. Effect of Advertising on Revenue** The revenue of Leisure Travel is given by $f(x)$ dollars, where $x$ is the dollar amount spent by the company on advertising. The amount spent by Leisure at time $t$ on advertising is given by $g(t)$ dollars. What does the function $f \circ g$ represent?

**59. Cost of Producing DVDs** TMI, a manufacturer of blank DVDs, has a monthly fixed cost of $12,100 and a variable cost of $.60/disc. Find a function $C$ that gives the total cost incurred by TMI in the manufacture of $x$ discs/month.

**60. Business Email** The average number of email messages sent and received per corporate user per day in year $t$ between 2011 ($t = 1$) and 2015 ($t = 5$) is projected to be

$$f(t) = 3t + 69 \qquad (1 \le t \le 5)$$

The average number of spam emails sent per corporate user per day for the period under consideration is projected to be

$$g(t) = -0.2t + 13.8 \qquad (1 \le t \le 5)$$

**a.** Find a function, $h$, giving the projected average number of legitimate (non spam) emails sent and received per corporate user per day in year $t$.
Hint: $h(t) = f(t) - g(t)$

**b.** Compute $f(5)$, $g(5)$, and $h(5)$. Is $h(5) = f(5) - g(5)$?

*Source:* Radicati Group.

**61. Public Transportation Budget Deficit** According to the Massachusetts Bay Transportation Authority (MBTA), the projected cumulative MBTA budget deficit with a $160 million rescue package (in billions of dollars) is given by

$$D_1(t) = 0.0275t^2 + 0.081t + 0.07 \qquad (0 \le t \le 3)$$

and the budget deficit without the rescue package is given by

$$D_2(t) = 0.035t^2 + 0.21t + 0.24 \qquad (0 \le t \le 3)$$

Find the function $D = D_2 - D_1$, and interpret your result.

*Source:* MBTA Review.

**62. Motorcycle Deaths** Suppose the fatality rate (deaths per 100 million miles traveled) of motorcyclists is given by $g(x)$, where $x$ is the percentage of motorcyclists who wear helmets. Next, suppose the percentage of motorcyclists who wear helmets at time $t$ ($t$ measured in years) is $f(t)$, with $t = 0$ corresponding to 2000.

**a.** If $f(0) = 0.64$ and $g(0.64) = 26$, find $(g \circ f)(0)$ and interpret your result.

**b.** If $f(6) = 0.51$ and $g(0.51) = 42$, find $(g \circ f)(6)$ and interpret your result.

**c.** Comment on the results of parts (a) and (b).

*Source:* National Highway Traffic Safety Administration.

**63. Fighting Crime** Suppose the reported serious crimes (crimes that include homicide, rape, robbery, aggravated assault, burglary, and car theft) that end in arrests or in the identification of suspects is $g(x)$ percent, where $x$ denotes the total number of detectives. Next, suppose the total number of detectives in year $t$ is $f(t)$, with $t = 0$ corresponding to 2001.

**a.** If $f(1) = 406$ and $g(406) = 23$, find $(g \circ f)(1)$ and interpret your result.

**b.** If $f(6) = 326$ and $g(326) = 18$, find $(g \circ f)(6)$ and interpret your result.

**c.** Comment on the results of parts (a) and (b).

*Source:* Boston Police Department.

**64. Profit from Sale of Smartphones** Apollo manufactures smartphones at a variable cost of

$$V(x) = 0.000003x^3 - 0.03x^2 + 200x$$

dollars, where $x$ denotes the number of units manufactured per month. The monthly fixed cost attributable to the division that produces them is $100,000. The total revenue realized by Apollo from the sale of $x$ smartphones is given by the total revenue function

$$R(x) = -0.1x^2 + 500x \qquad (0 \le x \le 5000)$$

where $R(x)$ is measured in dollars.

**a.** Find a function $C$ that gives the total cost incurred by the manufacture of $x$ smartphones.

**b.** Find the total profit function.

**c.** What is the profit when 1500 units are produced and sold each month?

HW from Section 2.3



**2. MARKET EQUILIBRIUM** The demand function for Mrs. Baker's cookies is given by

$$d(x) = -\frac{2}{15}x + 4$$

where $d(x)$ is the wholesale price in dollars per pound and $x$ is the quantity demanded each week, measured in thousands of pounds. The supply function for the cookies is given by

$$s(x) = \frac{1}{75}x^2 + \frac{1}{10}x + \frac{3}{2}$$

where $s(x)$ is the wholesale price in dollars per pound and $x$ is the quantity, in thousands of pounds, that will be made available in the market each week by the supplier.

a. Sketch the graphs of the functions $d$ and $s$.

b. Find the equilibrium quantity and price.

*Solutions to Self-Check Exercises 2.3 can be found on page 95.*

## 2.3 Concept Questions

1. Describe mathematical modeling in your own words.
2. Define (a) a polynomial function and (b) a rational function. Give an example of each.
3. a. What is a demand function? A supply function?

   b. What is market equilibrium? Describe how you would go about finding the equilibrium quantity and equilibrium price given the demand and supply equations associated with a commodity.

## 2.3 Exercises

In Exercises 1–8, determine whether the equation defines $y$ as a linear function of $x$. If so, write it in the form $y = mx + b$.

1. $2x + 3y = 6$
2. $-2x + 4y = 7$
3. $x = 2y - 4$
4. $2x = 3y + 8$
5. $2x - 4y + 9 = 0$
6. $3x - 6y + 7 = 0$
7. $2x^2 - 8y + 4 = 0$
8. $3\sqrt{x} + 4y = 0$

In Exercises 9–14, determine whether the function is a polynomial function, a rational function, or some other function. State the degree of each polynomial function.

9. $f(x) = 3x^6 - 2x^2 + 1$
10. $f(x) = \dfrac{x^2 - 9}{x - 3}$
11. $G(x) = 2(x^2 - 3)^3$
12. $H(x) = 2x^{-3} + 5x^{-2} + 6$
13. $f(t) = 2t^2 + 3\sqrt{t}$
14. $f(r) = \dfrac{6r}{r^3 - 8}$
15. Find the constants $m$ and $b$ in the linear function $f(x) = mx + b$ such that $f(0) = 2$ and $f(3) = -1$.
16. Find the constants $m$ and $b$ in the linear function $f(x) = mx + b$ such that $f(2) = 4$ and the straight line represented by $f$ has slope $-1$.
17. A manufacturer has a monthly fixed cost of \$40,000 and a production cost of \$8 for each unit produced. The product sells for \$12/unit.
    a. What is the cost function?
    b. What is the revenue function?
    c. What is the profit function?
    d. Compute the profit (loss) corresponding to production levels of 8000 and 12,000 units.
18. A manufacturer has a monthly fixed cost of \$100,000 and a production cost of \$14 for each unit produced. The product sells for \$20/unit.
    a. What is the cost function?
    b. What is the revenue function?
    c. What is the profit function?
    d. Compute the profit (loss) corresponding to production levels of 12,000 and 20,000 units.
19. **DISPOSABLE INCOME** Economists define the *disposable annual income* for an individual by the equation $D = (1 - r)T$, where $T$ is the individual's total income and $r$ is the net rate at which he or she is taxed. What is the disposable income for an individual whose income is \$60,000 and whose net tax rate is 28%?
20. **CHILDREN'S DRUG DOSAGES** A method sometimes used by pediatricians to calculate the dosage of medicine for children is based on the child's surface area. If $a$ denotes the adult dosage (in milligrams) and $S$ is the surface area of the child (in square meters), then the child's dosage is given by

$$D(S) = \frac{Sa}{1.7}$$

If the adult dose of a substance is 500 mg, how much should a child whose surface area is 0.4 $m^2$ receive?

HW from Section 2.3



**21. Cowling's Rule** Cowling's Rule is a method for calculating pediatric drug dosages. If $a$ denotes the adult dosage (in milligrams) and $t$ is the age of the child (in years), then the child's dosage is given by

$$D(t) = \left(\frac{t+1}{24}\right)a$$

If the adult dose of a substance is 500 mg, how much should a 4-year-old child receive?

**22. Drinking and Driving Among High School Students** The percentage of high school students who drink and drive stood at 17.5% at the beginning of 2001 and declined linearly to 10.3% at the beginning of 2011.

a. Find a linear function $f(t)$ giving the percentage of high school students who drink and drive in year $t$, where $t = 0$ corresponds to the beginning of 2001.
b. At what rate was the percentage of students who drink and drive dropping between 2001 and 2011?
c. If the trend continues, what will the percentage of high school students who drink and drive be at the beginning of 2014?

*Source:* Centers for Disease Control and Prevention.

**23. California Emissions Caps** The California emissions cap is set at 400 million metric tons of carbon dioxide equivalent in 2015 and is expected to drop by 13.2 million metric tons of carbon dioxide equivalent per year through 2020.

a. Find a linear function $f$ giving the California emissions cap in year $t$, where $t = 0$ corresponds to 2015.
b. If the same rate of decline of emissions cap is adopted through 2017, what will the emissions cap be in 2017?

*Source:* California Air Resource Board.

**24. U.S. Airplane Passenger Projections** A report issued by the U.S. Department of Transportation in 2012 predicted that the number of passengers boarding planes in the United States would grow steadily from the current 0.7 billion boardings/year to 1.2 billion boardings/year in 2032.

a. Find a linear function $f$ giving the projected boardings (in billions) in year $t$, where $t = 0$ corresponds to 2012.
b. What is the projected annual rate of growth of boardings between 2012 and 2032?
c. How many boardings per year are projected in 2022?

*Source:* U.S. Department of Transportation.

**25. Bounced-Check Charges** Overdraft fees have become an important part of a bank's total fee income. The following table gives the bank revenue from overdraft fees (in billions of dollars) from 2004 through 2009.

| Year, $t$ | 0 | 1 | 2 | 3 | 4 | 5 |
|---|---|---|---|---|---|---|
| Revenue, $y$ | 27.5 | 29 | 31 | 34 | 36 | 38 |

where $t$ is measured in years, with $t = 0$ corresponding to 2004. A mathematical model giving the approximate projected bank revenue from overdraft fees over the period under consideration is given by

$$f(t) = 2.19t + 27.12 \qquad (0 \le t \le 5)$$

a. Plot the six data points and sketch the graph of the function $f$ on the same set of axes.
b. Assuming that the projection holds and the trend continues, what was the projected bank revenue from overdraft fees in 2010 ($t = 6$)?
c. What was the rate of increase of the bank revenue from overdraft fees over the period from 2004 through 2009?

*Source: New York Times.*

**26. Worker Efficiency** An efficiency study showed that the average worker at Delphi Electronics assembled cordless telephones at the rate of

$$f(t) = -\frac{3}{2}t^2 + 6t + 10 \qquad (0 \le t \le 4)$$

phones/hr, $t$ hr after starting work during the morning shift. At what rate does the average worker assemble telephones 2 hr after starting work?

**27. Effect of Advertising on Sales** The quarterly profit of Cunningham Realty depends on the amount of money $x$ spent on advertising per quarter according to the rule

$$P(x) = -\frac{1}{8}x^2 + 7x + 30 \qquad (0 \le x \le 50)$$

where $P(x)$ and $x$ are measured in thousands of dollars. What is Cunningham's profit when its quarterly advertising budget is \$28,000?

**28. Baby Boomers and Social Security Benefits** Aging baby boomers will put a strain on Social Security benefits unless Congress takes action. The Social Security benefits to be paid out from 2010 through 2040 are projected to be

$$S(t) = 0.1375t^2 + 0.5185t + 0.72 \qquad (0 \le t \le 3)$$

where $S(t)$ is measured in trillions of dollars and $t$ is measured in decades, with $t = 0$ corresponding to 2010.

a. What was the amount of Social Security benefits paid out in 2010?
b. What is the amount of Social Security benefits projected to be paid out in 2040?

*Source:* Social Security and Medicare Trustees' 2010 report.

**29. Mobile Device Usage** The average time U.S. adults spent per day on mobile devices (in minutes) for the years 2009 through 2012 is approximated by

$$f(t) = 2.25t^2 + 13.41t + 21.76 \qquad (0 \le t \le 3)$$

where $t = 0$ corresponds to 2009.

a. What was the average time U.S. adults spent per day on mobile devices in 2009?
b. If the trend continued through 2013, what was the average time U.S. adults spent per day on mobile devices in 2013?

*Source:* eMarketer.

HW from Section 2.3



**30. U.S. GDP** The gross domestic product (GDP) of the United States, in trillions of dollars, from 2011 through 2015 is approximately

$$G(t) = 0.064t^2 + 0.473t + 15.0 \qquad (0 \le t \le 4)$$

where $t$ is measured in years, with $t = 0$ corresponding to 2011. In constructing this model, the government used actual GDP figures from 2011 and 2012 and estimates for the years 2013 through 2015.

**a.** What was the U.S. GDP in 2011?

**b.** What is the predicted U.S. GDP for 2015?

*Source:* World Bank.

**31. Sub-Saharan African GDP** The real GDP per capita of sub-Saharan Africa (in 2009 U.S. dollars) from 1990 through 2030 is projected to be

$$f(t) = 1.86251t^2 - 28.08043t + 884 \qquad (0 \le t \le 40)$$

where $t$ is measured in years, with $t = 0$ corresponding to 1990.

**a.** What was the real GDP per capita of sub-Saharan Africa in 2000?

**b.** Assuming that the projection holds true, what will be the GDP per capita of sub-Saharan Africa in 2030?

*Source:* IMF.

**32. Instant Messaging Accounts** The number of enterprise instant messaging (IM) accounts is projected to grow according to the function

$$N(t) = 2.96t^2 + 11.37t + 59.7 \qquad (0 \le t \le 5)$$

where $N(t)$ is measured in millions and $t$ in years, with $t = 0$ corresponding to 2006.

**a.** How many enterprise IM accounts were there in 2006?

**b.** What was the expected number of enterprise IM accounts in 2010?

*Source:* The Radical Group.

**33. Solar Power** More and more businesses and homeowners are installing solar panels on their roofs to draw energy from the sun's rays. According to the U.S. Department of Energy, the solar cell kilowatt-hour use in the United States (in millions) is projected to be

$$S(t) = 0.73t^2 + 15.8t + 2.7 \qquad (0 \le t \le 8)$$

in year $t$, with $t = 0$ corresponding to 2000. What was the number of projected solar cell kilowatt-hours used in the United States for 2006? For 2008?

*Source:* U.S. Department of Energy.

**34. U.S. Public Debt** The U.S. public debt (the outstanding amount owed by the federal government of the United States from the issue of securities by the U.S. Treasury and other federal government agencies) for the years 2005 through 2011 is modeled by the function

$$f(t) = -0.03817t^3 + 0.4571t^2 - 0.1976t + 8.246 \qquad (0 \le t \le 7)$$

where $f(t)$ is measured in trillions of dollars and $t$ is measured in years, with $t = 0$ corresponding to 2005. What was the U.S. public debt in 2005? In 2008?

*Source:* U.S. Department of the Treasury.

**35. Workers' Expectations** The percentage of workers who expect to work past age 65 has more than tripled in 30 years. The function

$$f(t) = 0.004545t^3 - 0.1113t^2 + 1.385t + 11 \qquad (0 \le t \le 22)$$

gives an approximation of the percentage of workers who expect to work past age 65 in year $t$, where $t$ is measured in years, with $t = 0$ corresponding to 1991. What was the percentage of workers who expected to work past age 65 in 1991? In 2013?

*Source:* PBS News.

**36. Aging Drivers** The number of fatalities due to car crashes, based on the number of miles driven, begins to climb after the driver is past age 65. Aside from declining ability as one ages, the older driver is more fragile. The number of fatalities per 100 million vehicle miles driven is approximately

$$N(x) = 0.0336x^3 - 0.118x^2 + 0.215x + 0.7 \qquad (0 \le x \le 7)$$

where $x$ denotes the age group of drivers, with $x = 0$ corresponding to those aged 50–54, $x = 1$ corresponding to those aged 55–59, $x = 2$ corresponding to those aged 60–64, . . . , and $x = 7$ corresponding to those aged 85–89. What is the fatality rate per 100 million vehicle miles driven for an average driver in the 50–54 age group? In the 85–89 age group?

*Source:* U.S. Department of Transportation.

**37. Total Global Mobile Data Traffic** In a 2009 report, equipment maker Cisco forecast the total global mobile data traffic to be

$$f(t) = 0.021t^3 + 0.015t^2 + 0.12t + 0.06 \qquad (0 \le t \le 5)$$

million terabytes/month in year $t$, where $t = 0$ corresponds to 2009.

**a.** What was the total global mobile data traffic in 2009?

**b.** According to Cisco, what will the total global mobile data traffic be in 2014?

*Source:* Cisco.

**38. Leveraged Return** The return on assets using borrowed money, called leveraged return, is given by

$$L = \frac{Y - (1 - D)R}{D}$$

where $Y$ is the return of the asset, $R$ is the cost of borrowed money, and $D$ is the percentage of money the investor must put down to secure the loan. Leanne wants to buy a bond returning 6%/year using borrowed money. The bank required her to pay 20% in cash with the

HW from section 2.3



remaining 80% borrowed at an interest rate of 5%/year. What is the leveraged return?
*Source:* Scientific American.

**39. Online Video Viewers** As broadband Internet grows more popular, video services such as YouTube will continue to expand. The number of online video viewers (in millions) is projected to grow according to the rule

$$N(t) = 52t^{0.531} \qquad (1 \le t \le 10)$$

where $t = 1$ corresponds to 2003.
**a.** Sketch the graph of $N$.
**b.** How many online video viewers were there in 2012?
*Source:* eMarketer.com.

**40. Infant Mortality Rates in Massachusetts** The deaths of children younger than 1 year old per 1000 live births is modeled by the function

$$R(t) = 162.8t^{-3.025} \qquad (1 \le t \le 3)$$

where $t$ is measured in 50-year intervals, with $t = 1$ corresponding to 1900.
**a.** Find $R(1)$, $R(2)$, and $R(3)$ and use your result to sketch the graph of the function $R$ over the domain $[1, 3]$.
**b.** What was the infant mortality rate in 1900? In 1950? In 2000?
*Source:* Massachusetts Department of Public Health.

**41. Outsourcing of Jobs** According to a study conducted in 2003, the total number of U.S. jobs (in millions) that are projected to leave the country by year $t$, where $t = 0$ corresponds to 2000, is

$$N(t) = 0.0018425(t + 5)^{2.5} \qquad (0 \le t \le 15)$$

What was the projected number of outsourced jobs for 2005 ($t = 5$)? For 2013 ($t = 13$)?
*Source:* Forrester Research.

**42. Chip Sales** The worldwide sales of flash memory chip (in billions of dollars) is approximated by

$$S(t) = 4.3(t + 2)^{0.94} \qquad (0 \le t \le 6)$$

where $t$ is measured in years, with $t = 0$ corresponding to 2002. Flash chips are used in cell phones, digital cameras, and other products.
**a.** What were the worldwide flash memory chip sales in 2002?
**b.** What were the estimated sales for 2010?
*Source:* Web-Feet Research, Inc.

**43. U.S. Health-Care Costs** The U.S. health-care costs per capita (in dollars) from 2001 through 2011 can be approximated by the linear function $f(t) = at + b$, where $t$ is measured in years, with $t = 1$ corresponding to 2001, and $a$ and $b$ are constants. The costs per capita in 2001 and 2011 were \$5240 and \$8680, respectively.
**a.** Find $a$ and $b$.
**b.** Use the model obtained in part (a) to find the approximate per capita costs for 2005.
*Source:* Centers for Medicare and Medicaid Services.

**44. Reaction of a Frog to a Drug** Experiments conducted by A. J. Clark suggest that the response $R(x)$ of a frog's heart muscle to the injection of $x$ units of acetylcholine (as a percent of the maximum possible effect of the drug) may be approximated by the rational function

$$R(x) = \frac{100x}{b + x} \qquad (x \ge 0)$$

where $b$ is a positive constant that depends on the particular frog.
**a.** If a concentration of 40 units of acetylcholine produces a response of 50% for a certain frog, find the "response function" for this frog.
**b.** Using the model found in part (a), find the response of the frog's heart muscle when 60 units of acetylcholine are administered.

**45. Digital Versus Film Cameras** The sales of digital cameras (in millions of units) in year $t$ are given by the function

$$f(t) = 3.05t + 6.85 \qquad (0 \le t \le 3)$$

where $t = 0$ corresponds to 2001. Over that same period, the sales of film cameras (in millions of units) are given by

$$g(t) = -1.85t + 16.58 \qquad (0 \le t \le 3)$$

**a.** Show that more film cameras than digital cameras were sold in 2001.
**b.** When did the sales of digital cameras first exceed those of film cameras?
*Source: Popular Science.*

**46. Walking Versus Running** The oxygen consumption (in milliliter per pound per minute) for a person walking at $x$ mph is approximated by the function

$$f(x) = \frac{5}{3}x^2 + \frac{5}{3}x + 10 \qquad (0 \le x \le 9)$$

whereas the oxygen consumption for a runner at $x$ mph is approximated by the function

$$g(x) = 11x + 10 \qquad (4 \le x \le 9)$$

**a.** Sketch the graphs of $f$ and $g$.
**b.** At what speed is the oxygen consumption the same for a walker as it is for a runner? What is the level of oxygen consumption at that speed?
**c.** What happens to the oxygen consumption of the walker and the runner at speeds beyond that found in part (b)?
*Source:* William McArdley, Frank Katch, and Victor Katch, *Exercise Physiology.*

**47. Cricket Chirping and Temperature** Entomologists have discovered that a linear relationship exists between the