1. There are N symmetric generators, each owned by a generation company. Each generator has a capacity K. For example, $K = 100$ Mega-Watts (MW) corresponds to a typical generator. The generator can produce electricity in any quantity between $[0, K]$. The generator has a constant marginal cost c in \$/MWh. For example, $c = \$10$/MWh means the generator's production cost to generation 1MWh of electric energy is \$10.

2. Each company i submits a bid for its own generator to the ISO. The bid consists a price b_i, at which the company wants to sell its electricity. According to the ISO's market rule, the bidding price b_i cannot exceed a price cap Φ to prevent the electricity from rising too high (political pressure would be too high at these times!), and $\Phi > c$. All generation companies submit the bids simultaneously and do not know each other's bid.

3. Electricity demand is represented by a demand function as $D(p) = d_0 - k(p - p_0)$. Here p is the electricity price, p_0 is a constant representing a nominal price level, d_0 is the demand corresponding to $p = p_0$, and $k \geq 0$ specifies the slope of the demand function. If $k > 0$, the demand function is called elastic, since it will change with respect to price p; if $k = 0$, the demand is called inelastic. In the traditional electricity markets, demand is usually quite inelastic, but there is a growing trend to make demand more responsive to the electricity price. In this project, we will analyze both elastic and inelastic demand. We also assume there is always enough production capacity to cover the largest possible demand, i.e. $NK > D(c)$. This is called resource adequacy. It is always ensured in real world electricity markets (in normal operating conditions. Google California Electricity Crisis if you want to learn more about market failure.)

4. Market clearing process: After all the companies submit their bids, the ISO will clear the market and compute the market price. In particular, the ISO constructs a system supply function using all the submitted bids and finds the intersection of the supply function and the demand function. The intersection gives the market price p_M and the cleared total electricity.

   Let us see an example of how the market clears.

   (a) Suppose there are $N = 5$ companies in the market, each having a capacity $K = 10$ and marginal cost $c = 1$ and price cap $\Phi = 10$. The demand function is $D(p) = 25 - 4.56(p - 5.5)$.

   (b) Suppose five companies submit offer bids: $\{4, 2, 1, 3, 5\}$, i.e. company 1 bids $b_1 = 4$, company 2 bids $b_2 = 2$, and $b_3 = 1$, $b_4 = 3$, $b_5 = 5$.

   (c) Then, the ISO constructs a system supply function by stacking the offer bids from the lowest price to the highest price. The blue step curve in Figure 1 shows the system supply function. The red curve is the demand function.

   (d) The intersection of supply and demand functions at the black dot determines the market price $p_M = 4$ and the total cleared demand is $D(p_M) = 25 - 4.56(4 - 5.5) = 31.8$.

   (e) All the companies bidding below the market price will produce at full capacity K, i.e. the production quantity $q_i = 10$ for companies $i = 2, 3, 4$. Any company bidding above p_M will produce nothing, i.e. $q_5 = 0$. Any company bidding at p_M is called the marginal player (because its bid sets the market price), and it will produce the remaining demand, i.e. $q_4 = D(p_M) - 3K = 31.8 - 30 = 1.8$. In case there are multiple marginal players, then

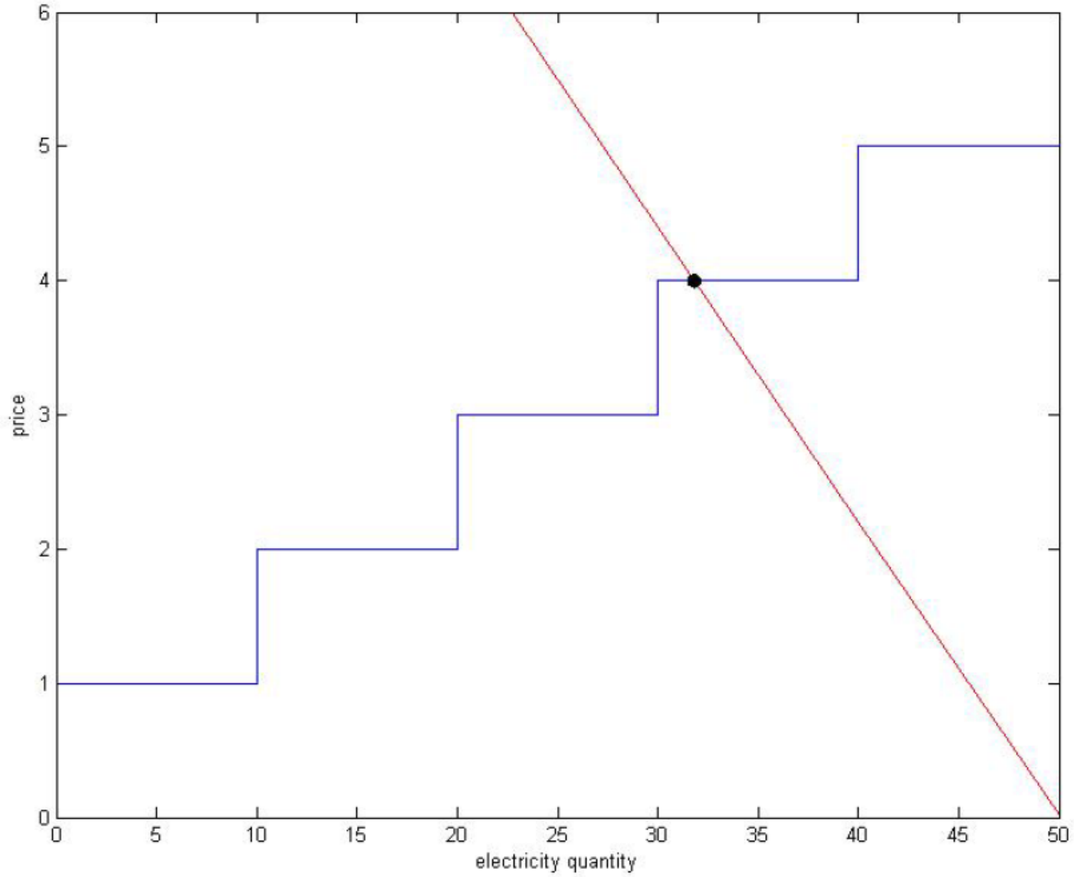

Figure 1: Market clearing example

one of them is selected at random to produce the residual demand, while the other marginal player(s) produces at full capacity. For example, if every company bids their marginal cost $b_i = 1$ in this example. The market price is $p_M = 1$ and the cleared demand is $D(c) = 45.5$. Then every company is marginal. One of them is selected as the residual producer, all the other four produce at full capacity K. The residual producer produces $45.5 - 40 = 5.5$.

(f) All generators that produce electricity are paid at the uniform market price p_M. Company i's profit is $u_i = (p_M - c)q_i$. This pricing scheme is called uniform pricing. Each generation company wants to maximize its profit.

(g) It is also possible that the demand function intersects with the supply function at one of its vertical segments. For example, if the demand function in Figure 1 is shifted downward a bit, it may intersect with the supply function at price 3.5 and demand 30, which is at the vertical segment between price 3 and 4. In this case, the market price $p_M = 3.5$ and cleared demand is 30. Generators 2,3,4 produce at full capacity, and the other two produce nothing.

5. This is a one-shot game. And all the above information is common knowledge to all companies. In reality, the same generation companies participate in electricity auctions every day. It would be more appropriate to model the game as a repeated game. But for our purpose, let us assume a one-shot game.

The above electricity market model is simple, yet captures some key features of the real-world electricity markets, such as generators bid prices (rather than quantity) through a sealed-bid auction, demand can be elastic, a price cap exists, and market price is determined by a uniform pricing scheme. Generation companies' profits depend on each other's bids. Therefore, this is a situation of strategic interaction. We want to analyze the behavior of the generation companies in this game.

Answer the following questions with detailed derivation and discussion, which are designed to guide you through a sequence of more and more complicated scenarios to get a better understanding of this one-shot game.

1. Suppose there is only one generation company in the market, i.e., it is a monopoly market. The generation company has enough capacity to satisfy the highest possible demand. The generator capacity K, marginal cost c, price cap Φ.
   (a) Inelastic demand $D = d_0$: Find the optimal bidding price.
   (b) Elastic demand function $D(p) = d_0 - k(p - p_0)$: Find the optimal bidding price.
   (c) Assume $p_0 + d_0/k > c$. In which case, inelastic demand or elastic demand, does the monopoly company have a higher profit?

2. Now suppose there are two generation companies in the market, companies 1 and 2, i.e., this is a duopoly market. Analyze the duopoly market when the demand is inelastic, i.e. $D(p) = d_0$, and both generators are needed to satisfy demand, i.e. $K < d_0 < 2K$.
   (a) Find company 1's profit $u_1(b_1, b_2)$. Plot it as a function of b_1 with b_2 fixed.
   (b) Find the best response of company 1 as a mapping of b_2. Plot it. Note the best response may be a set function, i.e., the best response to a bid can be a set, rather than a point. Specify this best response mapping.
   (c) Find all the pure strategy Nash equilibria of this game, if there is any. Otherwise, argue why there is no Nash equilibrium.

3. Still consider the duopoly market with inelastic demand, but now the demand can be satisfied by one generator, i.e. $0 < d_0 < K$. Is there a Nash equilibrium? If yes, find one.

4. Now consider a market of N companies. Here the number of players in the market and their capacity has quite significant implications on equilibrium behavior. First consider the situation where every generator is needed to satisfy the largest possible demand, i.e. $(N-1)K < D(c) < NK$.

   Define p^* to be the price that maximizes the following problem:

   $$p^* = \arg \max_{c \le b \le \Phi} (p - c)\left(D(p) - (N-1)K\right).$$

   That is, p^* is the best market price that maximizes the marginal player's profit to satisfy the residual demand $D(p) - (N-1)K$, while all other companies produce at full capacity.

   Consider the following strategy profile: one company bids p^*, and all other companies bid $b_i < p^*$. Can this strategy profile form a Nash equilibrium? What b_i's are needed in the Nash equilibrium, if there is any? Also, is there a unique Nash equilibrium or multiple Nash equilibria, if there is any?

5. Now consider a market of N companies where $D(c) \le (N-1)K$. Is there any Nash equilibrium? Find one if there is any. Is there a unique or multiple Nash equilibria, if any?