

## 2. Matlab problem:

Consider a continuous signal  $x(t) = t^2 - 6t + 5$  with  $t \in [0, 5]$ . Construct a discrete signal  $x[n]$  by sifting the continuous signal  $x(t)$  at discrete time points  $t = 0, 1, 2, 3, 4, 5$ . From the discrete signal  $x[n]$ , build an upsampled discrete signal  $g[n] = x[\frac{n}{10}]$  using Taylor expansion. Plot the signals  $x(t)$ ,  $x[n]$ , and  $g[n]$ . Write your observation about the accuracy of  $x[n]$  and  $g[n]$  with respect to the original continuous signal  $x(t)$ .

**Hint:** To evaluate  $x[\frac{n}{10}]$ , observe first that  $x[n]$  is a discrete signal and we know  $x$  at discrete integer levels of  $n$ . Let  $\frac{n}{10} \in [n_l, n_r]$  where  $n_l$  and  $n_r$  are the left and right integer boundaries of  $\frac{n}{10}$ . Now, with  $x[n_l]$  as an anchor, we can estimate  $x[\frac{n}{10}]$  using Taylor expansion as:

$$\begin{aligned}x[\frac{n}{10}] &= x[n_l + (\frac{n}{10} - n_l)] \\ &\approx x[n_l] + (\frac{n}{10} - n_l) \frac{x[n_r] - x[n_l]}{n_r - n_l}\end{aligned}$$

1.51. In this project the concept of *dual-tone multi-frequency (DTMF)* signaling will be explored. As the name implies, DTMF signals are mixtures of two sinusoids at distinct frequencies. They are used in communications over analog telephone lines. A particular version of DTMF signaling is utilized in dialing a number with push-button telephone handsets, a scheme known as *touch tone dialing*. When the caller dials a number, the DTMF generator produces a dual-tone signal for each digit dialed. The synthesized signal is in the form

$$x_k(t) = \sin(2\pi f_1 t) + \sin(2\pi f_2 t), \quad 0 \leq t \leq T_d$$

Frequency assignments for the digits on a telephone keypad are shown in Fig. P.1.51.

$f_2 \backslash f_1$	1209 Hz	1336 Hz	1477 Hz
697 Hz	1	2	3
770 Hz	4	5	6
852 Hz	7	8	9
941 Hz	*	0	#

Figure P. 1.51

The goal of this project is to develop a DTMF synthesizer function for MATLAB.

- a. Develop a function named `ss_dtmf1(...)` to produce the signal for one digit. The syntax of the function should be

$$x = \text{ss\_dtmf1}(n, t)$$

The first argument "n" is the digit for which the DTMF signal is to be generated. Let values  $n = 0$  through  $n = 9$  represent the corresponding keys on the keypad. Map the remaining two keys "\*" and "#" to values  $n = 10$  and  $n = 11$  respectively. Finally, the value  $n = 12$  should represent a pause, that is, a silent period. The vector "t" contains the time instants at which the DTMF signal  $x(t)$  is evaluated and returned in vector "x".

- b. Develop a function named `ss_dtmf(...)` with the syntax

$$x = \text{ss\_dtmf}(\text{number}, dt, nd, sp)$$

The arguments for the function `ss_dtmf(...)` are defined as follows:

**number:** The phone number to be dialed, entered as a vector. For example, to dial the number 555-1212, the vector "number" would be entered as

$$\text{number} = [5, 5, 5, 1, 2, 1, 2]$$

**dt:** The time increment  $\Delta t$  to be used in computing the amplitudes of the DTMF signal.

nd: Parameter to control the duration of the DTMF signal for each digit. The duration of each digit should be

$$T_d = n_d \Delta t$$

np: Parameter to control the duration of pause between consecutive digits. The duration of pause should be

$$T_p = n_p \Delta t$$

The function `ss_dtmf(...)` should use the function `ss_dtmf1(...)` to produce the signals  $x_k(t)$  for each digit (and the pauses between digits) and append them together to create the signal  $x(t)$ .

- c. Write a script to test the function `ss_dtmf(...)` with the number 555-1212. Use a time-increment of 125 microseconds corresponding to 8000 values per second. The duration of each digit should be 200 milliseconds with 80 millisecond pauses between digits.
- d. Play back the resulting signal  $x(t)$  using the `sound(...)` function.