

1. Consider each time-domain function shown below. **Plot** each function, and **write** the corresponding Laplace domain function:

a. $a(t) = -0.5 + 2t$

b. $b(t) = 3e^{-2t}$

c. $c(t) = 3(1 - e^{2t})$

d. $d(t) = e^{-0.5t} \sin(3t)$

2. Consider the Laplace-domain functions below. For each one, list all time domain functions that are included in the corresponding time-domain functions (e.g. sin function: $\sin(2t)$; exponential: e^{-2t} , ramp: $2t$; constant 2; a time delay by amount θ , etc.). This may require you to find some terms in the partial fraction expansion, but you do not need to find the coefficients of each function of time.

a. $A(s) = \frac{5}{s(s^2 + 6s + 5)}$

b. $B(s) = \frac{2}{s^2 + 4s}$

c. $C(s) = \frac{0.5}{s^2 + 0.25}$

d. $D(s) = \frac{1}{s+1} + \frac{1}{s-1}$

e. $E(s) = \frac{3e^{-5s}}{s(s+2)}$

3. For each function in the previous question, indicate whether the time-domain function converges to a final value. If it has a final value, show that it can be found using the *final value theorem*.
4. Use Laplace transforms to solve the ODE:

$$\frac{dy}{dt} + ay(t) = ct; \quad y(0) = 0$$

5. a. Consider the integral in the time domain:

$$\int_0^t f(t^*) dt^*$$

Use the definition of the Laplace transform and integration by parts to find

$$\mathcal{L} \left[\int_0^t f(t^*) dt^* \right] = \int_0^\infty \left[\int_0^t f(t^*) dt^* \right] e^{-st} dt$$

- b. Use your result from above to find the Laplace transform of

$$\tau_1 \frac{dy}{dt} + y(t) = K \left(u(t) + \frac{1}{\tau_a} \int_0^t u(t^*) dt^* \right); \quad y(0) = u(0) = 0$$

- c. Find the ratio:

$$\frac{Y(s)}{U(s)}$$

6. Use the MATLAB functions 'laplace' and 'ilaplace' to confirm the answers to problem 1(d) and problem 2(d).