Exercises

Directions: For each equation 1-6 below, determine its order. Also, name the independent variable, the dependent variable, and any parameters in the equation. If it is not clear what the independent variable is, choose one (generally t or x if it is not being used as a dependent variable). Assume any letters that are not t, x, or y (or the dependent variable) are parameters.

1.1.1 $y' = ky$

1.1.2 $dP/dt = rP(1 - P/N)$

1.1.3 $x'' + 2x' + 2x = \sin(t)$

1.1.4 $\theta'' + \theta' + k \sin\theta = 0$

1.1.5 $\frac{d^4y}{dx^4} + 4y = 0$

1.1.6 $T'(t) = k(A - T(t))$

Guess a solution to each DE below. It does not have to be a general solution. Check your guess to make sure that it works.

1.1.7 $y' = -y$

1.1.8 $y' = -5y$

1.1.9 $y'' = y$

1.1.10 $y'' = 3y$

1.1.11 $y'' = -3y$

1.1.12 $y^{(4)} = y$ (here $y^{(4)}$ means the fourth derivative of y)

For each of the equations, or system of equations below, show that the given function(s) form a solution.

1.1.13 Equation is $y' = y + 1$, solution is $y(t) = e^t - 1$

1.1.14 Equation is $y' = y + \sin(t)$, solution is $y(t) = e^t - \frac{1}{2}\sin t - \frac{1}{2}\cos t$

1.1.15 Equation is $x'' + 4x' + 4x = 0$, solution is $x(t) = e^{-2t}$

1.1.16 Equation is $x'' + 4x' + 4x = 0$, solution is $x(t) = te^{-2t}$

1.1.17 System is $\begin{matrix} x' = y, & y' = 4x \end{matrix}$, solution is $x(t) = -\frac{1}{2}e^{-2t}$, $y(t) = e^{-2t}$

1.1.18 System is $\begin{matrix} x' = y, & y' = 4x \end{matrix}$, solution is $x(t) = \frac{1}{2}e^{2t}$, $y(t) = e^{2t}$,

In each problem below, a differential equation, a general solution to the differential equation and one or more initial conditions are given. First show that the given function is in fact a solution, then use the initial condition(s) to determine the (integration) constants.

1.1.19 Equation is $y' = y + 1$, general solution is $y(t) = Ce^t - 1$, initial condition is $y(0) = 4$.

1.1.20 Equation is $y' = y + \sin(t)$, general solution is $y(t) = Ce^t - \frac{1}{2}\sin t - \frac{1}{2}\cos t$, initial condition is $y(0) = 0$.

1.1.21 Equation is $x'' - 4x = 0$, general solution is $x(t) = C_1e^{-2t} + C_2e^{2t}$, initial conditions are $x(0) = 1$, $x'(0) = 0$.

1.1.22 Equation is $x'' + 4x' + 4x = 0$, general solution is $x(t) = C_1e^{-2t} + C_2te^{-2t}$, initial conditions are $x(0) = 1$, $x'(0) = 0$.

1.1.23 System is $\begin{matrix} x' = y & y' = 4x \end{matrix}$, general solution is $x(t) = \frac{1}{2}C_1e^{2t} - \frac{1}{2}C_2e^{-2t}$, $y(t) = C_1e^{2t} + C_2e^{-2t}$, initial conditions are $x(0) = 1$, $y(0) = -1$.

1.1.24 System is $\begin{matrix} x' = y & y' = -4x \end{matrix}$, general solution is $x(t) = -\frac{1}{2}C_1\cos(2t) + \frac{1}{2}C_2\sin(2t)$, $y(t) = C_1\sin(2t) + C_2\cos(2t)$, initial conditions are $x(0) = 1$, $y(0) = -1$.

Exercises

2.1.1 Find a general solution to the basic population model $y' = ky$.

For each equation below state whether it is linear or not.

1. $x' = t\sin(x)$
2. $x' = t + \sin(x)$
3. $x' = x\sin(t)$
4. $x' = x + \sin(t)$
5. $y' + ty^2 = t$
6. $y' + x^2y = x^2$

   Solve each of the following linear equations.

7. $x' + 2x = e^{-2t}\cos(t)$
8. $x' = x + 1$
9. $tx' + 2x = 1 + t$

10. $x' = x + t$

Solve the following initial-value problems:

11. $x' = x + e^{3t}, \quad x(0) = 2$

12. $x' + 2x = e^{-2t}\cos(t), \quad x(0) = 1$

13. $tx' + x = t^2 + t, \quad x(1) = 0$

14. $x' + 2tx = t, \quad x(0) = 1$