

2) In the first class we investigated a problematic reconfiguration that turned a square array with area 64 into a rectangular shape with area 65 (please see pages 12-15 in the Powerpoint slides to remind yourself of the details of the cuts to make - you can find all the Powerpoint slides we use in the "Handouts and Slides" tab on the left). For this problem you might want to cut out your own Fibonacci Square puzzle pieces again if you didn't save them from class and investigate this strange phenomenon.

Come up with an explanation as to why it is that you can rearrange an area 64 square into an area 65 rectangle (hint, no, the extra square didn't magically appear!) - in fact, of course, it's not there at all - there must be some blank space somewhere...) The hints that were made in class about slopes being off from each other, i.e. that angles were not matching up is a good starting point, but it's not enough to just say that there's a problem with the angles - your explanation needs to cover specifically where the extra space is - what shape is it in?

Your explanation doesn't need to be lengthy, but please write it as if you're writing it for an average high school student - so that it includes enough information to thoroughly convince the student as to what's going on.

3) Next, explain how the reconfiguration yielding just 63 squares works - where did the square go? (you can see this configuration in Day1 Powerpoint slides on page 15)

4) Some of you might have picked up on the fact that there is a connection between the area puzzle in the last problem and the Fibonacci sequence (given that the numbers 3, 5, 8, and 13 all showed up as the various lengths involved in the puzzle). For this problem please create your own Fibonacci square to show off another erroneous reconfiguration, but this time instead of working with an 8 by 8 square to start with,

start with a larger 13 by 13 square - either use graph paper or print out this 13 by

13 grid [GridPaper13by13.pdf](#). Part of the problem is for you to figure out how to generalize the cuts

that we did for the 8 by 8 example. To do this you'll do the exact same thing we did before - you'll start by cutting the 13 by 13 square into two rectangles - but you'll need to think about where to make the cut (think about the numbers in the Fibonacci sequence). Next you'll cut the smaller rectangle into two triangles, and then you'll cut the larger rectangle into two trapezoids, again as we did in class, but now with larger dimensions that you need to figure out. Next, regroup the four pieces as we did with the smaller set in class and see what happens. Write down a short explanation about what's going on in this example as compared to the previous problem (you don't need to repeat the whole explanation you'd written in the last problem - just explain what's going on that's different in this new reconfiguration).

5) Euclid begins his classic book "The Elements" with a list of definitions of basic geometric terms along

with a list of axioms (before he begins the formal set of propositions - the constructions/theorems). During our first class we started the process of doing a similar thing - beginning with a list of geometric terms, but we didn't get a chance to actually define these terms yet. So now, please come up with your own definitions for the following basic geometric terms - the point here is that I'd like you to each try doing this *on your own with your own words*. Please do not look at the definitions Euclid gives before doing this yourself! In doing this try to use your own explanations as much as possible instead of just trying to remember definitions you might have learned at some point (and note that there are no perfect answers here - translating such mathematical concepts into English is an inexact science - simply do your best!): (1) point (2) line (3) angle (4) perpendicular (5) boundary (of a figure) (6) circle (7) diameter (8) square.

Picture what it must have been like to have one of the first to have to codify such a set of terms - this kind of exercise is not that straightforward as it's easy to fall into a trap of providing definitions that depend on other definitions that you might not have described yet (for instance, if after defining a point, one writes down a pseudodefinition for a line as "a collection of points that are connected in both directions" then this begs the questions what does "connected" mean and what does "in both directions" mean?)

This question might seem simple, but it is actually quite tough to do well - (and in some sense it's essentially impossible!) - just do your best - the point is to try to duplicate the sense of starting from scratch that Euclid (and others) faced. This marks one of the major differences between Geometry and Arithmetic, as I mentioned in class. Arithmetic deals with numbers - these are relatively to describe. Geometry, on the other hand, deals with a whole range of objects. Good luck!

6) Finally, it's time for Euclid! Note that there's nothing to turn in for these last several problems - and please write up your definition answers to the last question before opening up Euclid! Please do your best to get hold of a copy of the second edition of Sir Thomas Heath's translation of Books I and II of Euclid's Elements. If you're not able to get a copy of the book by next class, that's okay - I've scanned in the relevant parts for you to use for now if you don't have a copy of the book yet in the following file: [Euclid Definitions and Notes.pdf](#) Now try to study (read, reread and try to digest!) the beginning of the Elements, starting

with the 23 definitions, 5 postulates and 5 "common notions" on pages 153-155. Next do your best to read through the Notes on the first several definitions (at least 1 through 3, as well as the notes for definition 23). These Notes are pretty dense reading, so just do your best to get the general flavor of the arguments and issues raised along with noticing how the attitudes towards the definitions has developed over the past several thousand years.

We will continue building from these first principle concepts next week. Being armed with knowledge of these basic terms will serve you well as deal with the first several propositions in Euclid.

7) Continuing on with Euclid - again, nothing to turn in for this problem - time to get a bit of background context about the work and the times. In Heath's version of *The Elements* please read the Preface (the original preface, not the one marked "preface to the second edition"). Next read Chapter 1 on what little is known about Euclid. Finally in Chapter IX please read pages 114 through 124 on the set up of the Elements (and why its title makes sense). Here's a copy [The Elements Preface and History.pdf](#) of these sections if you haven't been able to get your own copy of the book yet. Note that this reading is really dense(!) so just do your best to pick your way through it and learn what you can. I might recommend writing down a brief summary as you read, pulling out key points from the reading. Please don't bother memorizing all the different characters (such as Hypsicles and Apollonius) that are mentioned in the text (although a number of them such as Aristotle and Archimedes should at least be familiar names). There won't be any quizzes on this information, but knowing at least a bit of the context surrounding *The Elements* is useful for understanding why it was considered such an important text for so many thousands of years. Note there's nothing to turn in for this problem.

8) Last but not least, please buy (if you don't already own) a small compass (as in a geometrical compass - available at many office supply stores or CVS, for example, for under \$2) and a cheap ruler (to use as a straightedge during class). Please try to remember to bring these to class each week (or have them on hand if you're watching the class) as we will often try out some constructions together during class.

Bonus question - we figured out equations for two of the means shown on page 20 of the Class One Slides (the Arithmetic Mean and the Geometric Mean, and the Harmonic Mean). What is the length of the Harmonic Mean? Try and use our strategy of using similar triangles to do this. Also, what is the length of the fourth one that's marked "Q"? Hint - try using the Pythagorean Theorem for an appropriate right triangle. Then do some internet sleuthing to find out what this length is typically called.

[illegible]

Nuts & Bolts...

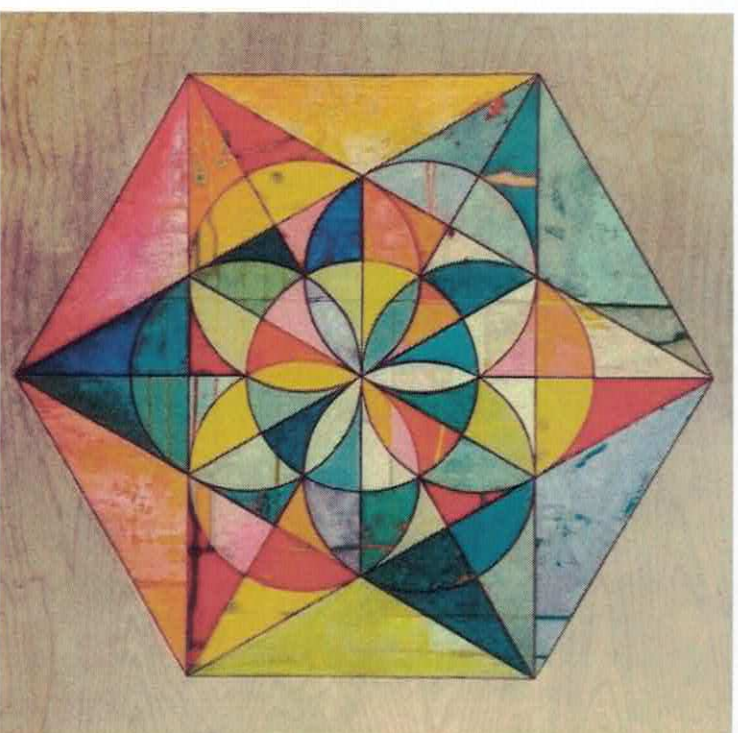
- The canvas site is your go-to: <https://canvas.harvard.edu/courses/35237>
- Textbook: *Euclid's Elements, Dover Edition*
- Homework will be posted on Friday, and will be due the next class.
- Academic integrity is crucial!
- Show your chops: *Midterm Mar 29th + Final on May 10th*
- Homework worth a large chunk of your course grade



LOOK BACK IN ORDER TO LOOK AHEAD



TAKE BACK TO YOUR CLASSROOM



REDISCOVER THE JOY OF GEOMETRY



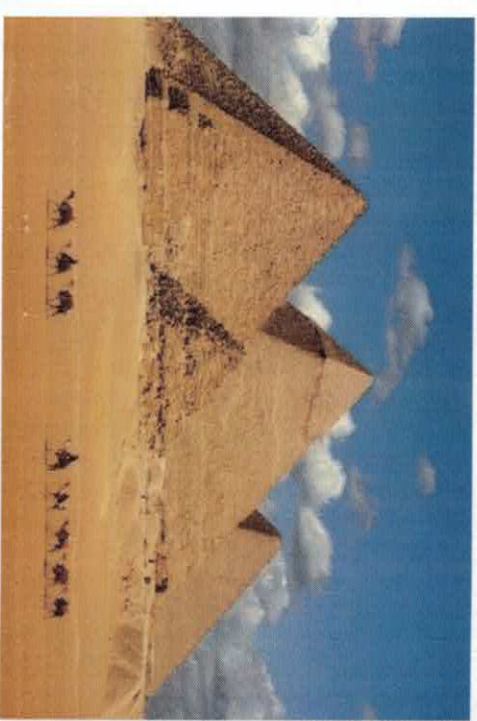
Origin Story



INDUS VALLEY CIVILIZATION



BABYLONIAN TABLETS



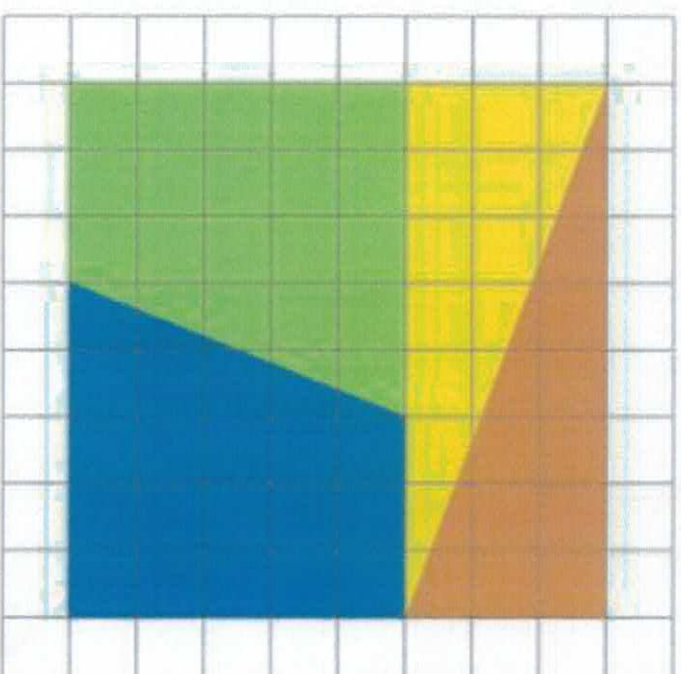
EGYPTIAN CIVILIZATION

Puzzle of the Day!

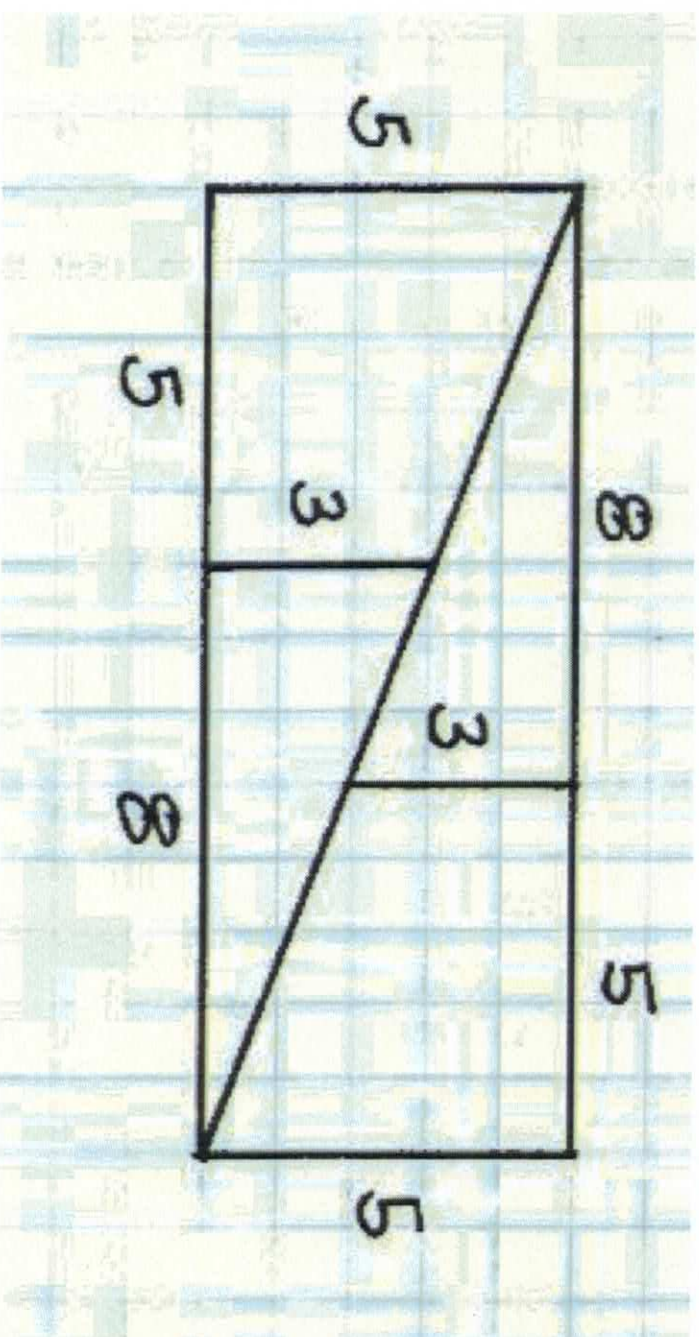
Go to:

<https://canvas.harvard.edu/courses/35237/>

and click on the "Handouts and Slides" tab
on the left

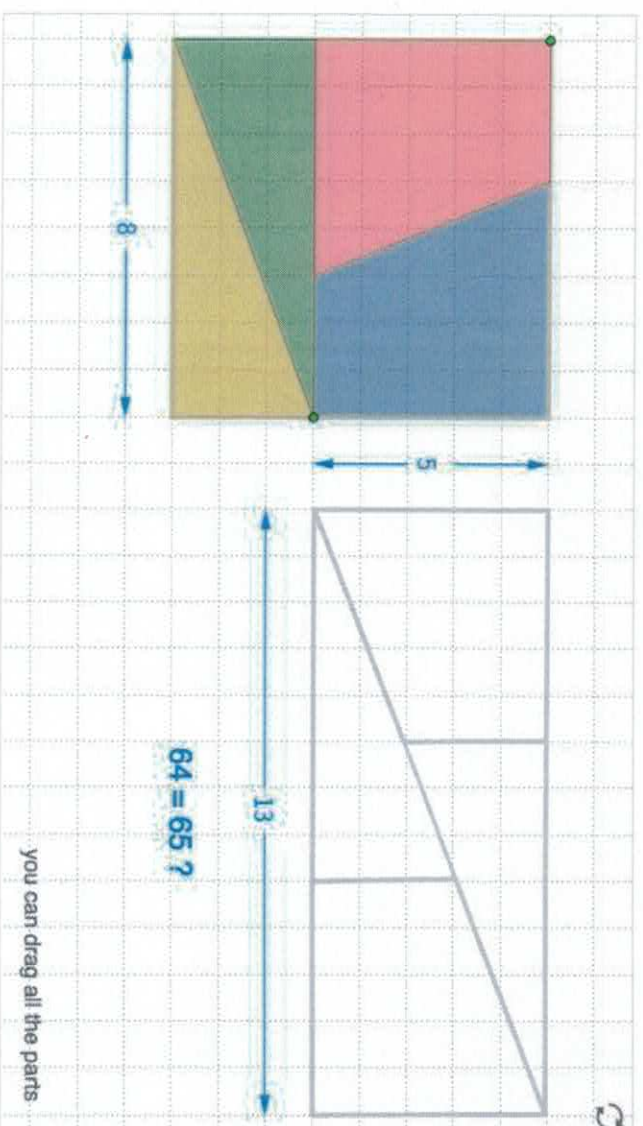


Rearrange and reassess...



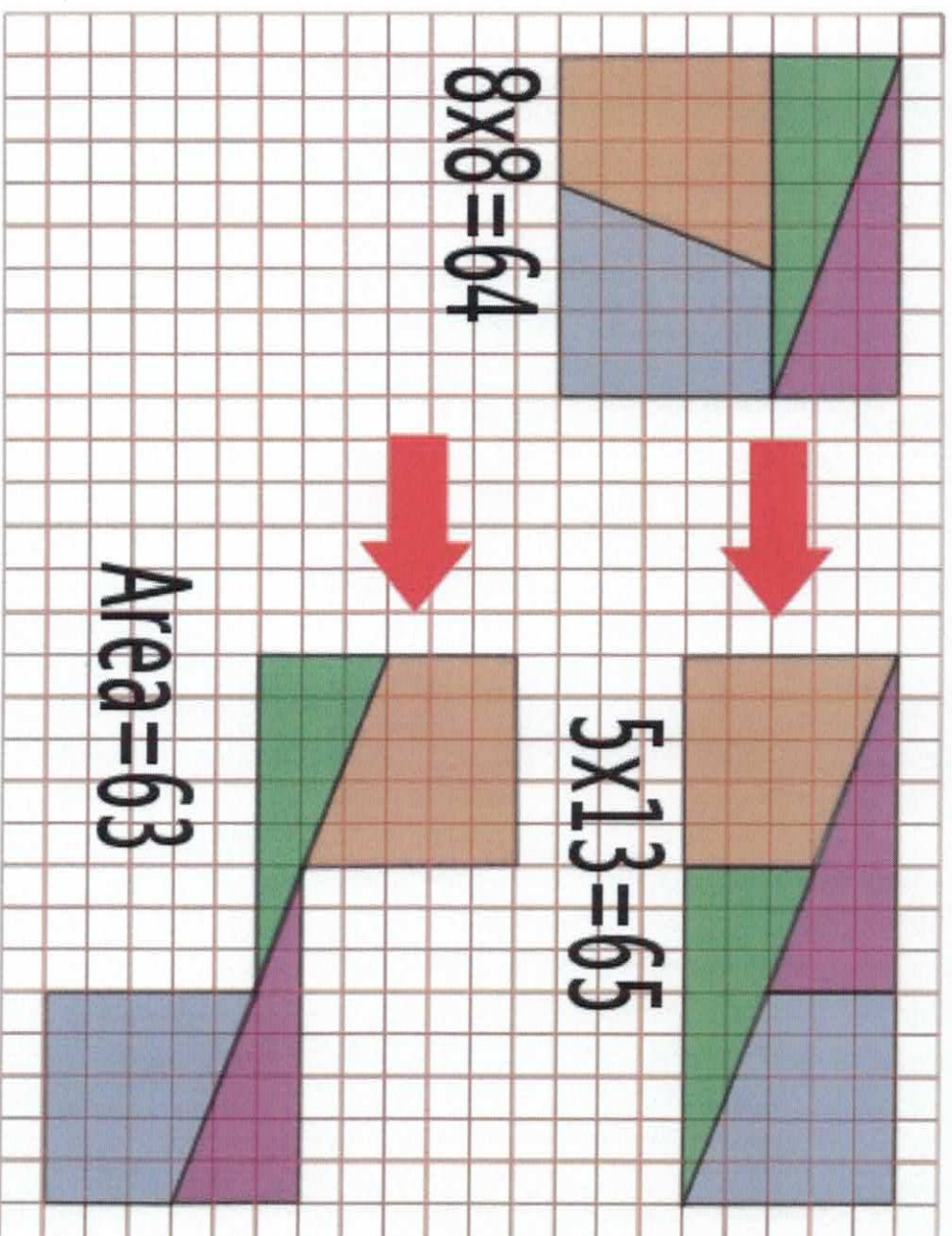
If your craft skills don't cut it... try this!

square-rectangle paradox



<http://www.geogebra.org/material/simple/id/113698>

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And we can
do more!

