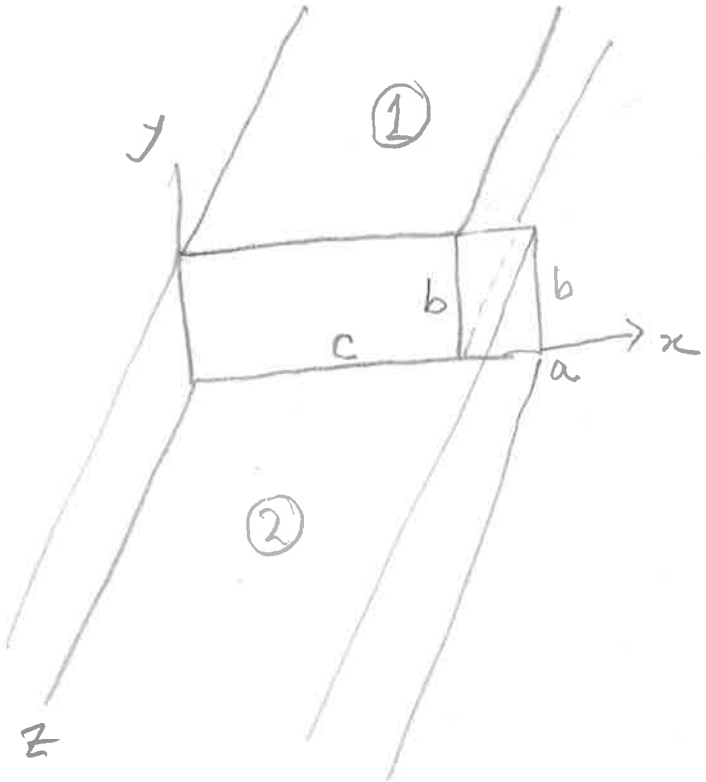

Incident Field in waveguide 1

$$E_y^i = \sin\frac{\pi x}{c} e^{-\gamma_{10}^1 z}$$

$$H_x^i = -\frac{1}{Z_{10}^1} \sin\frac{\pi x}{c} e^{-\gamma_{10}^1 z}$$

Reflected field

$$E_y^r = \rho_1 \sin\frac{\pi x}{c} e^{\gamma_{10}^1 z} + \rho_2 \sin\frac{2\pi x}{c} e^{\gamma_{20}^1 z} + \rho_3 \sin\frac{3\pi x}{c} e^{\gamma_{30}^1 z}$$

$$H_x^r = \frac{\rho_1}{Z_{10}^1} \sin\frac{\pi x}{c} e^{\gamma_{10}^1 z} + \frac{\rho_2}{Z_{20}^1} \sin\frac{2\pi x}{c} e^{\gamma_{20}^1 z}$$

$$+ \frac{\rho_3}{Z_{30}^1} \sin\frac{3\pi x}{c} e^{\gamma_{30}^1 z}$$

Transmitted Fields

$$E_y^t = \tau_1 \sin\frac{\pi x}{a} e^{-\gamma_{10}^2 z} + \tau_2 \sin\frac{2\pi x}{a} e^{-\gamma_{20}^2 z}$$

$$+ \tau_3 \sin\frac{3\pi x}{a} e^{-\gamma_{30}^2 z}$$

$$H_y^t = -\frac{\tau_1}{Z_{10}^2} \sin\frac{\pi x}{a} e^{-\gamma_{10}^2 z} - \frac{\tau_2}{Z_{20}^2} \sin\frac{2\pi x}{a} e^{-\gamma_{20}^2 z}$$

$$+ \tau_3 \sin\frac{3\pi x}{a} e^{-\gamma_{30}^2 z}$$

a.) Minimize the function (W.r.t. $\rho_1 .. \rho_3, \tau_1 .. \tau_3$)

$$J(\rho_1, \rho_2, \rho_3, \tau_1, \tau_2, \tau_3) \equiv J(X)$$

$$X = \{\rho_1, \rho_2, \rho_3, \tau_1, \tau_2, \tau_3\}$$

Where

$$J(X) = \int_0^c \left\{ \left[E^1(x) - E^2(x)\right]^2 + \eta_0^2 \left[H^1(x) - H^2(x)\right]^2 \right\} dx.$$

The fields are given at the interface $z = 0$, as

$$E^1(x) = \left[E^i(x,z) + E^r(x,z)\right]_{z=0}$$

$$H^1(x) = \left[H^i(x,z) + H^r(x,z)\right]_{z=0}$$

$$E^2(x) = E^t(x,z)\Big|_{z=0}$$

$$H^2(x) = H^t(x,z)\Big|_{z=0}$$

b.) Determine the reflection and transmission coefficients.