SKILLS WARM UP 1.5

The following warm-up exercises involve skills that were covered in a previous course or in earlier sections. You will use these skills in the exercise set for this section. For additional help, review Appendix A.3 and Section 1.4.

In Exercises 1–4, simplify the expression by factoring.

1. $\dfrac{2x^3 + x^2}{6x}$ **2.** $\dfrac{x^5 + 9x^4}{x^2}$ **3.** $\dfrac{x^2 - 3x - 28}{x - 7}$ **4.** $\dfrac{x^2 + 11x + 30}{x + 5}$

In Exercises 5–8, evaluate the expression and simplify.

5. $f(x) = x^2 - 3x + 3$

(a) $f(-1)$ (b) $f(c)$ (c) $f(x + h)$

6. $f(x) = \begin{cases} 2x - 2, & x < 1 \\ 3x + 1, & x \ge 1 \end{cases}$

(a) $f\left(-\frac{1}{2}\right)$ (b) $f(1)$ (c) $f(t^2 + 1)$

7. $f(x) = x^2 - 2x + 2$ $\dfrac{f(1 + h) - f(1)}{h}$

8. $f(x) = 4x$ $\dfrac{f(2 + h) - f(2)}{h}$

In Exercises 9–12, find the domain and range of the function and sketch its graph.

9. $h(x) = -\dfrac{5}{x}$ **10.** $g(x) = \sqrt{25 - x^2}$ **11.** $f(x) = |x - 3|$ **12.** $f(x) = \dfrac{2|x|}{x}$

In Exercises 13 and 14, determine whether y is a function of x.

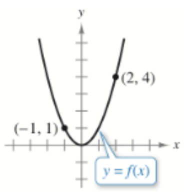
13. $9x^2 + 4y^2 = 49$ **14.** $2x^2y + 8x = 7y$

Exercises 1.5

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Finding Limits Graphically In Exercises 1–4, use the graph to find the limit. *See Examples 1 and 2.*

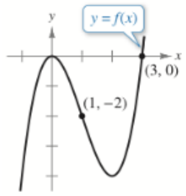
1.



(a) $\lim_{x \to 2} f(x)$

(b) $\lim_{x \to -1} f(x)$

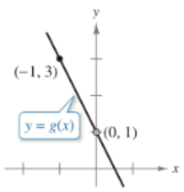
2.



(a) $\lim_{x \to 1} f(x)$

(b) $\lim_{x \to 3} f(x)$

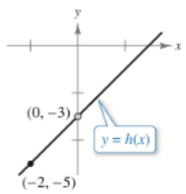
3.



(a) $\lim_{x \to 0} g(x)$

(b) $\lim_{x \to -1} g(x)$

4.



(a) $\lim_{x \to -2} h(x)$

(b) $\lim_{x \to 0} h(x)$

Finding Limits Numerically In Exercises 5–12, complete the table and use the result to estimate the limit. Use a graphing utility to graph the function to confirm your result. *See Examples 1 and 2.*

5. $\lim_{x \to 6} \dfrac{2x + 3}{5}$

x	5.9	5.99	5.999	6	6.001	6.01	6.1
$f(x)$				?			

6. $\lim_{x \to 1} (x^2 - 4x - 1)$

x	0.9	0.99	0.999	1	1.001	1.01	1.1
$f(x)$				?			

7. $\lim_{x \to 4} \dfrac{x - 4}{x^2 - 5x + 4}$

x	3.9	3.99	3.999	4	4.001	4.01	4.1
$f(x)$				?			

8. $\lim_{x \to 2} \frac{x-2}{x^2-4}$

x	1.9	1.99	1.999	2	2.001	2.01	2.1
$f(x)$				?			

9. $\lim_{x \to 0} \frac{\sqrt{x+16}-4}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

10. $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

x	-0.1	-0.01	-0.001	0	0.001	0.01	0.1
$f(x)$				?			

11. $\lim_{x \to -4} \frac{\frac{1}{x+4} - \frac{1}{4}}{x}$

x	-4.1	-4.01	-4.001	-4	-3.999	-3.99	-3.9
$f(x)$				?			

12. $\lim_{x \to -2} \frac{\frac{1}{2} - \frac{1}{x+2}}{2x}$

x	-2.1	-2.01	-2.001	-2	-1.999	-1.99	-1.9
$f(x)$				?			

Evaluating Basic Limits In Exercises 13–20, find the limit. *See Example 3.*

13. $\lim_{x \to 3} 6$ **14.** $\lim_{x \to 5} 4$

15. $\lim_{x \to -2} x$ **16.** $\lim_{x \to 10} x$

17. $\lim_{x \to 7} x^2$ **18.** $\lim_{x \to 3} x^3$

19. $\lim_{x \to 36} \sqrt{x}$ **20.** $\lim_{x \to -1} \sqrt[3]{x}$

Operations with Limits In Exercises 21 and 22, find the limit of (a) $f(x) + g(x)$, (b) $f(x)g(x)$, and (c) $f(x)/g(x)$, as x approaches c.

21. $\lim_{x \to c} f(x) = 3$, $\lim_{x \to c} g(x) = 9$

22. $\lim_{x \to c} f(x) = \frac{3}{2}$, $\lim_{x \to c} g(x) = \frac{1}{2}$

Operations with Limits In Exercises 23 and 24, find the limit of (a) $\sqrt{f(x)}$, (b) $3f(x)$, and (c) $[f(x)]^2$, as x approaches c.

23. $\lim_{x \to c} f(x) = 16$ **24.** $\lim_{x \to c} f(x) = 9$

Using Properties of Limits In Exercises 25–36, find the limit using direct substitution. *See Examples 3 and 4.*

25. $\lim_{x \to -3} (2x + 5)$ **26.** $\lim_{x \to -4} (4x + 3)$

27. $\lim_{x \to 1} (1 - x^2)$ **28.** $\lim_{x \to 2} (-x^2 + x - 2)$

29. $\lim_{x \to 3} \sqrt{x+6}$ **30.** $\lim_{x \to 5} \sqrt[3]{x-5}$

31. $\lim_{x \to -3} \frac{2}{x+2}$ **32.** $\lim_{x \to -2} \frac{3x+1}{2-x}$

33. $\lim_{x \to -2} \frac{x^2-1}{2x}$ **34.** $\lim_{x \to -8} \frac{3x}{x+2}$

35. $\lim_{x \to 5} \frac{\sqrt{x+11}+6}{x}$ **36.** $\lim_{x \to 12} \frac{\sqrt{x-3}-2}{x}$

Finding Limits In Exercises 37–58, find the limit (if it exists). *See Examples 5, 6, 7, 9, and 11.*

37. $\lim_{x \to -3} \frac{x^2-9}{x+3}$ **38.** $\lim_{x \to -1} \frac{2x^2-x-3}{x+1}$

39. $\lim_{x \to 2} \frac{x^2+3x-10}{x^2-4}$ **40.** $\lim_{t \to 1} \frac{t^2+t-2}{t^2-1}$

41. $\lim_{x \to -2} \frac{x^3+8}{x+2}$ **42.** $\lim_{x \to -3} \frac{x^3+27}{x+3}$

43. $\lim_{\Delta x \to 0} \frac{2(x+\Delta x) - 2x}{\Delta x}$ **44.** $\lim_{\Delta x \to 0} \frac{-3(x+\Delta x) + 3x}{\Delta x}$

45. $\lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 - 5(t+\Delta t) - (t^2 - 5t)}{\Delta t}$

46. $\lim_{\Delta t \to 0} \frac{(t+\Delta t)^2 - 4(t+\Delta t) + 2 - (t^2 - 4t + 2)}{\Delta t}$

47. $\lim_{x \to 4} \frac{\sqrt{x+5}-3}{x-4}$ **48.** $\lim_{x \to 3} \frac{\sqrt{x+1}-2}{x-3}$

49. $\lim_{x \to 0} \frac{\sqrt{x+5}-\sqrt{5}}{x}$ **50.** $\lim_{x \to 0} \frac{\sqrt{x+2}-\sqrt{2}}{x}$

51. $\lim_{x \to 2} f(x)$, where $f(x) = \begin{cases} 4 - x, & x \neq 2 \\ 0, & x = 2 \end{cases}$

52. $\lim_{x \to 1} f(x)$, where $f(x) = \begin{cases} x^2 + 2, & x \neq 1 \\ 1, & x = 1 \end{cases}$

53. $\lim_{x \to 3} f(x)$, where $f(x) = \begin{cases} \frac{1}{3}x - 5, & x \leq 3 \\ -3x + 7, & x > 3 \end{cases}$

54. $\lim_{s \to 4} f(s)$, where $f(s) = \begin{cases} 3s - 4, & s \leq 4 \\ 5 - \frac{1}{2}s, & s > 4 \end{cases}$

55. $\lim_{x\to -4} \frac{2}{x+4}$

56. $\lim_{x\to 5} \frac{4}{x-5}$

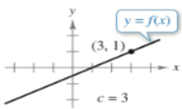
57. $\lim_{x\to 2} \frac{x-2}{x^2-4x+4}$

58. $\lim_{t\to -6} \frac{t+6}{t^2+12t+36}$

Finding Limits Graphically In Exercises 59–64, use the graph to find the limit (if it exists).

(a) $\lim_{x\to c^+} f(x)$ (b) $\lim_{x\to c^-} f(x)$ (c) $\lim_{x\to c} f(x)$

59.



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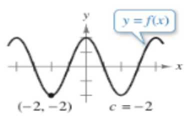

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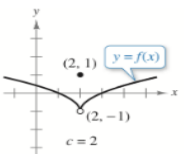

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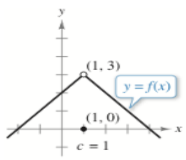
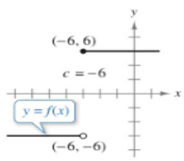
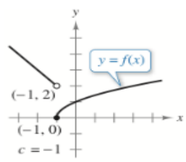


63.



64.



Finding One-Sided Limits In Exercises 65 and 66, use a graph to find the limit from the left and the limit from the right. *See Example 8.*

65. $\lim_{x\to -3^-} \frac{|x+3|}{x+3}$

$\lim_{x\to -3^+} \frac{|x+3|}{x+3}$

66. $\lim_{x\to 6^-} \frac{|x-6|}{x-6}$

$\lim_{x\to 6^+} \frac{|x-6|}{x-6}$

One-Sided Limits of Unbounded Functions In Exercises 67–70, use a graphing utility to graph the function and estimate the limit (if it exists). Use a table to reinforce your conclusion. If the limit does not exist, explain why the limit fails to exist.

67. $\lim_{x\to 2^+} \frac{3}{x^2-4}$

68. $\lim_{x\to -1^+} \frac{6}{x+1}$

69. $\lim_{x\to -2^-} \frac{1}{x+2}$

70. $\lim_{x\to 0^-} \frac{x+1}{x}$

Estimating Limits In Exercises 71–74, use a graphing utility to estimate the limit (if it exists).

71. $\lim_{x\to 2} \frac{x^2-5x+6}{x^2-4x+4}$

72. $\lim_{x\to 1} \frac{x^2+6x-7}{x^3-x^2+2x-2}$

73. $\lim_{x\to -4} \frac{x^3+4x^2+x+4}{2x^2+7x-4}$

74. $\lim_{x\to -2} \frac{4x^3+7x^2+x+6}{3x^2-x-14}$

75. Environment The cost C (in dollars) of removing $p\%$ of the pollutants from the water in a small lake is given by

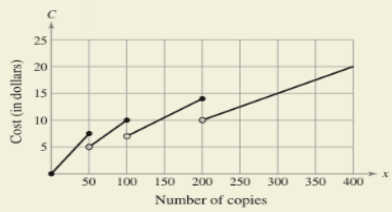
$$C = \frac{25{,}000p}{100-p}, \quad 0 \le p < 100.$$

(a) Find the cost of removing 50% of the pollutants.

(b) What percent of the pollutants can be removed for \$100,000?

(c) Evaluate $\lim_{p\to 100^-} C$. Explain your results.

76. HOW DO YOU SEE IT? The graph shows the cost C (in dollars) of making x photocopies at a copy shop.



(a) Does $\lim_{x\to 50} C$ exist? Explain your reasoning.

(b) Does $\lim_{x\to 150} C$ exist? Explain your reasoning.

(c) You have to make 200 photocopies. Would it be better to make 200 or 201? Explain your reasoning.

77. Compound Interest Consider a certificate of deposit that pays 10% (annual percentage rate) on an initial deposit of \$1000. The balance A after 10 years is $A = 1000(1 + 0.1x)^{10/x}$, where x is the length of the compounding period (in years).

(a) Use a graphing utility to graph A, where $0 \le x \le 1$.

(b) Use the *zoom* and *trace* features to estimate the balance for quarterly compounding and daily compounding.

(c) Use the *zoom* and *trace* features to estimate $\lim_{x\to 0^+} A$. What do you think this limit represents? Explain your reasoning.

SKILLS WARM UP 1.6

The following warm-up exercises involve skills that were covered in a previous course or in earlier sections. You will use these skills in the exercise set for this section. For additional help, review Appendices A.4 and A.5, and Section 1.5.

In Exercises 1–4, simplify the expression.

1. $\dfrac{x^2 + 6x + 8}{x^2 - 6x - 16}$ **2.** $\dfrac{x^2 - 5x - 6}{x^2 - 9x + 18}$ **3.** $\dfrac{2x^2 - 2x - 12}{4x^2 - 24x + 36}$ **4.** $\dfrac{x^3 - 16x}{x^3 + 2x^2 - 8x}$

In Exercises 5–8, solve for *x*.

5. $x^2 + 7x = 0$ **6.** $x^2 + 4x - 5 = 0$

7. $3x^2 + 8x + 4 = 0$ **8.** $3x^3 - x^2 - 24x = 0$

In Exercises 9 and 10, find the limit.

9. $\lim_{x \to 3} (2x^2 - 3x + 4)$ **10.** $\lim_{x \to -2} \sqrt{x^2 - x + 3}$

Exercises 1.6

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Determining Continuity In Exercises 1–10, determine whether the function is continuous on the entire real number line. Explain your reasoning. *See Examples 1 and 2.*

1. $f(x) = 5x^3 - x^2 + 2$ **2.** $f(x) = (x^2 - 1)^3$

3. $f(x) = \dfrac{3}{x^2 - 16}$ **4.** $f(x) = \dfrac{1}{9 - x^2}$

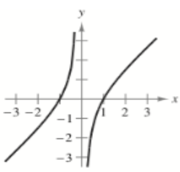
5. $f(x) = \dfrac{1}{4 + x^2}$ **6.** $f(x) = \dfrac{5x}{x^2 + 8}$

7. $f(x) = \dfrac{2x - 1}{x^2 - 8x + 15}$ **8.** $f(x) = \dfrac{x + 4}{x^2 - 6x + 5}$

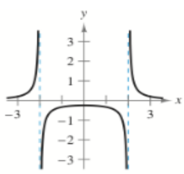
9. $g(x) = \dfrac{x^2 - 8x + 12}{x^2 - 36}$ **10.** $g(x) = \dfrac{x^2 - 11x + 30}{x^2 - 25}$

Determining Continuity In Exercises 11–40, describe the interval(s) on which the function is continuous. Explain why the function is continuous on the interval(s). If the function has a discontinuity, identify the conditions of continuity that are not satisfied. *See Examples 1, 2, 3, 4, and 5.*

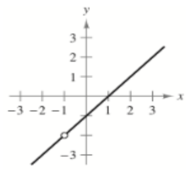
11. $f(x) = \dfrac{x^2 - 1}{x}$



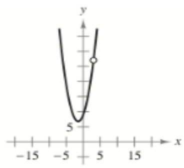
12. $f(x) = \dfrac{1}{x^2 - 4}$



13. $f(x) = \dfrac{x^2 - 1}{x + 1}$



14. $f(x) = \dfrac{x^3 - 27}{x - 3}$



15. $f(x) = x^2 - 9x + 14$ **16.** $f(x) = 3 - 2x - x^2$

17. $f(x) = \dfrac{x}{x^2 - 1}$ **18.** $f(x) = \dfrac{x - 3}{x^2 - 9}$

19. $f(x) = \dfrac{7x}{x^2 + 5}$ **20.** $f(x) = \dfrac{6}{x^2 + 3}$

21. $f(x) = \dfrac{x - 5}{x^2 - 9x + 20}$ **22.** $f(x) = \dfrac{x - 1}{x^2 + x - 2}$

23. $f(x) = \sqrt{4 - x}$ **24.** $f(x) = \sqrt{x - 1}$

25. $f(x) = \sqrt{x} + 2$ **26.** $f(x) = 3 - \sqrt{x}$

27. $f(x) = \begin{cases} -2x + 3, & -1 \le x \le 1 \\ x^2, & 1 < x \le 3 \end{cases}$

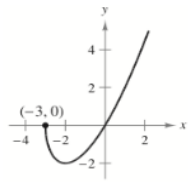
28. $f(x) = \begin{cases} \frac{1}{2}x + 1, & -3 \le x \le 2 \\ 3 - x, & 2 < x \le 4 \end{cases}$

29. $f(x) = \begin{cases} 4 - 2x, & x \le 2 \\ x^2 - 3, & x > 2 \end{cases}$

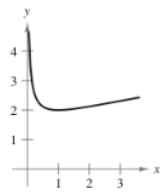
30. $f(x) = \begin{cases} x^2 - 2, & x \le -1 \\ 3x + 2, & x > -1 \end{cases}$

31. $f(x) = \dfrac{|x + 1|}{x + 1}$ **32.** $f(x) = \dfrac{|4 - x|}{4 - x}$

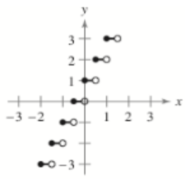
33. $f(x) = x\sqrt{x+3}$



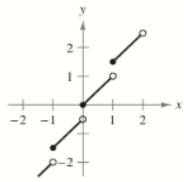
34. $f(x) = \dfrac{x+1}{\sqrt{x}}$



35. $f(x) = [\![2x]\!] + 1$



36. $f(x) = \dfrac{[\![x]\!]}{2} + x$



37. $f(x) = [\![x - 1]\!]$

38. $f(x) = x - [\![x]\!]$

39. $h(x) = f(g(x)),\quad f(x) = \dfrac{1}{\sqrt{x}},\quad g(x) = x - 1, x > 1$

40. $h(x) = f(g(x)),\quad f(x) = \dfrac{1}{x-1},\quad g(x) = x^2 + 5$

Determining Continuity In Exercises 41–46, sketch the graph of the function and describe the interval(s) on which the function is continuous. If there are any discontinuities, determine whether they are removable.

41. $f(x) = \dfrac{x^2 - 16}{x - 4}$

42. $f(x) = \dfrac{2x^2 + x}{x}$

43. $f(x) = \dfrac{x + 4}{3x^2 - 12}$

44. $f(x) = \dfrac{-x}{x^3 - x}$

45. $f(x) = \begin{cases} x^2 + 1, & x < 0 \\ x - 1, & x \geq 0 \end{cases}$

46. $f(x) = \begin{cases} x^2 - 3, & x \leq 0 \\ 2x + 3, & x > 0 \end{cases}$

Determining Continuity on a Closed Interval In Exercises 47–50, discuss the continuity of the function on the closed interval. If there are any discontinuities, determine whether they are removable.

Function	*Interval*
47. $f(x) = x^2 - 4x - 5$	$[-1, 5]$
48. $f(x) = \dfrac{5}{x^2 + 1}$	$[-2, 2]$
49. $f(x) = \dfrac{1}{x - 2}$	$[1, 4]$
50. $f(x) = \dfrac{x - 1}{x^2 - 4x + 3}$	$[0, 4]$

Finding Discontinuities In Exercises 51–56, use a graphing utility to graph the function. Use the graph to determine any x-value(s) at which the function is not continuous. Explain why the function is not continuous at the x-value(s).

51. $h(x) = \dfrac{1}{x^2 - x - 2}$

52. $k(x) = \dfrac{4 - x}{x^2 + x - 12}$

53. $f(x) = \begin{cases} 2x - 4, & x \leq 3 \\ x^2 - 2x, & x > 3 \end{cases}$

54. $f(x) = \begin{cases} 3x - 2, & x \leq 2 \\ x + 1, & x > 2 \end{cases}$

55. $f(x) = x - 2[\![x]\!]$

56. $f(x) = [\![2x - 1]\!]$

Making a Function Continuous In Exercises 57 and 58, find the constant a (Exercise 57) and the constants a and b (Exercise 58) such that the function is continuous on the entire real number line.

57. $f(x) = \begin{cases} x^3, & x \leq 2 \\ ax^2, & x > 2 \end{cases}$

58. $f(x) = \begin{cases} 2, & x \leq -1 \\ ax + b, & -1 < x < 3 \\ -2, & x \geq 3 \end{cases}$

Writing In Exercises 59 and 60, use a graphing utility to graph the function on the interval $[-4, 4]$. Does the graph of the function appear to be continuous on this interval? Is the function in fact continuous on $[-4, 4]$? Write a short paragraph about the importance of examining a function analytically as well as graphically.

59. $f(x) = \dfrac{x^2 + x}{x}$

60. $f(x) = \dfrac{x^3 - 8}{x - 2}$

61. Environmental Cost The cost C (in millions of dollars) of removing x percent of the pollutants emitted from the smokestack of a factory can be modeled by

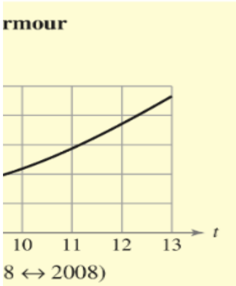
$$C = \frac{2x}{100 - x}.$$

(a) What is the implied domain of C? Explain your reasoning.

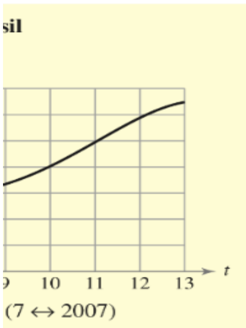
(b) Use a graphing utility to graph the cost function. Is the function continuous on its domain? Explain your reasoning.

(c) Find the cost of removing 75% of the pollutants from the smokestack.

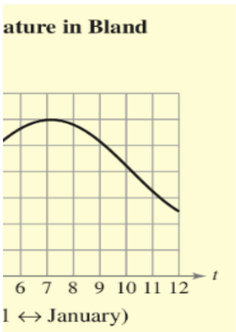
esents the revenue R (in nder Armour from 2008 sents the year, with $t = 8$ ate and interpret the slopes 2009 and 2011. *(Source:*



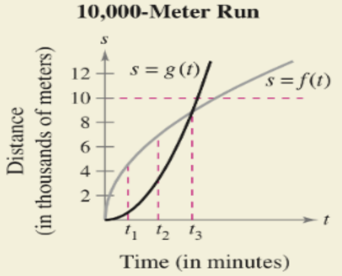
ts the sales S (in millions 007 through 2013, where t = 7 corresponding to 2007. lopes of the graph for the *rce: Fossil, Group)*



ı represents the average grees Fahrenheit) in Bland, t represents the month, with ary, $t = 2$ corresponding to te and interpret the slopes ıd 10. *(Source: National lministration)*



16. **HOW DO YOU SEE IT?** Two long distance runners starting out side by side begin a 10,000-meter run. Their distances are given by $s = f(t)$ and $s = g(t)$, where s is measured in thousands of meters and t is measured in minutes.



(a) Which runner is running faster at t_1?

(b) What conclusion can you make regarding their rates at t_2?

(c) What conclusion can you make regarding their rates at t_3?

(d) Which runner finishes the race first? Explain.

Finding the Slope of a Graph In Exercises 17–26, use the limit definition to find the slope of the graph of f at the given point. *See Examples 3, 4, and 5.*

17. $f(x) = -1;\ (0, -1)$

18. $f(x) = 6;\ (-2, 6)$

19. $f(x) = 13 - 4x;\ (3, 1)$

20. $f(x) = 6x + 3;\ (1, 9)$

21. $f(x) = 2x^2 - 3;\ (2, 5)$

22. $f(x) = 11 - x^2;\ (3, 2)$

23. $f(x) = x^3 - 4x;\ (-1, 3)$

24. $f(x) = 7x - x^3;\ (-3, 6)$

25. $f(x) = 2\sqrt{x};\ (4, 4)$

26. $f(x) = \sqrt{x + 1};\ (8, 3)$

Finding a Derivative In Exercises 27–40, use the limit definition to find the derivative of the function. *See Examples 6 and 7.*

27. $f(x) = 3$

28. $f(x) = -2$

29. $f(x) = -5x$

30. $f(x) = 4x + 1$

31. $g(s) = \frac{1}{3}s + 2$

32. $h(t) = 6 - \frac{1}{2}t$

33. $f(x) = 4x^2 - 5x$

34. $f(x) = 2x^2 + 7x$

35. $h(t) = \sqrt{t - 3}$

36. $f(x) = \sqrt{x + 2}$

37. $f(t) = t^3 - 12t$

38. $f(t) = t^3 + t^2$

39. $f(x) = \dfrac{1}{x + 2}$

40. $g(s) = \dfrac{1}{s - 4}$

Finding an Equation of a Tangent Line In Exercises 41–48, find an equation of the tangent line to the graph of f at the given point. Then verify your results by using a graphing utility to graph the function and its tangent line at the point.

41. $f(x) = \frac{1}{2}x^2;\ (2, 2)$

42. $f(x) = -\frac{1}{8}x^2;\ (-4, -2)$

43. $f(x) = (x - 1)^2;\ (-2, 9)$

44. $f(x) = 2x^2 - 5;\ (-1, -3)$

45. $f(x) = \sqrt{x} + 1;\ (4, 3)$

46. $f(x) = \sqrt{x + 3};\ (6, 3)$

47. $f(x) = \dfrac{1}{5x};\ \left(-\dfrac{1}{5}, -1\right)$

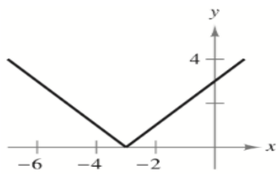
48. $f(x) = \dfrac{1}{x - 3};\ (2, -1)$

Finding an Equation of a Tangent Line In Exercises 49–52, find an equation(s) of the line(s) that is tangent to the graph of f and parallel to the given line.

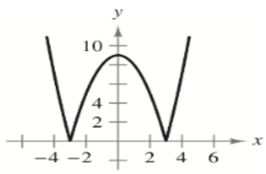
	Function	*Line*
49.	$f(x) = -\frac{1}{4}x^2$	$x + y = 0$
50.	$f(x) = x^2 - 7$	$2x + y = 0$
51.	$f(x) = -\frac{1}{3}x^3$	$9x + y - 6 = 0$
52.	$f(x) = x^3 + 2$	$3x - y - 4 = 0$

Determining Differentiability In Exercises 53–58, describe the x-values at which the function is differentiable. Explain your reasoning.

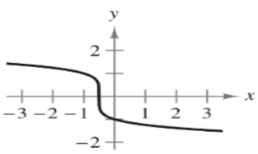
53. $y = |x + 3|$



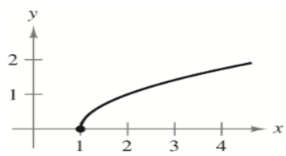
54. $y = |x^2 - 9|$



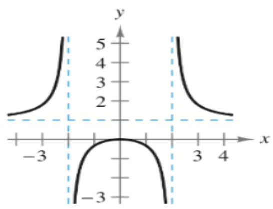
55. $y = -\sqrt[5]{2x + 1}$



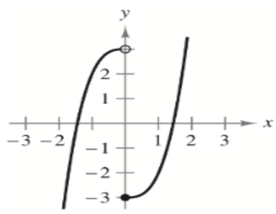
56. $y = \sqrt{x - 1}$



57. $y = \dfrac{x^2}{x^2 - 4}$



58. $y = \begin{cases} x^3 + 3, & x < 0 \\ x^3 - 3, & x \geq 0 \end{cases}$



Writing a F Exercises 59 has the given function.

59. $f(0) = 2;\ f'(x) = -$

60. $f(-2) = f(4) = 0;$ $f'(x) < 0$ for $x < 1;$

Graphical, Numerica Exercises 61–64, use the interval $[-2, 2]$. Co estimating the slopes o Then evaluate the slope results with those obtai

x	-2	$-\frac{3}{2}$
$f(x)$		
$f'(x)$		

61. $f(x) = \frac{1}{4}x^2$

63. $f(x) = -\frac{1}{2}x^3$

Graphing a Function a 65–68, find the derivativ to graph f and its deriva What does the x-inter about the graph of f?

65. $f(x) = x^2 - 4x$

67. $f(x) = x^3 - 3x$

True or False? In Exe the statement is true or give an example that sh

69. The slope of the gra point on the graph of

70. If a function is co differentiable at that

71. If a function is dif continuous at that po

72. A tangent line to a gr than one point.

73. Writing Use a gr functions

$f(x) = x^2 + 1$ and

in the same viewing features to analyze What do you observe at this point? Write geometric significanc

SKILLS WARM UP 2.2

The following warm-up exercises involve skills that were covered in a previous course. You will use these skills in the exercise set for this section. For additional help, review Appendices A.3 and A.4.

In Exercises 1 and 2, evaluate each expression when $x = 2$.

1. (a) $2x^2$ (b) $(5x)^2$ (c) $6x^{-2}$

2. (a) $\dfrac{1}{(3x)^2}$ (b) $\dfrac{1}{4x^3}$ (c) $\dfrac{(2x)^{-3}}{4x^{-2}}$

In Exercises 3–6, simplify the expression.

3. $4(3)x^3 + 2(2)x$

4. $\frac{1}{2}(3)x^2 - \frac{3}{2}x^{1/2}$

5. $\left(\frac{1}{4}\right)x^{-3/4}$

6. $\frac{1}{3}(3)x^2 - 2\left(\frac{1}{2}\right)x^{-1/2} + \frac{1}{3}x^{-2/3}$

In Exercises 7–10, solve the equation.

7. $3x^2 + 2x = 0$

8. $x^3 - x = 0$

9. $x^2 + 8x - 20 = 0$

10. $3x^2 - 10x + 8 = 0$

Exercises 2.2

See *CalcChat.com* for tutorial help and worked-out solutions to odd-numbered exercises.

Finding Derivatives In Exercises 1–24, find the derivative of the function. *See Examples 1, 2, 4, 5, and 8.*

1. $y = 3$

2. $f(x) = -8$

3. $y = x^5$

4. $f(x) = \dfrac{1}{x^6}$

5. $h(x) = 3x^3$

6. $h(x) = 6x^5$

7. $y = \dfrac{5x^4}{6}$

8. $g(t) = \dfrac{3t^2}{4}$

9. $f(x) = 4x$

10. $g(x) = \dfrac{x}{3}$

11. $y = 8 - x^3$

12. $y = t^2 - 6$

13. $f(x) = 4x^2 - 3x$

14. $g(x) = 3x^2 + 5x^3$

15. $f(t) = -3t^2 + 2t - 4$

16. $y = 7x^3 - 9x^2 + 8$

17. $s(t) = 4t^4 - 2t^2 + t + 3$

18. $y = 2x^3 - x^2 + 3x - 1$

19. $g(x) = x^{2/3}$

20. $h(x) = x^{5/2}$

21. $y = 4t^{4/3}$

22. $f(x) = 12x^{1/6}$

23. $y = 4x^{-2} + 2x^2$

24. $s(t) = 8t^{-4} + t$

Using Parentheses When Differentiating In Exercises 25–30, find the derivative of the function. *See Example 6.*

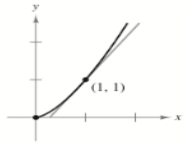
Function	*Rewrite*	*Differentiate*	*Simplify*
25. $y = \dfrac{2}{7x^4}$			
26. $y = \dfrac{2}{3x^2}$			
27. $y = \dfrac{1}{(4x)^3}$			
28. $y = \dfrac{\pi}{(2x)^6}$			
29. $y = \dfrac{4}{(2x)^{-5}}$			
30. $y = \dfrac{4x}{x^{-3}}$			

Differentiating Radical Functions In Exercises 31–36, find the derivative of the function. *See Example 7.*

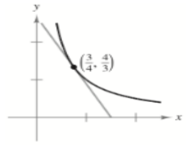
Function	*Rewrite*	*Differentiate*	*Simplify*
31. $y = 6\sqrt{x}$			
32. $y = \dfrac{3\sqrt{x}}{4}$			
33. $y = \dfrac{1}{7\sqrt[6]{x}}$			
34. $y = \dfrac{3}{2\sqrt[4]{x^3}}$			
35. $y = \sqrt[5]{8x}$			
36. $y = \sqrt[3]{6x^2}$			

Finding the Slope of a Graph In Exercises 37–44, find the slope of the graph of the function at the given point. Use the *derivative* feature of a graphing utility to confirm your results. *See Examples 3 and 9.*

37. $y = x^{3/2}$



38. $y = x^{-1}$



39. $f(t) = t^{-4}; \left(\frac{1}{2}, 16\right)$

40. $f(x) = x^{-1/3}; \left(8, \frac{1}{2}\right)$

41. $f(x) = 2x^3 + 8x^2 - x - 4; (-1, 3)$

42. $f(x) = x^4 - 2x^3 + 5x^2 - 7x; (-1, 15)$

43. $f(x) = -\frac{1}{2}x(1 + x^2); (1, -1)$

44. $f(x) = 3(5 - x)^2; (5, 0)$

Finding an Equation of a Tangent Line In Exercises 45–50, (a) find an equation of the tangent line to the graph of the function at the given point, (b) use a graphing utility to graph the function and its tangent line at the point, and (c) use the *tangent* feature of a graphing utility to confirm your results. *See Example 10.*

45. $y = -2x^4 + 5x^2 - 3; (1, 0)$

46. $y = x^3 + x + 4; (-2, -6)$

47. $f(x) = \sqrt[3]{x} + \sqrt[5]{x}; (1, 2)$

48. $f(x) = \dfrac{1}{\sqrt[3]{x^2}} - x; (-1, 2)$

49. $y = 3x\left(x^2 - \dfrac{2}{x}\right); (2, 18)$

50. $y = (2x + 1)^2; (0, 1)$

Finding Derivatives In Exercises 51–62, find $f'(x)$.

51. $f(x) = x^2 - \dfrac{4}{x} - 3x^{-2}$

52. $f(x) = 6x^2 - 5x^{-2} + 7x^{-3}$

53. $f(x) = x^2 - 2x - \dfrac{2}{x^4}$

54. $f(x) = x^2 + 4x + \dfrac{1}{x}$

55. $f(x) = x^{4/5} + x$

56. $f(x) = x^{1/3} - 1$

57. $f(x) = x(x^2 + 1)$

58. $f(x) = (x^2 + 2x)(x + 1)$

59. $f(x) = \dfrac{2x^3 - 4x^2 + 3}{x^2}$

60. $f(x) = \dfrac{2x^2 - 3x + 1}{x}$

61. $f(x) = \dfrac{4x^3 - 3x^2 + 2x + 5}{x^2}$

62. $f(x) = \dfrac{-6x^3 + 3x^2 - 2x + 1}{x}$

Finding Horizontal Tangent Lines In Exercises 63–66, determine the point(s), if any, at which the graph of the function has a horizontal tangent line.

63. $y = x^4 - 2x^2 + 3$

64. $y = x^3 + 3x^2$

65. $y = \frac{1}{2}x^2 + 5x$

66. $y = x^2 + 2x$

Using the Derivative In Exercises 67 and 68, determine the point(s), if any, at which the graph of the function has a tangent line with the given slope.

	Function	*Slope*
67.	$y = x^2 + 3$	$m = 4$
68.	$y = x^2 + 2x$	$m = -10$

Exploring Relationships In Exercises 69 and 70, (a) sketch the graphs of f and g, (b) find $f'(1)$ and $g'(1)$, (c) sketch the tangent line to each graph at $x = 1$, and (d) explain the relationship between f' and g'.

69. $f(x) = x^3$
$g(x) = x^3 + 3$

70. $f(x) = x^2$
$g(x) = 3x^2$

Exploring Relationships In Exercises 71–74, the relationship between f and g is given. Explain the relationship between f' and g'.

71. $g(x) = f(x) + 6$

72. $g(x) = 2f(x)$

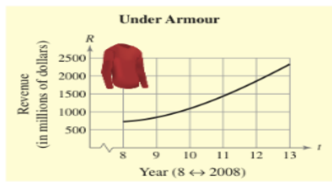
73. $g(x) = -5f(x)$

74. $g(x) = 3f(x) - 1$

75. Revenue The revenue R (in millions of dollars) for Under Armour from 2008 through 2013 can be modeled by

$$R = -4.1685t^3 + 175.037t^2 - 1950.88t + 7265.3$$

where t is the year, with $t = 8$ corresponding to 2008. *(Source: Under Armour, Inc.)*



(a) Find the slopes of the graph for the years 2009 and 2011.

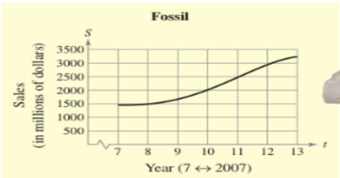
(b) Compare your results with those obtained in Exercise 13 in Section 2.1.

(c) Interpret the slope of the graph in the context of the problem.

76. Sales The sales S (in millions of dollars) for Fossil from 2007 through 2013 can be modeled by

$$S = -2.67538t^4 + 94.0568t^3 - 1155.203t^2 + 6002.42t - 9794.2$$

where t is the year, with $t = 7$ corresponding to 2007. *(Source: Fossil, Group)*

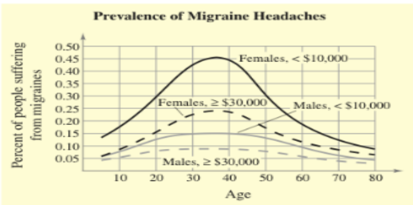


(a) Find the slopes of the graph for the years 2010 and 2012.

(b) Compare your results with those obtained in Exercise 14 in Section 2.1.

(c) Interpret the slope of the graph in the context of the problem.

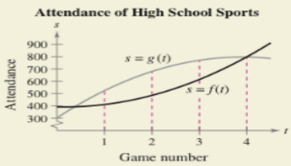
77. Psychology: Migraine Prevalence The graph illustrates the prevalence of migraine headaches in males and females in selected income groups. *(Source: Adapted from Sue/Sue/Sue, Understanding Abnormal Behavior, Seventh Edition)*



(a) Write a short paragraph describing your general observations about the prevalence of migraines in females and males with respect to age group and income bracket.

(b) Describe the graph of the derivative of each curve, and explain the significance of each derivative. Include an explanation of the units of the derivatives, and indicate the time intervals in which the derivatives would be positive and negative.

78. HOW DO YOU SEE IT? The attendance for four high school basketball games is given by $s = f(t)$, and the attendance for four high school football games is given by $s = g(t)$, where $t = 1$ corresponds to the first game.



(a) Which attendance rate, f' or g', is greater at game 1?

(b) What conclusion can you make regarding the attendance rates, f' and g', at game 3?

(c) What conclusion can you make regarding the attendance rates, f' and g', at game 4?

(d) Which sport do you think would have a greater attendance for game 5? Explain your reasoning.

79. Cost The marginal cost for manufacturing an electrical component is \$7.75 per unit, and the fixed cost is \$500. Write the cost C as a function of x, the number of units produced. Show that the derivative of this cost function is a constant and is equal to the marginal cost.

80. Political Fundraiser A politician raises funds by selling tickets to a dinner for \$500. The politician pays \$150 for each dinner and has fixed costs of \$7000 to rent a dining hall and wait staff. Write the profit P as a function of x, the number of dinners sold. Show that the derivative of the profit function is a constant and is equal to the increase in profit from each dinner sold.

Finding Horizontal Tangent Lines In Exercises 81 and 82, use a graphing utility to graph f and f' over the given interval. Determine any points at which the graph of f has a horizontal tangent line.

	Function	*Interval*
81.	$f(x) = 4.1x^3 - 12x^2 + 2.5x$	$[0, 3]$
82.	$f(x) = x^3 - 1.4x^2 - 0.96x + 1.44$	$[-2, 2]$

True or False? In Exercises 83 and 84, determine whether the statement is true or false. If it is false, explain why or give an example that shows it is false.

83. If $f'(x) = g'(x)$, then $f(x) = g(x)$.

84. If $f(x) = g(x) + c$, then $f'(x) = g'(x)$.

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