HW 7

(1) Consider two functions $\psi_n(x)$ and $\psi_m(x)$ for a particle in a 1-D box. Bound state eigenfunctions (wave functions) of the Schrödinger equation are orthogonal to each other. Demonstrate this general rule for $\psi_n(x)$ and $\psi_m(x)$, i.e. show that

$$\int_{-\infty}^{\infty} \psi_n^*(x)\, \psi_m(x)\, dx = 0 \quad \text{if } n \neq m$$

(2) a) By direct substitution into the Schrödinger equation, show that the wave function

$$\psi(x) = C\left(\alpha^{3/2} x^3 - \frac{3}{4}\sqrt{\alpha}\, x\right) e^{-\alpha x^2}$$

is a solution of the 1-D harmonic oscillator. Here $\alpha = \frac{m\omega}{2\hbar}$ and C is a normalization constant.

b) From part a), calculate the corresponding energy for $\psi(x)$

(3) a) Calculate the ground state energy of an electron that is confined to be in a cube of volume $2.7 \cdot 10^{-29}$ m^3.
Find your result numerically in eV unit.

b) Calculate the maximum value of the wavelength of a photon that can be absorbed by the electron in part a). (Find your result numerically.)

(4) a) Calculate Δx and Δp_x for the ground state of a particle in a 1-D harmonic oscillator.

b) Show that the uncertainty principle is satisfied, i.e.

$$\Delta x \cdot \Delta p_x \geq \frac{\hbar}{2}$$

[Note: $$\int_{-\infty}^{\infty} y^{2k} e^{-\alpha y^2} dy = (-1)^k \frac{\partial^k}{\partial \alpha^k}\left(\frac{\sqrt{\pi}}{\sqrt{\alpha}}\right)$$

$$\int_{-\infty}^{+\infty} y^{2k+1} e^{-\alpha y^2} dy = 0$$

for $k = 0, 1, \ldots$

(5) Consider the following wave function for a particle in a 1-D box with $V(x)=0$ between $x=0$ and $x=L$, $V=+\infty$ otherwise:

$$\psi(x) = \begin{cases} C \sin^2\left(\frac{\pi}{L}x\right) & \text{for } 0 \leq x \leq L \\ 0 & \text{otherwise} \end{cases}$$

a) Normalize $\psi(x)$, i.e. find C

b) Calculate the probability of finding a particle (mass m) at the ground state.

c) Calculate the probability of finding a particle at the first excited state.