1. (10 points) Given $f(\vec{r}) = 4x^2 + 5xy + 7y^3 + 8z + 9$. Find the following:

(a). $\frac{\partial f}{\partial x}$, (b). $\frac{\partial f}{\partial y}$, (c). $\frac{\partial f}{\partial z}$ (d). $\vec{\nabla} f$, (e). $\mathrm{d}f$

(f). Laplacian of function f, $\Delta f = \vec{\nabla}^2 f$

2. (5 points) Given $\vec{T} = kxy^2\vec{i} + 3yz\vec{j} + mxyz^3\vec{k}$. Find the following:

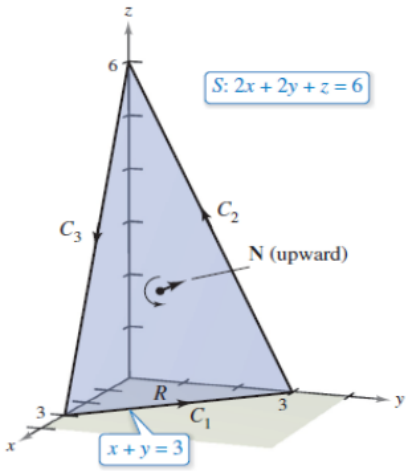
(a). Divergence of function $\vec{T}$, $\vec{\nabla} \bullet \vec{T}$

(b). Curl of function $\vec{T}$, $\vec{\nabla} \times \vec{T}$

3. (5 points) Let *C* be the oriented triangle lying in the plane 2*x* + 2*y* + *z* = 6, as shown in the Figure.

Evaluate $\int_C \vec{F} \bullet d\vec{r}$ using Stokes's theorem, where

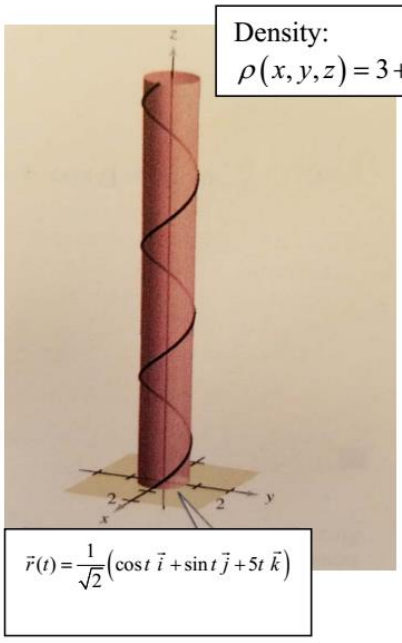
$\vec{F}(x, y, z) = -y^2\vec{i} + z\vec{j} + x\vec{k}$



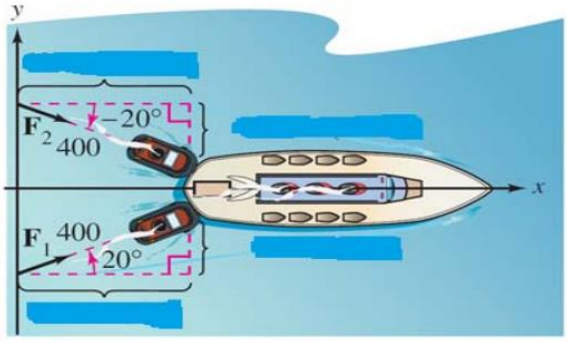
4. (10 points) Find the mass of a spring in the shape of the circular helix

$$\vec{r}(t)=\frac{1}{\sqrt{2}}\left(\cos t\ \vec{i}+\sin t\ \vec{j}+5t\ \vec{k}\right)$$

Where $0\le t\le 6\pi$ and the density of the spring is $\rho(x,y,z)=3+z$.



5. (5 points) Two tugboats are pushing an ocean liner, as shown in the figure. Each boat is exerting a force of 400 pounds. What is the resultant force on the ocean liner?



6. (10 points) The temperature at any point (x, y) in a steel plate is

$$T = 350 - 0.3x^2 - 0.4y^2$$

Where x and y are measured in meters. At the point $(1, 2)$, find the rates of changes of the temperature with respect to the distances moved along the plate in the directions of the $x-$ and $y-$ axes.

7. (10 points) The temperature in degrees Celsius on the surface of a metal plate is $T(x, y) = 50 - 3x^2 - 5y$, where x and y are measured in centimeters. Estimate the average temperature when x varies between 0 and 1 and y varies between 0 and 2 centimeters.

8. (10 points) Find the mass of the solid of density $\rho(x, y, z) = kxyz$ bounded by the graphs of the equations $x = 0, x = a, y = 0, y = b, z = 0, z = c$.

9. (5 points) Write the following laws in integral and differential forms:
(a). Faraday-Maxwell
(b). Ampere-Maxwell
(c). Gauss-Electric Flux
(d). Gauss-Magnetic Flux

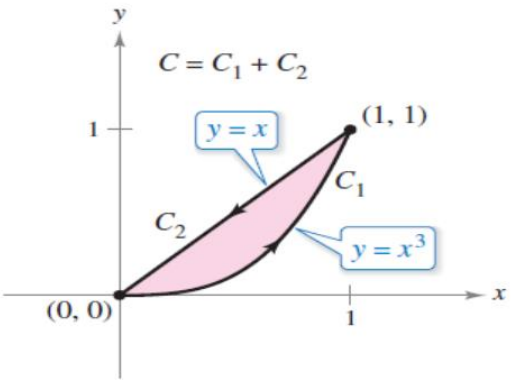
10. (5 points) The ideal gas law states that $PV = nRT$, where R is pressure of the gas, V is volume taken up by the gas, n is number of moles of the gas , T is the temperature of the gas. Find the following:

(a). $\frac{\partial P}{\partial V}$, (b). $\frac{\partial V}{\partial T}$, (c). $\frac{\partial T}{\partial P}$ (d). Show that $\left(\frac{\partial P}{\partial V}\right)\left(\frac{\partial P}{\partial V}\right)\left(\frac{\partial V}{\partial T}\right) = -1$

11. (5) points) Use Green's Theorem to evaluate the line integral

$$\int_C y^3\,dx + (x^3 + 3xy^2)\,dy$$

where *C* is the path from (0, 0) to (1, 1) along the graph of $y = x^3$ and from (1, 1) to (0,0) along the graph of $y = x$, as shown in below Figure .



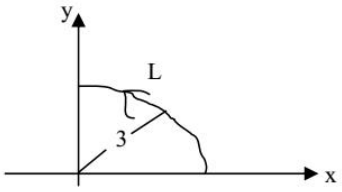
C is simple and closed, and the region *R* always lies to the left of *C*.

12. (5 points) Given the velocity potential by $f(x,y)$ of a flow, find the velocity $\vec{V} = \vec{\nabla} f$ of the field and its value $\vec{V}(P)$ at P. Where $f(x,y) = 3x^2 - 8x + y^3, P:(-2,3)$.

13. (5 points) Find the force on a charge Q_1, $10\mu C$, due to charge, Q_2, $-200\mu C$, where Q_1 is at $(0,1,2)$ m and Q_2 is at $(2,0,0)$ m.

14. (5 points) Find $\vec{E}$ at (0,2,3)m in Cartesian coordinates due to a point charge $Q = 0.5\mu C$ at the origin.

15. (5 points) Find the line integral of $\vec{F} = 9xy^2\vec{i} + (x^2+3)y\vec{j}$ over the path L.



1. To close a sliding door, a person pulls on a rope with a constant force of 60 pounds at a constant angle of 45°. Find the work done in moving the door 4 feet to its closed position.

2. A vertical force of 40 pounds is applied to the end of a 2 feet lever that is attached to an axle at point P as shown in the Figure. Find the moment of this force about the point P when $\theta = 60^\circ$.