EXERCISE SET 2.6

Do,
* Question
1, 3, 5

Practice Exercises

In Exercises 1–8, find the domain of each rational function.

1. $f(x) = \dfrac{5x}{x - 4}$

2. $f(x) = \dfrac{7x}{x - 8}$

3. $g(x) = \dfrac{3x^2}{(x - 5)(x + 4)}$

4. $g(x) = \dfrac{2x^2}{(x - 2)(x + 6)}$

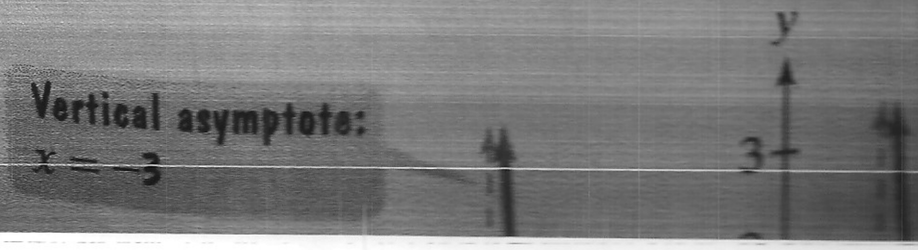
5. $h(x) = \dfrac{x + 7}{x^2 - 49}$

6. $h(x) = \dfrac{x + 8}{x^2 - 64}$

7. $f(x) = \dfrac{x + 7}{x^2 + 49}$

8. $f(x) = \dfrac{x + 8}{x^2 + 64}$

Use the graph of the rational function in the figure shown to complete each statement in Exercises 9–14.



22. $2x^3 - 5x^2 - 6x + 4 = 0$

23. $x^4 - 2x^3 - 5x^2 + 8x + 4 = 0$

24. $x^4 - 2x^2 - 16x - 15 = 0$

Do. Question 25, 27, 29

In Exercises 25–32, find an nth-degree polynomial function with real coefficients satisfying the given conditions. If you are using a graphing utility, use it to graph the function and verify the real zeros and the given function value.

25. $n = 3$; 1 and $5i$ are zeros; $f(-1) = -104$

26. $n = 3$; 4 and $2i$ are zeros; $f(-1) = -50$

27. $n = 3$; -5 and $4 + 3i$ are zeros; $f(2) = 91$

28. $n = 3$; 6 and $-5 + 2i$ are zeros; $f(2) = -636$

29. $n = 4$; i and $3i$ are zeros; $f(-1) = 20$

30. $n = 4$; -2, $-\frac{1}{2}$, and i are zeros; $f(1) = 18$

31. $n = 4$; -2, 5, and $3 + 2i$ are zeros; $f(1) = -96$

32. $n = 4$; -4, $\frac{1}{3}$, and $2 + 3i$ are zeros; $f(1) = 100$

In Exercises 33–38, use Descartes's Rule of Signs to determine the possible number of positive and negative real zeros for each given function.

33. $f(x) = x^3 + 2x^2 + 5x + 4$

34. $f(x)$ ...

EXERCISE SET 2.6

Question
2, 4, 6

Practice Exercises

In Exercises 1–8, find the domain of each rational function.

1. $f(x) = \dfrac{5x}{x - 4}$

* 2. $f(x) = \dfrac{7x}{x - 8}$

3. $g(x) = \dfrac{3x^2}{(x - 5)(x + 4)}$

* 4. $g(x) = \dfrac{2x^2}{(x - 2)(x + 6)}$

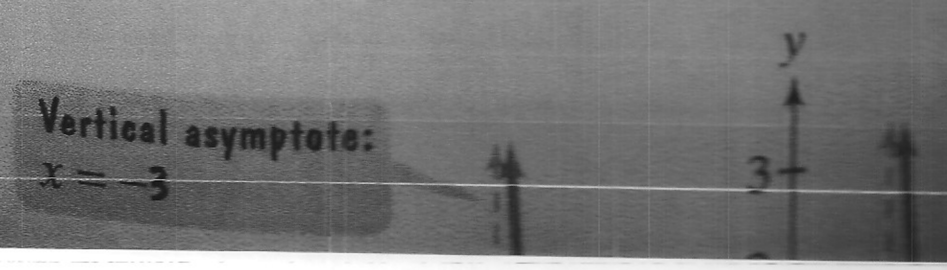
5. $h(x) = \dfrac{x + 7}{x^2 - 49}$

* 6. $h(x) = \dfrac{x + 8}{x^2 - 64}$

7. $f(x) = \dfrac{x + 7}{x^2 + 49}$

8. $f(x) = \dfrac{x + 8}{x^2 + 64}$

Use the graph of the rational function in the figure shown to complete each statement in Exercises 9–14.



18. As $x \to -2^-$, $f(x) \to$ ______.

19. As $x \to \infty$, $f(x) \to$ ______.

20. As $x \to -\infty$, $f(x) \to$ ______.

In Exercises 21–36, find the vertical asymptotes, if any, values of x corresponding to holes, if any, of the graph rational function.

* 21. $f(x) = \dfrac{x}{x+4}$

22. $f(x) = \dfrac{x}{x-3}$

* 23. $g(x) = \dfrac{x+3}{x(x+4)}$

24. $g(x) = \dfrac{x+3}{x(x-3)}$

25. $h(x) = \dfrac{x}{x(x+4)}$

26. $h(x) = \dfrac{x}{x(x-3)}$

Question 21, 23

Questions
31, 32, 37
39, 41

27. $r(x) = \dfrac{x}{x^2 + 4}$

28. $r(x) = \dfrac{x}{x^2 + 3}$

29. $f(x) = \dfrac{x^2 - 9}{x - 3}$

30. $f(x) = \dfrac{x^2 - 25}{x - 5}$

X **31.** $g(x) = \dfrac{x - 3}{x^2 - 9}$

X **32.** $g(x) = \dfrac{x - 5}{x^2 - 25}$

33. $h(x) = \dfrac{x + 7}{x^2 + 4x - 21}$

34. $h(x) = \dfrac{x + 6}{x^2 + 2x - 24}$

35. $r(x) = \dfrac{x^2 + 4x - 21}{x + 7}$

36. $r(x) = \dfrac{x^2 + 2x - 24}{x + 6}$

In Exercises 37–44, find the horizontal asymptote, if there is one, of the graph of each rational function.

X **37.** $f(x) = \dfrac{12x}{3x^2 + 1}$

38. $f(x) = \dfrac{15x}{3x^2 + 1}$

X **39.** $g(x) = \dfrac{12x^2}{3x^2 + 1}$

40. $g(x) = \dfrac{15x^2}{3x^2 + 1}$

X **41.** $h(x) = \dfrac{12x^3}{3x^2 + 1}$

42. $h(x) = \dfrac{15x^3}{3x^2 + 1}$

43. $f(x) = \dfrac{-2x + 1}{3x + 5}$

44. $f(x) = \dfrac{-3x + 7}{5x - 2}$

In Exercises 45–56, use transformations of $f(x) = \frac{1}{x}$ or $f(x) =$ to graph each rational function.

45. $g(x) = \dfrac{1}{}$

46. $g(x) = \dfrac{1}{}$

75. $f(x) = \dfrac{x^4}{x^2 + 2}$

76. $f(x) = \dfrac{\quad}{x^2 + 1}$

77. $f(x) = \dfrac{x^2 + x - 12}{x^2 - 4}$

78. $f(x) = \dfrac{x^2}{x^2 + x - 6}$

79. $f(x) = \dfrac{3x^2 + x - 4}{2x^2 - 5x}$

80. $f(x) = \dfrac{x^2 - 4x + 3}{(x + 1)^2}$

In Exercises 81–88, **a.** *Find the slant asymptote of the graph of each rational function and* **b.** *Follow the seven-step strategy and use the slant asymptote to graph each rational function.*

81. $f(x) = \dfrac{x^2 - 1}{x}$

82. $f(x) = \dfrac{x^2 - 4}{x}$

83. $f(x) = \dfrac{x^2 + 1}{x}$

84. $f(x) = \dfrac{x^2 + 4}{x}$

85. $f(x) = \dfrac{x^2 + x - 6}{x - 3}$

86. $f(x) = \dfrac{x^2 - x + 1}{x - 1}$

87. $f(x) = \dfrac{x^3 + 1}{x^2 + 2x}$

88. $f(x) = \dfrac{x^3 - 1}{x^2 - 9}$

Practice Plus

Questions 81, 85, 86

In Exercises 89–94, the equation for f is given by the simplified expression that results after performing the indicated operation. Write the equation for f and then graph the function.

89. $\dfrac{5x^2}{x^2 - 4} \cdot \dfrac{x^2 + 4x + 4}{10x^3}$

90. $\dfrac{x - 5}{10x - 2} \div \dfrac{x^2 - 10x + \ldots}{25x^2 - \ldots}$

91. $\dfrac{x}{2x + 6} - \dfrac{9}{x^2 - 9}$

92. $\dfrac{2}{\ldots}$

Practice Exercises

Solve each polynomial inequality in Exercises 1–42 and graph the solution set on a real number line. Express each solution set in interval notation.

Question, 1, 9, 17

1. $(x - 4)(x + 2) > 0$
2. $(x + 3)(x - 5) > 0$
3. $(x - 7)(x + 3) \le 0$
4. $(x + 1)(x - 7) \le 0$
5. $x^2 - 5x + 4 > 0$
6. $x^2 - 4x + 3 < 0$
7. $x^2 + 5x + 4 > 0$
8. $x^2 + x - 6 > 0$
9. $x^2 - 6x + 9 < 0$
10. $x^2 - 2x + 1 > 0$
11. $3x^2 + 10x - 8 \le 0$
12. $9x^2 + 3x - 2 \ge 0$
13. $2x^2 + x < 15$
14. $6x^2 + x > 1$
15. $4x^2 + 7x < -3$
16. $3x^2 + 16x < -5$
17. $5x \le 2 - 3x^2$
18. $4x^2 + 1 \ge 4x$
19. $x^2 - 4x \ge 0$
20. $x^2 + 2x < 0$
21. $2x^2 + 3x > 0$
22. $3x^2 - 5x \le 0$
23. $-x^2 + x \ge 0$
24. $-x^2 + 2x \ge 0$
25. $x^2 \le 4x - 2$
26. $x^2 \le 2x + 2$
27. $9x^2 - 6x + 1 < 0$
28. $4x^2 - 4x + 1 \ge 0$
29. $(x - 1)(x - 2)(x - 3) \ge 0$
30. $(x + 1)(x + 2)(x + 3) \ge 0$
31. $x(3 - x)(x - 5) \le 0$
32. $x(4 - x)(x - 6) \le 0$
33. $(2 - x)^2\left(x - \frac{7}{2}\right) < 0$
34. $(5 - x)^2\left(x - \frac{13}{2}\right) < 0$
35. $x^3 + 2x^2 - x - 2 \ge 0$
36. $x^3 + 2x^2 - 4x - 8 \ge 0$
37. $x^3 - 3x^2 - 9x + 27 < 0$
38. $x^3 + 7x^2 - x - 7 < 0$
39.

$x^3 - x^2 + 9x - 9 > 0$

47. $f(x) = 3^x$ and $g(x) = 3$

48. $f(x) = 3^x$ and $g(x) = -3^x$

49. $f(x) = 3^x$ and $g(x) = \frac{1}{3} \cdot 3^x$

50. $f(x) = 3^x$ and $g(x) = 3 \cdot 3^x$

51. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1} + 1$

52. $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \left(\frac{1}{2}\right)^{x-1} + 2$

Use the compound interest formulas $A = P\left(1 + \frac{r}{n}\right)^{nt}$ *and* $A = Pe^{rt}$ *to solve Exercises 53–56. Round answers to the nearest cent.*

Questions 54, 55

53. Find the accumulated value of an investment of $10,000 for 5 years at an interest rate of 5.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

54. Find the accumulated value of an investment of $5000 for 10 years at an interest rate of 6.5% if the money is **a.** compounded semiannually; **b.** compounded quarterly; **c.** compounded monthly; **d.** compounded continuously.

55. Suppose that you have $12,000 to invest. Which investment yields the greater return over 3 years: 7% compounded monthly or 6.85% compounded continuously?

56. Suppose that you have $6000 to invest. Which investment yields the greater return over 4 years: 8.25% compounded quarterly or 8.3% compounded semiannually?

Practice Plus

In Exercises 57–58, graph f and g in the same rectangular coordinate system. Th

CONCEPT AND VOCABULARY CHECK

Fill in each blank so that the resulting statement is true.

Question 1, 5, 9

1. $y = \log_b x$ is equivalent to the exponential form ________, $x > 0, b > 0, b \neq 1$.
2. The function $f(x) = \log_b x$ is the ________ function with base ________.
3. $\log_b b =$ ________
4. $\log_b 1 =$ ________
5. $\log_b b^x =$ ________
6. $b^{\log_b x} =$ ________
7. Using interval notation, the domain of $f(x) = \log_b x$ is ________ and the range is ________.
8. The graph of $f(x) = \log_b x$ approaches, but does not touch, the ________-axis. This axis, whose equation is ________, is a/an ________ asymptote.
9. The graph of $g(x) = 5 + \log_2 x$ is the graph of $f(x) = \log_2 x$ shifted ________.

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Question 11, 14

10. The graph of $g(x) = \log_3(x + 5)$ is the graph of $f(x) = \log_3 x$ shifted __________.

11. The graph of $g(x) = -\log_4 x$ is the graph of $f(x) = \log_4 x$ reflected about the __________.

12. The graph of $g(x) = \log_5(-x)$ is the graph of $f(x) = \log_5 x$ reflected about the __________.

13. The domain of $g(x) = \log_2(5 - x)$ can be found by solving the inequality __________.

14. The logarithmic function with base 10 is called the __________ logarithmic function. The function $f(x) = \log_{10} x$ is usually expressed as $f(x) =$ __________.

15. The logarithmic function with base e is called the __________ logarithmic function. The function $f(x) = \log_e x$ is usually expressed as $f(x) =$ __________.

Questions 21, 25, 27, 31, 24, 40

In Exercises 21–42, evaluate each expression without using a calculator.

21. $\log_4 16$ **22.** $\log_7 49$ **23.** $\log_2 64$ **24.** $\log_3 27$

25. $\log_5 \frac{1}{5}$ **26.** $\log_6 \frac{1}{6}$ **27.** $\log_2 \frac{1}{8}$ **28.** $\log_3 \frac{1}{9}$

29. $\log_7 \sqrt{7}$ **30.** $\log_6 \sqrt{6}$ **31.** $\log_2 \frac{1}{\sqrt{2}}$ **32.** $\log_3 \frac{1}{\sqrt{3}}$

33. $\log_{64} 8$ **34.** $\log_{81} 9$ **35.** $\log_5 5$ **36.** $\log_{11} 11$

37. $\log_4 1$ **38.** $\log_6 1$ **39.** $\log_5 5^7$ **40.** $\log_4 4^6$

41. $8^{\log_8 19}$ **42.** $7^{\log_7 23}$

43. Graph $f(x) = 4^x$ and $g(x) = \log_4 x$ in the same rectangular coordinate system.

44. Graph $f(x) = 5^x$ and $g(x) = \log_5 x$ in the same rectangular coordinate system.

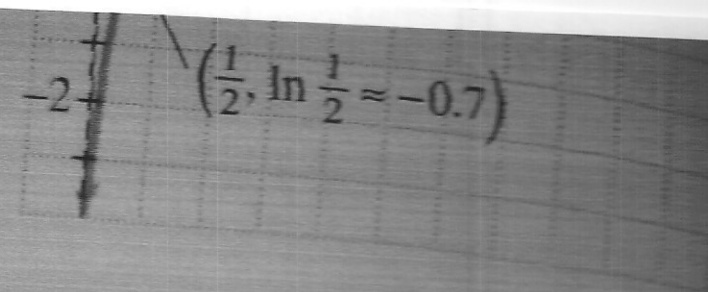
45. Graph $f(x) = \left(\frac{1}{2}\right)^x$ and $g(x) = \log_{\frac{1}{2}} x$ in the same rectangular coordinate system.

46. Graph $f(x) = \left(\frac{1}{4}\right)^x$ and $g(x) = \log_{\frac{1}{4}} x$ in the same rectangular coordinate system.

In Exercises 47–52, the graph of a logarithmic function is given. Select the function for each graph from the following options:

$$f(x) = \log_3 x,\ g(x) = \log_3(x-1),\ h(x) = \log_3 x - 1,$$

$$F(x) = -\log_3 x,\ G(x) = \log_3(-x),\ H(x) = 1 - \log_3 x$$



65. $g(x) = \ln(x+2)$

66. $g(x) = \ln(x+1)$

67. $h(x) = \ln(2x)$

68. $h(x) = \ln\left(\frac{1}{2}x\right)$

69. $g(x) = 2\ln x$

70. $g(x) = \frac{1}{2}\ln x$

71. $h(x) = -\ln x$

72. $h(x) = \ln(-x)$

73. $g(x) = 2 - \ln x$

74. $g(x) = 1 - \ln x$

In Exercises 75–80, find the domain of each logarithmic function.

75. $f(x) = \log_5(x+4)$

76. $f(x) = \log_5(x+6)$

77. $f(x) = \log(2-x)$

78. $f(x) = \log(7-x)$

79. $f(x) = \ln(x-2)^2$

80. $f(x) = \ln(x-7)^2$

In Exercises 81–100, evaluate or simplify each expression without using a calculator.

X **81.** $\log 100$ | **82.** $\log 1000$ | **83.** $\log 10^7$ | X **84.** $\log 10^8$

X **85.** $10^{\log 33}$ | **86.** $10^{\log 53}$ | **87.** $\ln 1$ | **88.** $\ln e$

89. $\ln e^6$ | **90.** $\ln e^7$ | X **91.** $\ln\frac{1}{e^6}$ | **92.** $\ln\frac{1}{e^7}$

93. $e^{\ln 125}$ | **94.** $e^{\ln 300}$ | X **95.** $\ln e^{9x}$ | **96.** $\ln e^{13x}$

97. $e^{\ln 5x^2}$ | **98.** $e^{\ln 7x^2}$ | **99.** $10^{\log\sqrt{x}}$ | **100.** $10^{\log\sqrt[3]{x}}$

Questions, 81, 84, 85, 91, 95

Practice Exercises

Question 10, 12, 23, 28, 33, 40

In Exercises 1–40, use properties of logarithms to expand each logarithmic expression as much as possible. Where possible, evaluate logarithmic expressions without using a calculator.

1. $\log_5(7 \cdot 3)$
2. $\log_8(13 \cdot 7)$
3. $\log_7(7x)$
4. $\log_9(9x)$
5. $\log(1000x)$
6. $\log(10{,}000x)$
7. $\log_7\left(\frac{7}{x}\right)$
8. $\log_9\left(\frac{9}{x}\right)$
9. $\log\left(\frac{x}{100}\right)$
10. $\log\left(\frac{x}{1000}\right)$ *
11. $\log_4\left(\frac{64}{y}\right)$
12. $\log_5\left(\frac{125}{y}\right)$ *
13. $\ln\left(\frac{e^2}{5}\right)$
14. $\ln\left(\frac{e^4}{8}\right)$
15. $\log_b x^3$
16. $\log_b x^7$
17. $\log N^{-6}$
18. $\log M^{-8}$
19. $\ln\sqrt[5]{x}$
20. $\ln\sqrt[7]{x}$
21. $\log_b(x^2y)$
22. $\log_b(xy^3)$
23. $\log_4\left(\frac{\sqrt{x}}{64}\right)$ *
24. $\log_5\left(\frac{\sqrt{x}}{25}\right)$
25. $\log_6\left(\frac{36}{\sqrt{x+1}}\right)$
26. $\log_8\left(\frac{64}{\sqrt{x+1}}\right)$
27. $\log_b\left(\frac{x^2y}{z^2}\right)$
28. $\log_b\left(\frac{x^3y}{z^2}\right)$ *
29. $\log\sqrt{100x}$
30. $\ln\sqrt{ex}$
31. $\log\sqrt[3]{\frac{x}{y}}$
32. $\log\sqrt[5]{\frac{x}{y}}$
33. $\log_b\left(\frac{\sqrt{x}y^3}{z^3}\right)$ *
34. $\log_b\left(\frac{\sqrt[3]{x}y^4}{z^5}\right)$
35. $\log_5\sqrt[3]{\frac{x^2y}{25}}$
36. $\log_2\sqrt[5]{\frac{xy^4}{16}}$
37. $\ln\left[\frac{x^3\sqrt{x^2+1}}{(x+1)^4}\right]$
38. $\ln\left[\frac{x^4\sqrt{x^2+3}}{(x+3)^5}\right]$
39. $\log\left[\frac{10x^2\sqrt[3]{1-x}}{7(x+1)^2}\right]$
40. $\log\left[\frac{100x^3\sqrt[3]{5-x}}{3(x+7)^2}\right]$ *

In Exercises 41–70, use properties of logarithms to condense each logarithmic expression as a single logarithm.

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74. log
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89. ln e
91. log
93. x log
95. ln(5
97. log

69. $\log x + \log(x^2 - 1) - \log 7 - \log(x + 1)$

70. $\log x + \log(x^2 - 4) - \log 15 - \log(x + 2)$

In Exercises 71–78, use common logarithms or natural logarithms and a calculator to evaluate to four decimal places.

71. $\log_5 13$
72. $\log_6 17$
73. $\log_{14} 87.5$
74. $\log_{16} 57.2$
75. $\log_{0.1} 17$
76. $\log_{0.3} 19$
77. $\log_\pi 63$
78. $\log_\pi 400$

In Exercises 79–82, use a graphing utility and the change-of-base property to graph each function.

79. $y = \log_3 x$
80. $y = \log_{15} x$
* **81.** $y = \log_2(x + 2)$
82. $y = \log_3(x - 2)$

Practice Plus

In Exercises 83–88, let $\log_b 2 = A$ *and* $\log_b 3 = C$. *Write each expression in terms of A and C.*

83. $\log_b \frac{3}{2}$
* **84.** $\log_b 6$
* **85.** $\log_b 8$
86. $\log_b 81$
87. $\log_b \sqrt{\frac{2}{27}}$
88. $\log_b \sqrt{\frac{3}{16}}$

In Exercises 89–102, determine whether each equation is true or false. Where possible, show work to support your conclusion. If the statement is false, make the necessary change(s) to produce a true statement.

3. If $e^{0.6x} = 6$, then $0.6x =$ ________

4. If $\log_5(x + 1) = 3$, then ________ $= x + 1$.

5. If $\log_3 x + \log_3(x + 1) = 2$, then $\log_3$ ________ $= 2$.

6. If $\ln\left(\frac{7x - 23}{x + 1}\right) = \ln(x - 3)$, then ________ $= x - 3$.

9. True or false: $4^x = 15$ is an exponential equation. ________

10. True or false: -3 is a solution of $\log_5 9 = 2\log$ ________

11. True or false: -10 is a solution of $\log_5(x + 3$ ________

EXERCISE SET 3.4

Practice Exercises

Solve each exponential equation in Exercises 1–22 by expressing each side as a power of the same base and then equating exponents.

1. $2^x = 64$
2. $3^x = 81$
3. $5^x = 125$
4. $5^x = 625$
5. $2^{2x-1} = 32$
6. $3^{2x+1} = 27$
7. $4^{2x-1} = 64$
8. $5^{3x-1} = 125$
9. $32^x = 8$
10. $4^x = 32$
11. $9^x = 27$
12. $125^x = 625$
13. $3^{1-x} = \frac{1}{27}$
14. $5^{2-x} = \frac{1}{125}$