Fundamentals of Real Estate Appraisal

BUSN 221
Fall [illegible]

William L. Ventolo, Jr. & Martha R. Williams, JD

Twelfth Edition

Dearborn
Real Estate Education

6. The purpose of *USPAP* is to
   a. delay government regulation.
   b. present information that will be meaningful to the client and will not be misleading in the marketplace.
   c. guarantee professionalism in appraisers.
   d. present information that will be useful to appraisers.

7. A certified appraiser is required for federally related transactions involving property valued at more than
   a. $250,000.
   b. $500,000.
   c. $750,000.
   d. $1,000,000.

8. The law that Congress passed to assist borrowers affected by the subprime lending crisis is the
   a. Financial Institutions Reform, Recovery, and Enforcement Act of 1989.
   b. Housing and Economic Recovery Act of 2008.
   c. Tax Reform Act of 1986.
   d. Fair Housing Amendments Act of 1988.

9. The appraisal coursework required by the AQB for a certified general real estate appraiser totals
   a. 75 hours.
   b. 150 hours.
   c. 300 hours.
   d. 500 hours.

Check your answers against those in the answer key at the back of the book.

CHAPTER TWO

APPRAISAL MATH AND STATISTICS

KEY TERMS

aggregate
area
array
average deviation
bell curve
compound interest
decimal
fraction
frequency distribution
mean
measures of central tendency
median
mode
normal distribution
outlier
percent
parameter
population
random sample
range
sample
simple interest
skewness
square foot
standard deviation
variate

LEARNING OBJECTIVES

After successfully completing this chapter, you should be able to

convert percentages to decimals, decimals to percentages, and fractions to decimals;

solve percentage problems;

calculate the area of squares, rectangles, triangles, and irregular closed figures;

convert various units of measure;

compute the amount of living area in a house;

compute the volume of triangular prisms;

define terms used in the study of statistics;

interpret and analyze statistical data when presented in an array and frequency distribution;

calculate the mean, median, and mode;

distinguish between the formula for finding the standard deviation of an entire population and the formula for finding the standard deviation of a sample;

calculate the range, average deviation, and standard deviation;

prepare statistical data in the form of tables, bar charts, histograms, and line graphs; and

describe regression analysis.

■ OVERVIEW

Math is used every day in the real estate business. Sometimes it involves a simple measurement of land area. At other times, it involves a complex investment analysis that requires sophisticated computers programmed with compound interest schedules and multiple regression tables.

This chapter covers some of the basics of real estate math. You will learn to work with percentages, decimals, and fractions, as is routine with the appraiser. In addition, this chapter provides a review of the mathematics involved in computing area and volume. A home appraisal generally requires the use of area in some way. When comparing properties, for example, the appraiser must be able to determine the size of a lot in square feet, the amount of floor space in a room or house, and the construction cost per square foot of various components of the house.

Many boundaries, lots, and houses are irregularly shaped; that is, they are not rectangular. Appraisers should know how to compute the area (and volume, where applicable) of just about any shape they may encounter.

Other mathematical computations with which the appraiser should be familiar, such as compound interest, are also discussed. The chapter ends with a brief introduction to statistics. Statistical analysis has particular application to the work of the appraiser because appraisers are continually drawing inferences about populations or markets from samples.

Because readers will have varying degrees of knowledge and experience in math computations, the questions at the end of this chapter may be used to review such computations. They will indicate whether all the material in this chapter should be studied.

CALCULATORS

Calculators are a great aid in the real estate business, and you should know how to use one. State licensing exams generally allow you to use a silent, handheld calculator, as long as it does not have a printout tape. A common four-function calculator (+, –, ×, ÷, plus % and √ keys) is sufficient for most problems you will encounter on the job or on exams. Most models have the ability to store and recall numbers in a built-in memory, which reduces the need to write down answers to be used in subsequent calculations.

If you plan to buy a new calculator, however, a good-quality financial calculator might be a better choice. This type of calculator has a memory and is also capable of performing all the mathematical and financial operations you will probably need. A financial calculator will include these additional function keys:

Key	Function
n	Number of interest compounding periods
i	Amount of interest per compounding period
PMT	Payment
PV	Present value
FV	Future value

These five keys on the financial calculator can handle almost every conceivable problem dealing with finance. Although proficiency with the calculator does not guarantee success on exams or on the job, there is little doubt that it does provide a critical edge.

PERCENTAGES

Percent (%) means per hundred or per hundred parts. For example, 50% means 50 parts out of a total of 100 parts (100 parts equals 1 whole), and 100% means all 100 of the 100 total parts, or 1 whole unit.

Converting Percentages to Decimals

To change a percentage to a **decimal**, move the decimal point two places to the left and drop the percent sign (%). All numbers have a decimal point, although it is usually not shown when only zeros follow it.

IN PRACTICE

99 is really 99.0
6 is really 6.0
$1 is the same as $1.00

So, percentages can be readily converted to decimals.

99% = 99.0% = 0.990 = 0.99
6% = 6.0% = 0.060 = 0.06
70% = 70.0% = 0.700 = 0.70

Note: Adding zeros to the right of a decimal point after the last digit does not change the value of the number.

Converting Decimals to Percentages

This process is the reverse of the one you just completed. Move the decimal point two places to the right and add the % sign.

IN PRACTICE

0.10 = 10%
1.00 = 100%
0.98 = 98%
0.987 = 98.7%

Converting Fractions to Decimals

A proper **fraction** is one whose top number is less than its bottom number. Its value is always less than 1.

Examples: $^1/_4$ $^1/_2$ $^3/_{17}$ $^9/_{100}$

The top number in a fraction is called the numerator. The bottom number in a fraction is called the denominator.

To convert a proper fraction to a decimal, divide the fraction's numerator by its denominator.

$^5/_8$

$5 \div 8 = 0.625$

Exercise 2-1

Convert the following fractions to decimals.

$^{2}/_{5}$

$^{37}/_{100}$

$^{1}/_{6}$

Check your answers against those in the answer key at the back of the book.

Percentage Problems

Percentage problems usually involve three elements: percent (rate), total, and part.

IN PRACTICE

Percent		Total		Part
10%	of	100	is	10

IN PRACTICE

Look at the example that follows. A problem involving percentages is really a multiplication problem. To solve this problem, you first convert the percentage to a decimal, and then multiply.

What is 20% of 150?

$150 \times 20\% = ?$
$20\% = 0.20$
$150 \times 0.20 = 30$

Answer: 20% of 150 is 30.

A generalized formula for solving percentage problems is

$$\text{Total} \times \text{Percent} = \text{Part}$$

To solve a percentage problem, you must know the value of two of the elements of this formula. The value that you must find is called the unknown (most often shown in the formula as x).

IN PRACTICE

If 25% of the houses in your area are less than 10 years old and there are 600 houses, how many houses are less than 10 years old?

Total × Percent = Part
$600 \times 25\% = x$
$600 \times 0.25 = 150$

Two additional formulas can be derived from the basic formula percent × total = part. The following diagram will help you remember them:

$$\frac{\text{Part}}{\text{Total} \times \text{Percent}}$$

Because part is over percent in the diagram, you make a fraction of these two elements when looking for a total:

$$\text{Total} = \frac{\text{Part}}{\text{Percent}}$$

Because part is over total, you make a fraction of these two elements when looking for a percentage:

$$\text{Percent} = \frac{\text{Part}}{\text{Total}}$$

Because percent and total are both in the lower part of the diagram, multiply these two to get the part:

$$\text{Part} = \text{Total} \times \text{Percent}$$

IN PRACTICE

If 25% of the houses in your area are less than 10 years old and this amounts to 150 houses, how many houses are there in your area?

Solution: State the formula.

$$\text{Total} = \frac{\text{Part}}{\text{Percent}}$$

Substitute values.

$$\text{Total} = \frac{150}{0.25}$$

Solve the problem.
Total = 600

IN PRACTICE

There are 600 houses in your area, and 150 of them are less than 10 years old. What percentage of the houses are less than 10 years old?

Solution:

$$\text{Percent} = \frac{\text{Part}}{\text{Total}}$$

$$\text{Total} = \frac{150}{600}$$

Percent = 0.25 or 25%

Problem-Solving Strategy

Let's take a look at how to solve word problems. Here's the five-step strategy you should use:

1. Read the problem carefully.
2. Analyze the problem, pick out the important factors, and put those factors into a simplified question, disregarding unimportant factors.
3. Choose the proper formula for the problem.
4. Substitute the figures for the elements of the formula.
5. Solve the problem.

If you use this strategy throughout this chapter and wherever it applies in this text, you'll have an easier time with word problems.

IN PRACTICE

Consider an example that applies the five-step strategy to a typical real estate problem.

1. Read the following problem carefully.

   A house sold for \$140,600, which was 95% of the original list price. At what price was the house originally listed?

2. Analyze the problem, pick out the important factors, and put those factors into a question.

   \$140,600 is 95% of what?

3. Choose the proper formula for the problem.

   $$\text{Total} = \frac{\text{Part}}{\text{Percent}}$$

4. Substitute the figures for the elements of the formula.

   $$\text{Total} = \frac{\$140{,}600}{0.95}$$

5. Solve the problem.
   Total = \$148,000

Let's try another problem, one that you'll deal with in chapters on income capitalization. Assume a property earns a net operating income of \$26,250 per year. What percentage of net operating income (rate) is this, if the property is valued at \$210,000?

You can solve this problem using these capitalization formulas:

$$\frac{\text{Income}}{\text{Rate} \times \text{Value}}$$

$$\frac{\text{Part}}{\text{Total} \times \text{Percent}}$$

Both formulas above are really the same. Think of total as value, part as income, and percent as rate.

Solution: Restate the problem:

What percentage of \$210,000 is \$26,250?

What formula will you use?

$$\text{Rate} = \frac{\text{Income}}{\text{Value}} \quad \text{or} \quad R = \frac{I}{V}$$

Solve the problem:

$$\$26{,}250 \div \$210{,}000 = 0.125 \text{ or } 12.5\%$$

As you have seen, in working with problems involving percentages, there are several ways of stating the same relationship.

For example, you can say that

$$P = B \times R$$

where
P is the percentage of the whole
B is the base, or whole
R is the rate

Or

$$I = RV$$

where
I is the amount of income
R is the rate
V is the value or whole

Or

$$\text{Part} = \text{Whole} \times \text{Rate}$$

Or

$$\text{Part} = \text{Total} \times \text{Percent}$$

IN PRACTICE

To find out how many square feet are in 20% of an acre, you must first know how many square feet are in an acre. This number is 43,560. The problem can be solved by using any of the preceding formulas.

Part = Whole × Rate
Part = 43,560 sq ft × 20%
Part = 43,560 sq ft × 0.20
Part = 8,712 sq ft

Exercise 2-2

If houses in your area have increased in value 8% during the past year and the average price of houses sold last year was $190,000, what is the average price of houses sold today?

A house purchased for $250,000 sold a few years later. The seller received $340,000 after deducting sale expenses. What percentage profit did the seller make on the investment?

Check your answers against those in the answer key at the back of the book.

INTEREST

Interest is the cost of using someone else's money. A person who borrows money is required to repay the loan plus a charge for interest. This charge will depend on the amount borrowed (principal), the length of time the money is used (time), and the percentage of interest agreed on (rate). Repayment, then, involves the return of the principal plus a return on the principal, called interest.

There are two types of interest: simple interest and compound interest.

Simple Interest

Simple interest is interest earned on only the original principal, not on the accrued interest. Interest is always payable for a particular period, whether daily, monthly, annually, or based on some other schedule. With simple interest, the amount earned at the end of each period is withdrawn or placed in a separate account.

The formula for computing simple interest is

$$\text{Principal} \times \text{Rate} \times \text{Time} = \text{Interest}$$

For example, the interest owed on a loan of \$1,000 for 180 days at a rate of 10% per year is \$1,000 × 0.10 × ($^{180}/_{360}$), which can be simplified to \$1,000 × 0.10 × ½, which equals \$50.

The formula for computing simple interest is used to determine the amount of up-front points (also called discount points) that a lender may require as part of the fee for a mortgage loan.

For example, a loan in the amount of \$160,000 may require payment of two points at the time of closing. In that case, the borrower must be prepared to pay \$160,000 × 2% , which is \$160,000 × 0.02, or \$3,200, to the lender as a condition of receiving the loan.

In computing a loan's annual percentage rate (APR) for federal mortgage loan disclosure purposes, one point (1%) paid at the time of origination is considered roughly equal to a one-eighth increase in the rate charged over the life of the loan. This means that if a loan of \$160,000 at an interest rate of 5% requires a payment of two points up front, the annual percentage rate—the actual interest rate paid—is closer to 5¼% . This figure is computed by adding 5% + (2 × $^{1}/_{8}$%), which is 5% + $^{2}/_{8}$%, which can be simplified to 5¼%.

Compound Interest

The term **compound interest** means that the interest is periodically added to the principal, and in effect the new balance (principal plus interest) draws interest. The annual interest rate may be calculated at different intervals, such as annually, semiannually, quarterly, monthly, or daily. When the interest comes due during the compounding period (for instance, at the end of the month), the interest is calculated and accrues, or is added to the principal.

IN PRACTICE

A savings account with a starting balance of \$20,000 earns compound interest at 2% per year compounded annually. To determine how much money will be in the account at the end of two years, follow these steps:

1. Calculate the interest for the first earning period
   \$20,000 × 0.02 = \$400

2. Add the first period's interest to the principal balance
   \$20,000 + \$400 = \$20,400

3. Calculate the interest for the next earning period on the new principal balance
   \$20,400 × 0.02 = \$408

The balance at the end of two years is \$20,000 + \$400 + \$408 = \$20,808.

The annuity and other capitalization techniques covered in Chapter 14 make use of tables of factors derived from formulas that represent the change in return over time at various interest rates. Annuity and reversion factors can be used to determine the amount that must be invested at the required interest rate to yield the desired return. The annuity factors shown in Figure 14.2 represent the present value of \$1 per period. The reversion factors shown in Figure 14.3 represent the present value of \$1.

The factors shown in Figure 2.1 represent the future value of \$1.

The formula for computing future investment value when interest is compounded is

$$\text{Principal} \times (1 + \text{Interest rate})^n = \text{Future value}$$

or

$$P(1 + i)^n = S$$

In the formula, i stands for the interest rate, n stands for the number of compounding periods (such as daily or monthly), and S stands for the sum of principal and all accumulated interest. The interest rate calculation $(1 + i)^n$ does not have to be computed separately for every problem. Instead, it can be found in a table of compound interest factors, such as the one shown in Figure 2.1, which indicates the factor to be applied to find the future value of \$1 at the interest rate and for the period specified.

For example, the factor for computing the future value of \$1 at the end of 10 years at 10% interest, compounded daily, is 2.718282. Using the preceding formula, the future value of an investment of \$1,000 at 10% interest compounded daily will be \$1,000 × 2.718282, or \$2,718.28. An initial investment of \$1,000 at 10%

FIGURE 2.1
10% Compound Interest (Future Value of $1)

	Conversion Period				
Year	Daily	Monthly	Quarterly	Semiannual	Annual
1	1.105171	1.104713	1.103813	1.102500	1.100000
2	1.221403	1.220391	1.218403	1.215506	1.210000
3	1.349859	1.348182	1.344889	1.340096	1.331000
4	1.491825	1.489354	1.484506	1.477455	1.464100
5	1.648721	1.645309	1.638616	1.628895	1.610510
6	1.822119	1.817594	1.808726	1.795856	1.771561
7	2.013753	2.007920	1.996495	1.979932	1.948717
8	2.225541	2.218176	2.203757	2.182875	2.143589
9	2.459603	2.450448	2.432535	2.406619	2.357948
10	2.718282	2.707041	2.685064	2.653298	2.593742

interest compounded annually for 10 years would require a factor of 2.593742, for a total future value of $2,593.74. The more frequent the compounding (in these examples, daily rather than annually), the greater the ultimate return.

Another factor table useful to the investor indicates the individual payment amounts needed to amortize a loan at the required interest rate over the desired period. Figure 2.2 shows the monthly payment required to amortize a loan of $1 at the interest rates indicated. The monthly payment needed to amortize a loan of $100,000 at 10% interest over 20 years can be found by applying the factor of 0.0097 to the loan amount:

$$\$100,000 \times 0.0097 = \$970 \text{ monthly installment}$$

Values in monthly installment tables are frequently expressed in terms of the required payment per $1,000 of loan value. The factor for a loan of $1,000 at 9% interest for a term of five years is 20.80. (The decimal point in each factor shown in Figure 2.2 would be moved three places to the right to show the factor for a loan of $1,000.)

Exercise 2-3

An account has a balance of $35,500 at the end of three years. If interest was compounded at an annual rate of 10% , what amount was originally deposited?

Check your answer against the one in the answer key at the back of the book.

FIGURE 2.2
Monthly Installment Required to Amortize $1

	Interest Rate								
Term of Years	6%	6½%	7%	7½%	8%	8½%	9%	9½%	10%
5	0.0193	0.0196	0.0198	0.0200	0.0203	0.0205	0.0208	0.0210	0.0213
8	0.0131	0.0134	0.0136	0.0139	0.0141	0.0144	0.0147	0.0149	0.0152
10	0.0111	0.0114	0.0116	0.0119	0.0121	0.0124	0.0127	0.0129	0.0133
12	0.0098	0.0100	0.0103	0.0106	0.0108	0.0111	0.0114	0.0117	0.0120
15	0.0083	0.0087	0.0090	0.0093	0.0096	0.0098	0.0101	0.0104	0.0108
18	0.0076	0.0077	0.0082	0.0084	0.0087	0.0091	0.0094	0.0097	0.0100
20	0.0072	0.0075	0.0078	0.0081	0.0084	0.0087	0.0090	0.0093	0.0097
25	0.0064	0.0067	0.0071	0.0074	0.0077	0.0081	0.0084	0.0087	0.0091
30	0.0060	0.0063	0.0067	0.0070	0.0073	0.0077	0.0080	0.0084	0.0088
35	0.0057	0.0060	0.0064	0.0067	0.0071	0.0075	0.0078	0.0082	0.0086
40	0.0055	0.0059	0.0062	0.0066	0.0070	0.0073	0.0077	0.0081	0.0085

Exercise 2-4

A house you like can be bought with a $900,000 loan and $180,000 cash. The interest rate is 7% , and the lender requires a loan fee of 1½ points. What amount of interest must be paid to obtain the loan, and what will the monthly payment be if the loan term is 30 years? (Use the appropriate table in this chapter.)

Check your answer against the one in the answer key at the back of the book.

AREA AND VOLUME

Many mathematical computations occur throughout the appraisal of any structure, from the simplest one-room warehouse to the most complex apartment or office building. The appraiser must find the square footage of the appraised site, the number of square feet of usable structure space, and the total square feet of the ground area the structure covers. In addition, one of the methods used in the cost approach to appraisal requires the measurement of cubic feet of space occupied by the structure.

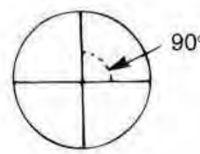
The appraiser should know how to compute the area (and volume, where applicable) of any shape. Property boundaries, particularly those measured by the method known as metes and bounds (also called courses and distances), often are not regularly shaped, and structures usually are not.

Area of Squares and Rectangles

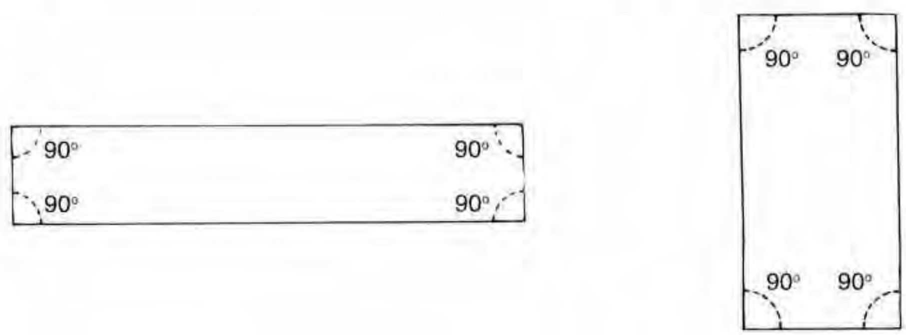
To review some basics about shapes and measurements:

The space inside a two-dimensional shape is called its **area**.

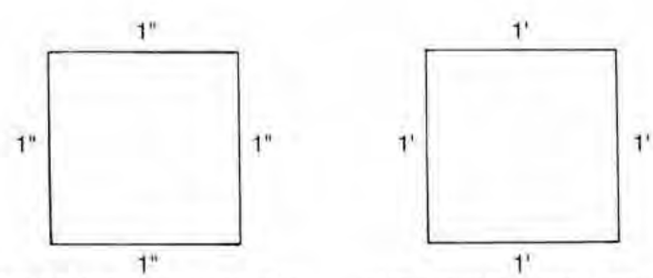
A right angle is the angle formed by one-fourth of a circle. Because a full circle is 360 degrees and one-fourth of 360 degrees is 90 degrees, a right angle is a 90-degree angle.



A rectangle is a closed figure with four sides that are at right angles to each other.



A square is a rectangle with four sides of equal length. A square with sides each one inch long is a square inch. A square with sides each one foot long is a **square foot**.



ONE SQUARE INCH (SQ. IN.)   ONE SQUARE FOOT (SQ. FT.)

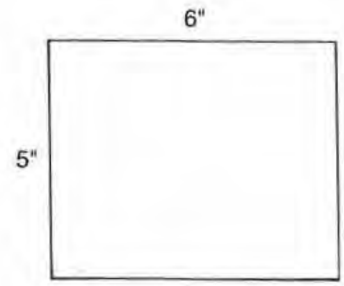
Note: The symbol for inch is ". The symbol for foot is '. The abbreviations are in. and ft.

The following formula may be used to compute the area of any rectangle:

$$\text{Area} = \text{Length} \times \text{Width}$$

$$A = L \times W$$

The area of the following rectangle, using the formula, is 5" × 6", or 30 square inches.

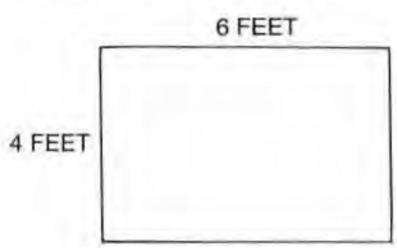


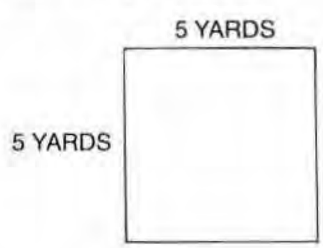
The term *30 inches* refers to a straight line 30 inches long. The term *30 square inches* refers to the area of a specific figure. When inches are multiplied by inches, the answer is in square inches. Likewise, when feet are multiplied by feet, the answer is in square feet.

Square feet are sometimes expressed by using the exponent ^{2}; for example, 10 ft^2 is read 10 feet squared and means 10' × 10', or 100 square feet.

An exponent indicates how many times the number, or unit of measurement, is multiplied by itself. This is called the power of the number or unit of measure. The exponent is indicated at the upper right of the original number or unit of measurement (for example, 10^3 would equal $10 \times 10 \times 10$; 10^4 is $10 \times 10 \times 10 \times 10$).

The area of the rectangle at the left, below, is 4' × 6', or 24 square feet. The area of the square at the right, below, is 5 yards × 5 yards, or 25 square yards.





IN PRACTICE

To provide storage space for its vehicles, a business has leased a vacant lot that measures 60 feet by 160 feet. How much rent will the business pay per year if the lot rents for $0.35 per square foot per year?

To solve this problem, the area of the lot must be computed first.

$$A = L \times W = 160' \times 60' = 9{,}600 \text{ sq ft}$$

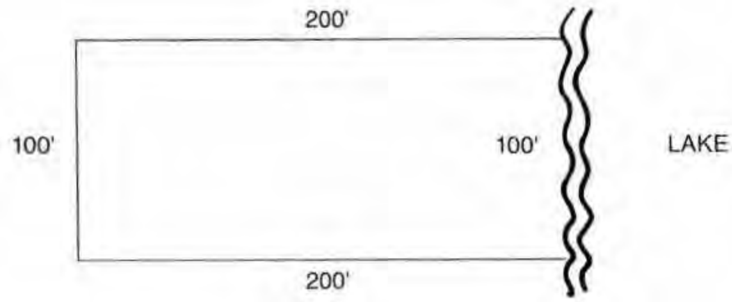
The number of square feet is then multiplied by the price per square foot to get the total rent.

$$9{,}600 \times \$0.35 = \$3{,}360$$

Front foot versus area

In certain situations, a tract of land may be priced at $X per front foot. Typically, this occurs when the land faces something desirable, such as a main street, a river, or a lake, thus making the frontage the major element of value.

For example, consider the following tract of land facing (fronting on) a lake.



The area of the lot is 20,000 square feet (100' × 200'). If this lot sells for $100,000, its price could be shown as $5 per square foot ($100,000 ÷ 20,000 square feet) or $1,000 per front foot ($100,000 ÷ 100 front feet).

Conversions—using like measures for area

When area is computed, all dimensions used must be given in the same kind of unit. When a formula is used to find an area, units of the same kind must be used for each element of the formula, with the answer as square units of that kind. So inches must be multiplied by inches to arrive at square inches, feet must be multiplied by feet to arrive at square feet, and yards must be multiplied by yards to arrive at square yards.

If the two dimensions to be multiplied are in different units of measure, one of the units of measure must be converted to the other. The following chart shows how to convert one unit of measure to another.

12 inches = 1 foot
36 inches = 1 yard
3 feet = 1 yard

To convert feet to inches,
multiply the number of feet by 12. (*ft* × 12 = in.)

To convert inches to feet,
divide the number of inches by 12. (*in.* ÷ 12 = ft)

To convert yards to feet,
multiply the number of yards by 3. (*yd* × 3 = ft)

To convert feet to yards,
divide the number of feet by 3. (*ft* ÷ 3 = yd)

To convert yards to inches,
multiply the number of yards by 36. (*yd* × 36 = in.)

To convert inches to yards,
divide the number of inches by 36. (*in.* ÷ 36 = yd)

Exercise 2-5

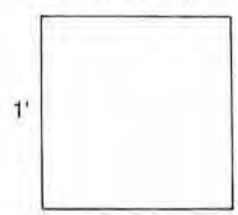
Solve the following problems:

12" × 3'= ____________ square feet or ____________ square inches

15" × 1.5' = ____________ square feet or ____________ square inches

72" × 7' = ____________ square feet or ____________ square inches

What is the area of the square below in square inches?



1,512 square inches = ____________ square feet.

Mr. Johnson's house is on a lot that is 75 feet by 125 feet. What is the area of his lot?

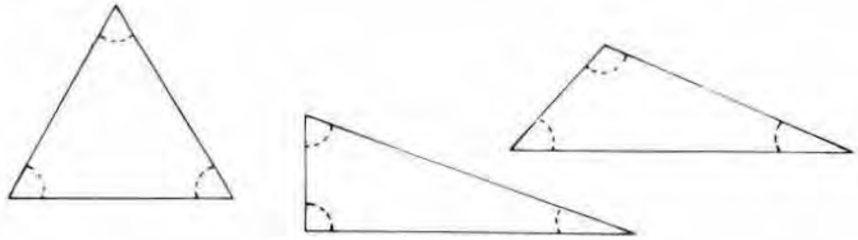
Check your answers against those in the answer key at the back of the book.

To convert square inches, square feet, and square yards, use the following chart:

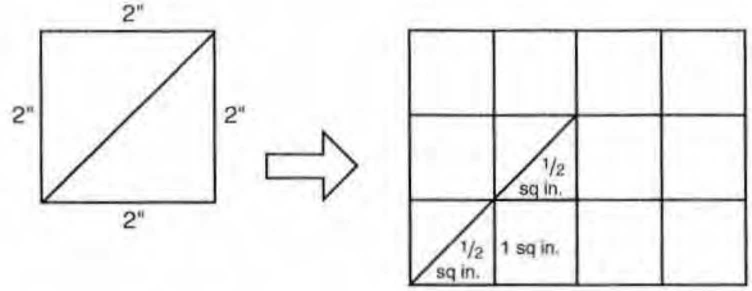
To convert square feet to square inches, *multiply the number of square feet by 144.*	(*sq ft* × 144 = sq in.)
To convert square inches to square feet, *divide the number of square inches by 144.*	(*sq in.* ÷ 144 = sq ft)
To convert square yards to square feet, *multiply the number of square yards by 9.*	(*sq yd* × 9 = sq ft)
To convert square feet to square yards, *divide the number of square feet by 9.*	(*sq ft* ÷ 9 = sq yd)
To convert square yards to square inches, *multiply the number of square yards by 1,296.*	(*sq yd* × 1,296 = sq in.)
To convert square inches to square yards, *divide the number of square inches by 1,296.*	(*sq in.* ÷ 1,296 = sq yd)

Area of Triangles

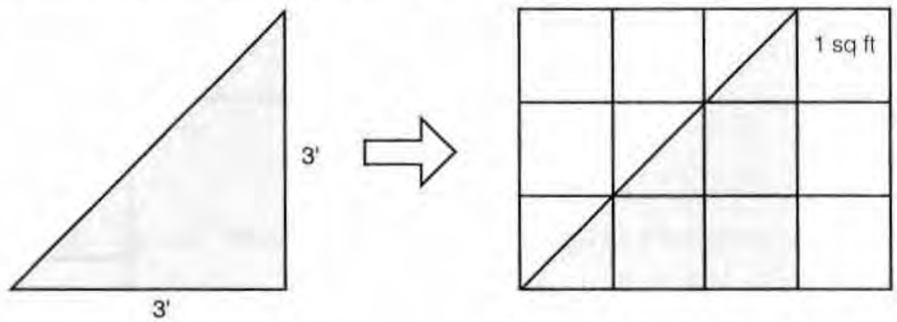
A triangle is a closed figure with three straight sides and three angles. The word *tri* means three.



The 4-square-inch figure at the left below has been cut in half by a straight line drawn between its opposite corners to make two equal triangles. When one of the triangles is placed on a square-inch grid, it is seen to contain ½ sq in. + ½ sq in. + 1 sq in., or 2 sq in.



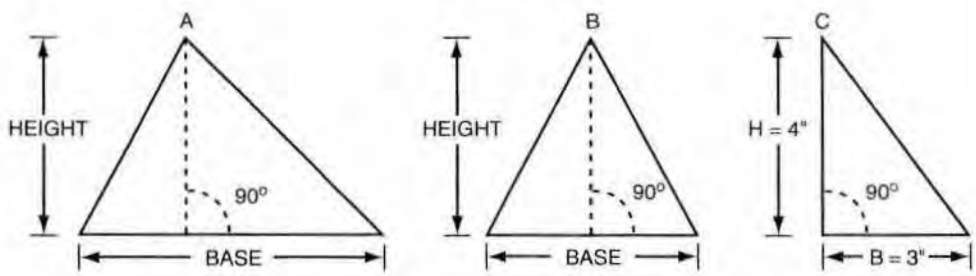
The area of the triangle below is 4.5 sq ft.



The square-unit grid is too cumbersome for computing large areas. It is more convenient to use the following formula for finding the area of a triangle:

Area of triangle A = ½ (Base × Height)

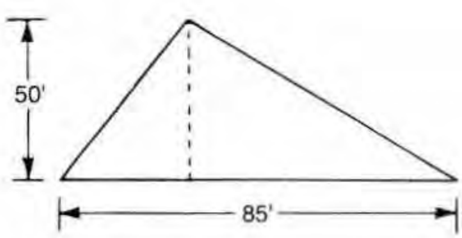
$A = ½ (BH)$



The base is the side on which the triangle sits. The height is the straight-line distance from the tip of the uppermost angle to the base. The height line must form a 90-degree angle to the base. The area of triangle C above is:

$A = ½(BH) = ½(3" \times 4") = ½(12 \text{ sq in.}) = 6 \text{ sq in.}$

Exercise 2-6

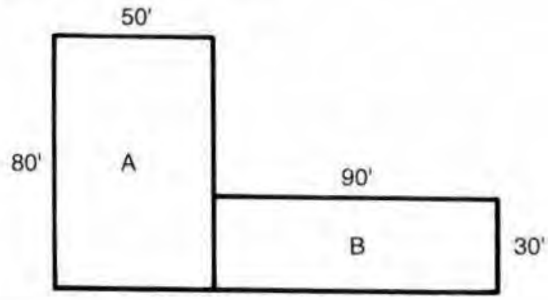


The diagram above shows a lakefront lot. Compute its area.

Check your answer against the one in the answer key at the back of the book.

Area of Irregular Closed Figures

Here is a drawing of two neighboring lots.



To find the total area of both lots:

Lot A = 50' × 80' = 4,000 sq ft

Lot B = 90' × 30' = 2,700 sq ft

Both lots = 4,000 sq ft + 2,700 sq ft = 6,700 sq ft

Two rectangles can be made by drawing one straight line inside Figure 1 below. There are two possible positions for the added line, as shown in Figures 2 and 3.

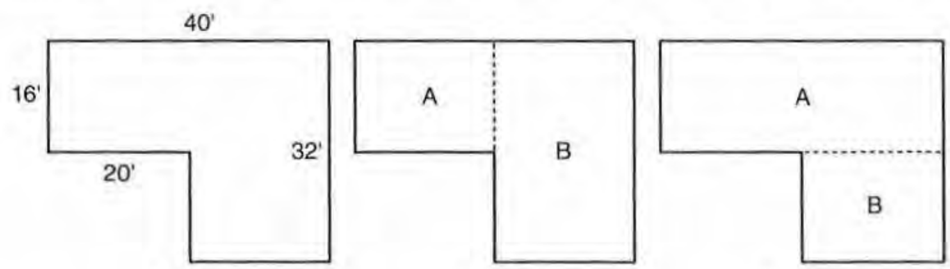


Figure 1 Figure 2 Figure 3

Using the measurements given in Figure 1, the total area of the figure may be computed in one of two ways:

Area of A = 20' × 16' = 320 sq ft

Area of B = 32' × (40' – 20') = 32' × 20' = 640 sq ft

Total area = 320 sq ft + 640 sq ft = 960 sq ft

Or

Area of A = 40' × 16' = 640 sq ft

Area of B = (40' – 20') × (32' – 16') = 20' × 16' = 320 sq ft

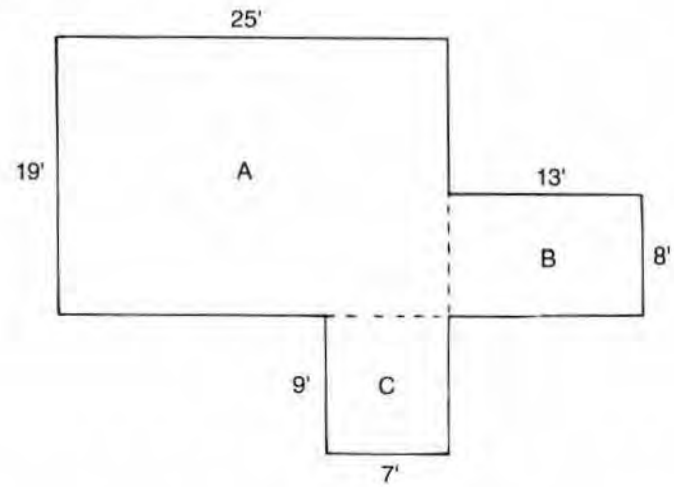
Total area = 640 sq ft + 320 sq ft = 960 sq ft

The area of an irregular figure can be found by dividing it into regular figures, computing the area of each, and adding all of the areas together to obtain the total area.

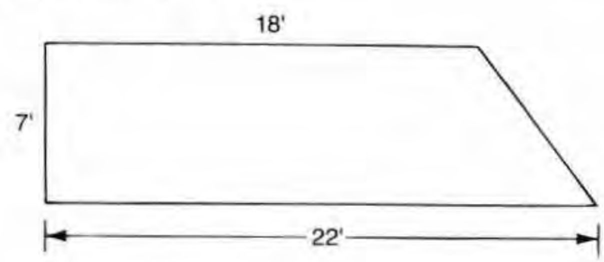
Exercise 2-7

This figure has been divided into rectangles, as shown by the broken lines.

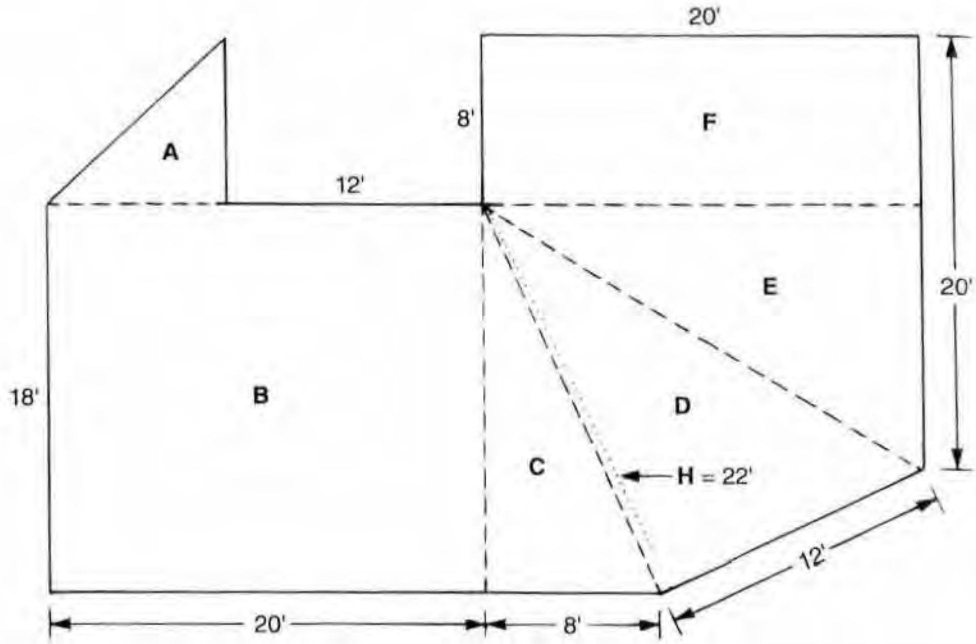
Compute the area of the figure.



Make a rectangle and a triangle by drawing a single line through the figure below, and then compute the area of the figure.



Compute the area of each section of the figure below. Then, compute the total area.



Check your answers against those in the answer key at the back of the book.

Living Area Calculations

Real estate appraisers frequently must compute the amount of living area in a house. The living area of a house is the area enclosed by the outside dimensions of the heated and air-conditioned portions of the house. This excludes open porches, garages, and such.

When measuring a house in preparation for calculating the living area, these steps should be followed:

1. Draw a sketch of the foundation.
2. Measure all outside walls.
3. If the house has an attached garage, treat the inside garage walls that are common to the house as outside walls of the house.
4. Measure the garage.
5. Convert inches to tenths of a foot (so that the same units of measurement are used in the calculations).

   Many appraisers use a tape measure marked with feet and tenths of a foot rather than feet and inches, which saves time and reduces the risk of errors.

6. Before leaving the house, check to see that net dimensions of opposite sides are equal. If not, remeasure.

7. Section off your sketch into rectangles.

8. Calculate the area of each rectangle.

9. Add up the areas, being careful to subtract the area of the garage, if necessary.

10. Before leaving the house, always recheck the dimensions.

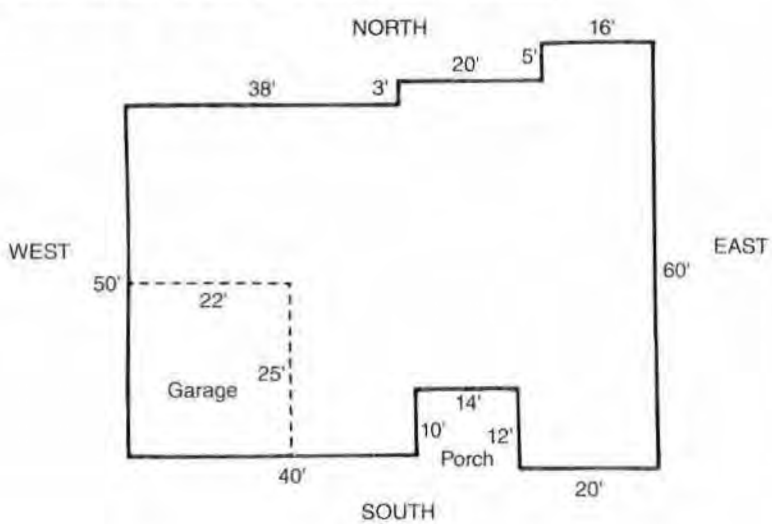
In 1996, the American National Standards Institute (ANSI) adopted the standard for measurement of floor area of single-family houses and condominiums that was developed by the NAHB Research Center (now called the National Innovation Labs), a subsidiary of the National Association of Home Builders. Before then, there was no uniform standard for home measurement, though there was a standard for measurement of floor area in office buildings. The text of the most recent version of the standard applicable to single-family houses and condominiums, known as ANSI Z765-2003, can be found at http://krec.ky.gov/legal/legal%20docs/calc_sqfootage.pdf. The appraiser should be familiar with this document, as well as the method commonly used by local real estate agents and appraisers.

WEB LINK

http://krec.ky.gov/legal/legal%20docs/calc_sqfootage.pdf

Exercise 2-8

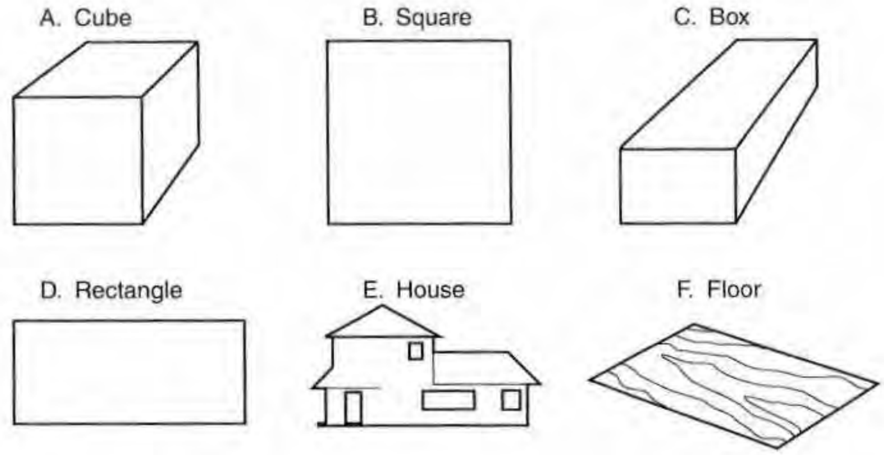
What is the living area of the house shown in the sketch below? Follow the steps above and remember to compute each area separately.



Check your answer against the one in the answer key at the back of the book.

Volume

When a shape has more than one side and encloses a space, the shape has volume, defined as the space that a three-dimensional object occupies.



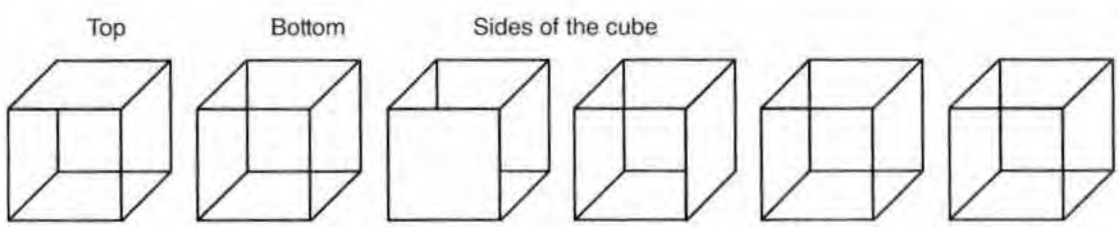
Of the above shapes, A, C, and E have volume; B, D, and F have area only.

Flat shapes—squares, rectangles, triangles, et cetera—do not have volume. Flat shapes have two dimensions (length and width or height), and shapes with volume have three dimensions (length, width, and height).

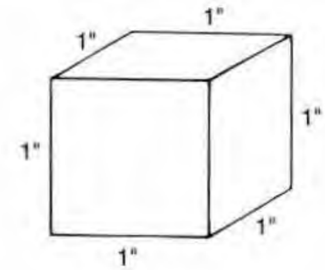
Technically speaking, each shape with three dimensions can also be measured in terms of its surface area. For example, a bedroom has volume because it has three dimensions—length, width, and height; however, one wall can be measured as surface area, or Area = Length × Width.

Cubic units

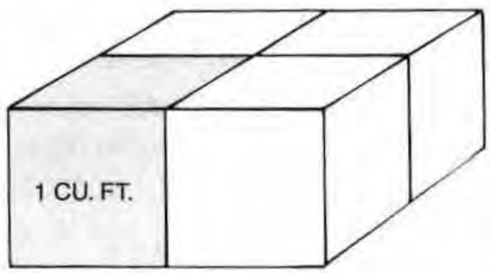
A cube is made up of six squares. Look at the six sides of the following cube.



Volume is measured in cubic units. Each side of the cube below measures one inch, so the figure is one cubic inch, or 1 cu in.



There are four cubic feet in the figure below. The exponent ^{3} may also be used to express cubic feet; that is, 1 ft^3 is one cubic foot.

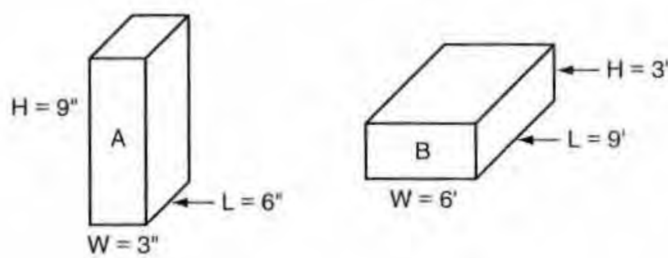


Using the formula for computing volume:

Volume of box A = L × W × H = 6" × 3" × 9" = 162 cu in.

Volume of box B = L × W × H = 9' × 6' × 3' = 162 cu ft

V (volume) = L (length) × W (width) × H (height)



IN PRACTICE

A building's construction cost was $450,000. The building is 60 feet long, 45 feet wide, and 40 feet high, including the basement. What was the cost of this building per cubic foot?

V = L × W × H = 60' × 45' × 40' = 108,000 cu ft

$$\text{Cost per cubic foot} = \frac{\text{Total cost}}{\text{Volume}} = \frac{\$450{,}000}{108{,}000 \text{ cu ft}} = \$4.166$$

The cost per cubic foot of this building is $4.17 (rounded).

Conversions—using like measures for volume

To convert cubic inches, cubic feet, and cubic yards, use the chart below.

To convert cubic feet to cubic inches, *multiply the number of cubic feet by 1,728.*	(*cu ft* × 1,728 = cu in.)
To convert cubic inches to cubic feet, *divide the number of cubic inches by 1,7728.*	(*cu in.* ÷ 1,728 = cu ft)
To convert cubic yards to cubic feet, *multiply the number of cubic yards by 27.*	(*cu yd* × 27 = cu ft)
To convert cubic feet to cubic yards, *divide the number of cubic feet by 27.*	(*cu ft* ÷ 27 = cu yd)
To convert cubic yards to cubic inches, *multiply the number of cubic yards by 46,656.*	(*cu yd* × 46,656 = cu in.)
To convert cubic inches to cubic yards, *divide the number of cubic inches by 46,656.*	(*cu in.* ÷ 46,656 = cu yd)

IN PRACTICE

How many cubic yards of space are there in a flat-roofed house that is 30 feet long, 18 feet wide, and 10 feet high?

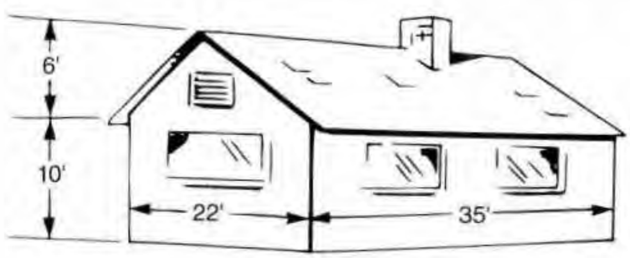
$$V = L \times W \times H = 30' \times 18' \times 10' = 5{,}400 \text{ cu ft}$$
$$\text{cu yd} = \text{cu ft} \div 27 = 5{,}400 \text{ cu ft} \div 27 = 200 \text{ cu yd}$$

Volume of triangular prisms

To compute the volume of a three-dimensional triangular figure, called a prism (e.g., an A-frame house), use the following formula:

$$\text{Volume} = \tfrac{1}{2}(B \times H \times W)$$

To compute the volume of the following house, first divide the house into two shapes, S and T.



Find the volume of S.

$$V = ½(B \times H \times W) = ½(22' \times 8' \times 35') =$$

$$½(6{,}160 \text{ cu ft}) = 3{,}080 \text{ cu ft}$$

Find the volume of T.

$$V = 22' \times 35' \times 10' = 7{,}700 \text{ cu ft}$$

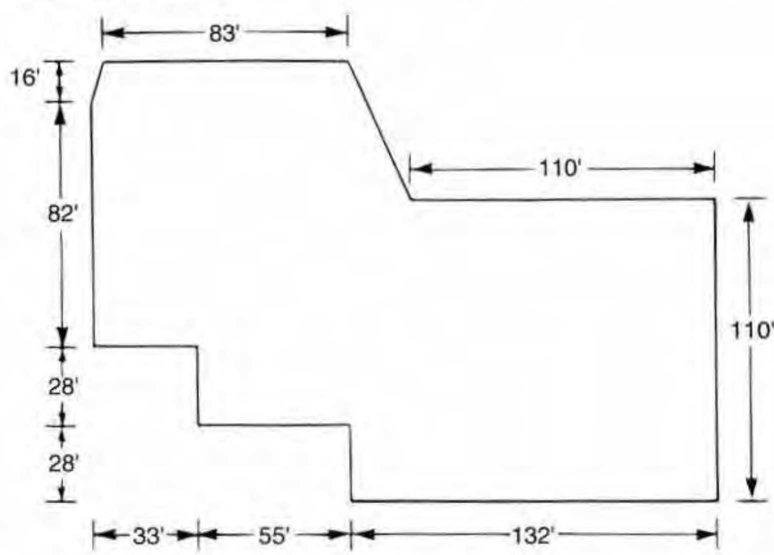
Total volumes S and T.

$$3{,}080 \text{ cu ft} + 7{,}700 \text{ cu ft} = 10{,}780 \text{ cu ft}$$

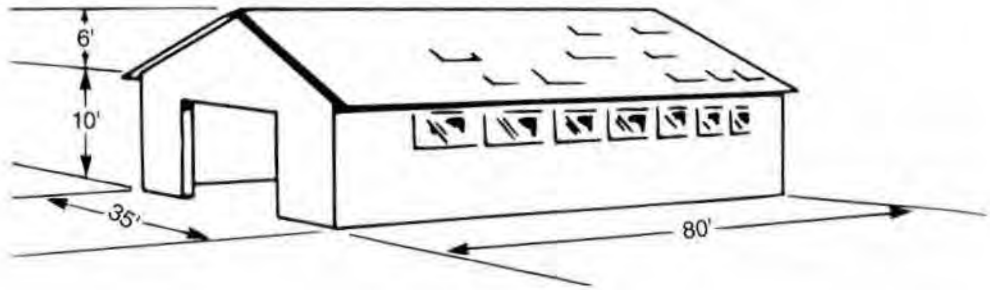
Exercise 2-9

Complete the following problems:

1. 8' × 7' = __________ sq ft = __________ sq in. = __________ sq yd
2. 9' × 3' × 2' = __________ cu ft = __________ cu in. = __________ cu yd
3. Find the total ground area covered by a building with the perimeter measurements shown below.



4. The building shown here has a construction cost of $45 per cubic yard. What is the total cost of this building?



Check your answers against those in the answer key at the back of the book.

■ STATISTICS

Statistics is the science of collecting, classifying, and interpreting information based on the number of things. Economists often use statistics to support theories and conclusions. Appraisers, too, can use statistics to support the assumptions that allow an estimate of value. Some of the commonly used statistical terms are defined in the following paragraphs.

Population and Sample

In the language of statistics, a **variate** is a single item in a group. One single-family home is a variate. All variates in a group make up a **population.** For example, in Pittsburgh, Pennsylvania, the population of single-family homes consists of all the single-family homes in that city.

Appraisers are rarely, if ever, able to deal with the total population. It may be too expensive or too time-consuming to gather the details on all the population. The appraiser must, therefore, rely on a **sample** of the population. A sample is defined as some of the population or some of the variates in that population. A sample must be large enough to accurately represent the population but small enough to be manageable.

Random sampling

A **random sample** is one in which every variate in the population is chosen by chance and therefore has an equal probability of being picked. By using random sampling, the likelihood of bias is reduced. A biased sample is one in which some variates in the population are more likely to be included than others.

Sample size

The size of the sample is simply the number of variates in the sample. For example, if 20 single-family houses were used by an appraiser to represent the market, then the sample size is equal to 20. The larger the sample, the more certain you can be that it accurately reflects the population.

Parameter and aggregate

A single number or attribute, called a **parameter**, can be used to describe an entire group or population of variates. For example, all the house sales in a community in a given year can be described by the total dollar amount of all the sales. The total, or sum, of all variates is called an **aggregate.**

Organization of Numerical Data

To facilitate interpretation and analysis, statistical data must be arranged or grouped in an orderly way. There are two principal methods of arranging numerical data. In one method, each of the various values or variates is presented in order of size. This arrangement is called an **array**. In the second method of arranging numerical data, called a **frequency distribution**, the values are grouped to show the frequency with which each size or class occurs.

The dollar amounts \$100,000, \$102,000, \$150,000, \$175,000, \$150,000, \$165,000, \$150,000, \$100,000, and \$150,000, which represent home sales in a neighborhood, have less meaning when presented in this unorganized manner than when presented in an array (see Figure 2.3) or in a frequency distribution (see Figure 2.4).

Measures of Central Tendency

In analyzing data, the first task is to describe the information in precise terms of measurement. Basic concepts of measurement used in statistics include measures of central tendency. A **measure of central tendency** describes the typical variate in a population. For example, the typical sales price for a single-family home and the typical rent for a square foot of office space are measures of central tendency.

: 2.3
ray

House Sales
\$175,000
165,000
150,000
150,000
150,000
150,000
102,000
100,000
100,000

Number of sales 9

FIGURE 2.4
Frequency Distribution

Sales	Frequency
$175,000	1
165,000	1
150,000	4
102,000	1
100,000	2
Number of sales	9

Three common statistical measures are the mean, the median, and the mode. All three measure central tendency and are used to identify the typical item or variate in a population or sample.

Mean

The **mean** is the average—that is, the sum of the variates divided by the number of variates. For example, in Figure 2.3, the mean price of the houses sold is $1,242,000 divided by 9, or $138,000.

Median

The **median** is found by dividing the number of variates into two equal groups. If the number of variates is odd, the median is the single variate at the middle. If the number of variates is even, the median is the arithmetic mean of the two variates closest to the middle from each end. In Figure 2.3, the median home price is $150,000.

Mode

The **mode** is the most frequently occurring variate. In Figure 2.4, the mode is $150,000.

Selecting a measure of central tendency

Before appraisers select one measure of central tendency, they should consider the following:

- The arithmetic mean is the most familiar measure of central tendency and can be conveyed to a client quickly and easily.
- The arithmetic mean is affected by extreme values called **outliers**, and might not represent any of the variates in a population. For example, the arithmetic mean of 5, 10, 15, 20, and 500 (outlier) is 110.
- When the outlier value of 500 is taken out of the calculation, the mean of the sample is only 12.5.
- The median is not affected by extreme variates as is the arithmetic mean. For example, the median in the above item is 15. So, in samples of popula-

tion where the variances are great, the median is a much better indicator than the average, or mean.

- The mode might represent an actual situation. For example, the population of apartments per building might have an arithmetic mean of 11.25, a median of 12.5, and a mode of 12. It is more realistic to discuss 12 units per building than 11.25 or 12.5. The mode, therefore, is most useful when a number of units have the same value.

Measures of Dispersion

Obtaining representative values other than measures of central tendency is often desirable. Measures of dispersion may be computed to measure the spread of the data; that is, to determine whether variates are grouped closely about the mean or median or are widely scattered or dispersed. In other words, measures of dispersion describe the variance in a data set. Three common measures of dispersion are the range, the average deviation, and the standard deviation.

Range

The **range** is a measure of the difference between the highest and lowest variates. The range of prices in Figure 2.3 is $175,000 minus $100,000, or $75,000.

Average deviation

The deviation is the measure of how widely the individual variates in a population vary. The **average deviation** measures how far the average variate differs from the mean. The formula for the average deviation finds the mean of the sum of the absolute differences (plus and minus signs are ignored) of each of the variates from the mean of the variates. The average deviation is also called the average absolute deviation because absolute differences are used, making all the values positive.

In Figure 2.5, the average deviation is 5.78 (52 ÷ 9). This means that, on the average, the test scores deviated from the mean by 5.78 points.

Standard deviation

The **standard deviation** measures the differences between individual variates and the entire population by taking the square root of the sum of the squared differences between each variate and the mean of all the variates in the population, divided by the number of variates in the population. The formula for standard deviation is

$$\sigma = \sqrt{\frac{\sum(x-\mu)^2}{N}}$$

and is read,

> the standard deviation (σ) equals the square root ($\sqrt{}$) of the sum (Σ) of X minus the mean (μ) squared, divided by the number of items or variates (N).

Substituting figures in the formula (see Figure 2.5):

The sum of X minus the mean squared is 428 divided by 9 equals 47.56; the square root of 47.56 is 6.896, rounded to 6.90 (σ).

Note: The formula for computing the standard deviation in this example holds true only when the entire population is considered. If a sample of a population is used, as is typically the case in real estate appraising, the sum of the squared differences from the mean is divided by the number of variates in the sample minus one. One is subtracted from the number of variates in a sample to adjust for the one degree of freedom that is lost when the mean is computed. The formula for finding the standard deviation of a sample is

Sample

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Population

$$\sigma = \sqrt{\frac{\sum(x - \mu)^2}{N}}$$

Notice the use of the different symbols in the formula when a sample rather than the total population is used:

s = the sample standard deviation
x = an individual variate in a sample
$\bar{x}$ = the sample mean
n = the number of variates in a sample

FIGURE 2.5

Average Deviation and Standard Deviation

	Scores on Real Estate Test	Deviations from Mean	Squares of Deviation
	94	12	144
	91	9	81
	85	3	9
	84	2	4
	82	0	0
	78	–4	16
	77	–5	25
	75	–7	49
	72	–10	100
Total	738	52	428

Mean = 82 (738 ÷ 9)

Median = 82

Mode: In the illustrated population of test scores there is no mode.

Range = 22

Average Deviation = 5.78 (52 ÷ 9)

Standard Deviation = 6.90 (√ of 428 ÷ 9)

Graphic Presentation of Data

Statistical presentation of data is generally in the form of tables, bar charts, histograms, and line graphs. Figure 2.6 lists the square foot construction costs for single-family homes. The first column shows dollars per square foot, and the second column the number of homes built in the square foot cost range indicated. The figures are totaled.

The same statistical data is presented in Figure 2.7 in a bar chart. Charts are often more informative than tables. A table is simply a list of the data; the chart gives a picture, and the data is more easily interpreted. The bar chart in Figure 2.7 and the histogram in Figure 2.8 both present a picture. All three are graphic presentations of statistics.

The histogram in Figure 2.8 is like a bar chart in a vertical position, but the bars or blocks on the histogram are contiguous and all are equal in width.

In the line graph shown in Figure 2.9, each point in the series of information on square foot construction costs is plotted, and the points are connected by a line. The line may be solid or dotted, and several items may be shown on the same graph. For example, in Figure 2.9 a dotted line could also be plotted to present construction costs per square foot in the year 2008 and 2007.

These are all simple illustrations. Tables, bar charts, histograms, and line graphs can be used to illustrate a variety of complex data pertaining to real estate. These may include population, operating statements, building permits issued, real estate taxes, traffic count, expense ratios, rents, land costs, comparable sales data, and other items of information.

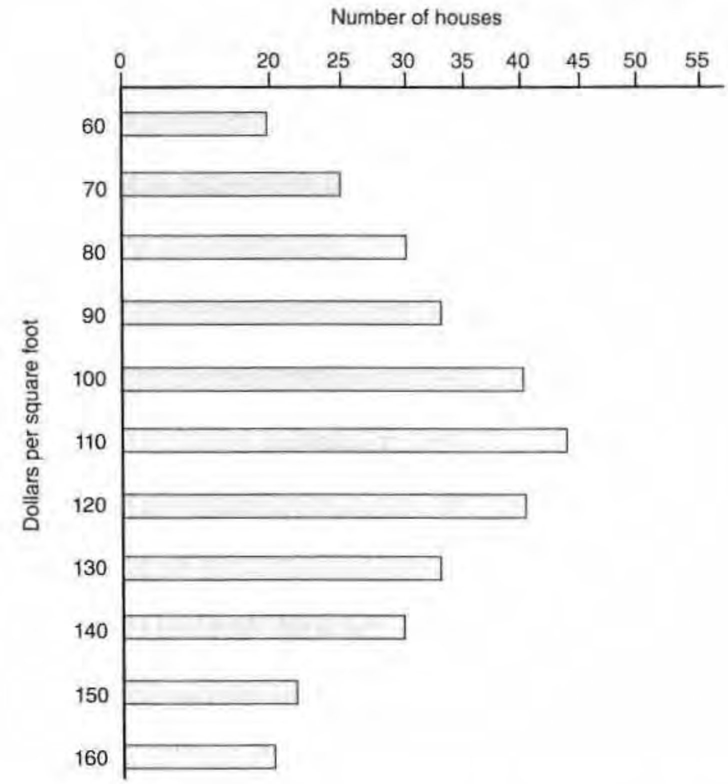
Normal Distribution

Suppose an appraiser wants to indicate the range of sales prices or other data lying between any given numbers. In ordinary computation, range merely describes the difference between the highest and lowest price; it does not lend itself to further

GURE 2.6
ıble Showing Square ›ot Construction osts for Single-Family omes—2006

Dollars per Square Foot		Number of Houses
60		20
70		25
80		30
90		35
100		40
110		45
120		40
130		35
140		30
150		25
160		20
	Total	345

FIGURE 2.7
Bar Chart

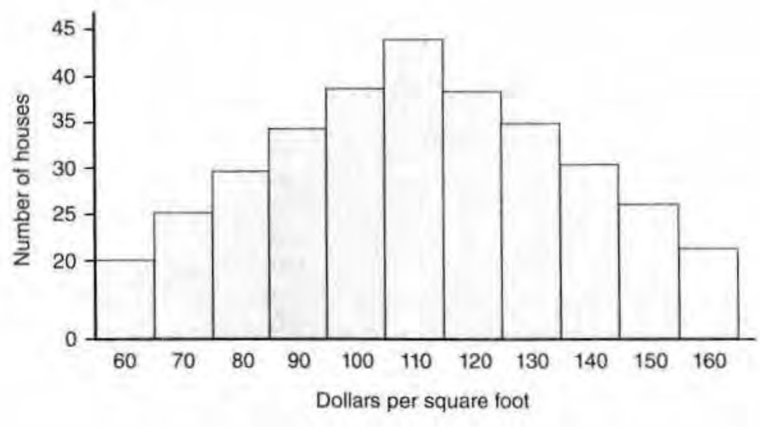


analysis. This limitation can be overcome by the use of a statistical approach; the appraiser calculates the standard deviation of dispersion within the population.

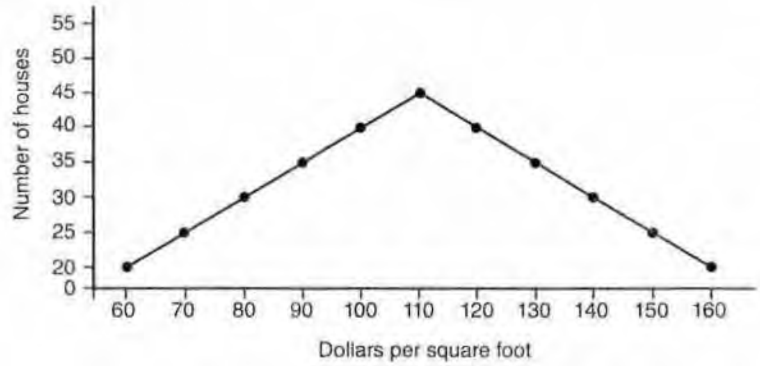
To illustrate, in a **normal distribution** of values, the shape assumed would be a normal curve, or **bell curve**, as shown in Figure 2.10.

In the normal curve, we know how much of any given area lies within given standard deviations from the mean. The mean deviation tells how much on the average each individual score varies from its own mean. The usefulness of standard deviation lies in the fact that approximately 34% of the population will fall

FIGURE 2.8
Histogram



URE 2.9
ie Graph



between the mean and one standard deviation. Therefore, 68% of the population will fall between plus or minus one standard deviation, approximately 95% between plus or minus two standard deviations, and almost 100% between plus or minus three standard deviations. With this information, the appraiser can apply the normal distribution to many situations.

IN PRACTICE

A sample of five houses has sales prices of $175,000, $170,000, $185,000, $180,000, and $180,000. The standard deviation is $5,701, and the mean is $178,000. Therefore, on the average, each house differs from the mean by $5,701.

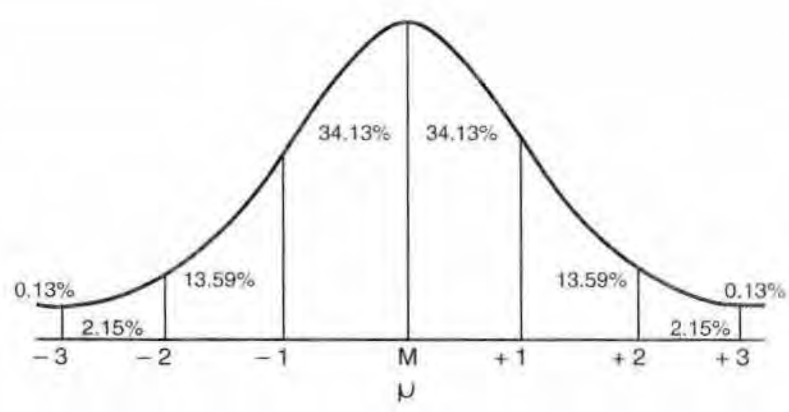
The formula for finding the standard deviation based on these facts is

$$s = \sqrt{\frac{\sum(x - \bar{x})^2}{n - 1}}$$

Where,

s = the sample standard deviation
$\sum$ = sum of
x = an individual variate in sample
$\bar{x}$ = mean of sample
n = number of variates in sample

IRE 2.10
rmal Curve



One is subtracted from the number of observations in a sample to adjust for the one degree of freedom that is lost when the mean is calculated.

In the first column below, the sum of the house sales represented by x is \$890,000. This total is then divided by 5 (the number of xs), resulting in a mean of \$178,000. The second column is found by subtracting the mean from x. For example, on the first line \$170,000 minus \$178,000 equals minus \$8,000. The third column is x minus the mean squared. On the first line, for example, x minus the mean squared (minus \$8,000) when squared equals \$64,000,000. The third column is then totaled to obtain the sum of x minus the mean squared, or \$130,000,000.

x	**$(x - \bar{x})$**	**$(x - \bar{x})^2$**
\$170,000	–\$8,000	\$64,000,000
175,000	–3,000	9,000,000
180,000	2,000	4,000,000
180,000	2,000	4,000,000
185,000	7,000	49,000,000
890,000		Total = \$130,000,000

$$\text{Mean} = \$890{,}000 \div 5 = \$178{,}000$$

Substituting figures in the formula:

$$s = \sqrt{130{,}000{,}000 \div 4}$$

$$s = \sqrt{32{,}500{,}000}$$

$$s = \$5{,}701$$

Exercise 2.10

Compute the sample standard deviation of the following data set:

25, 15, 18, 12, 28

Check your answer against the one in the answer key at the back of the book.

Skewness

Skewness is a measure of symmetry in a distribution, or more accurately, the lack of symmetry. A distribution is symmetric if it looks the same to the left or right of the center point of a bell-shaped curve, as shown in Figure 2.11. If the data is perfectly symmetrical, the mean, median, and mode are of equal value.

A positively skewed distribution has a longer tail to the right, and the mean is greater than the median, which is greater than the mode (see Figure 2.12). If the distribution has a longer tail to the left, it is negatively skewed, and the mode is greater than the median, which is greater than the mean.

Regression Analysis—An Old Technique with a New Use

In the past few years, advances in personal computer technology have brought about a profound revolution in the way appraisals are performed. The availability of ever cheaper and more powerful personal computers has elevated an old theoretical valuation technique to the cutting edge of appraisal technology—one that is within the means of even the smallest appraisal office. This technique is called multiple linear regression analysis.

Regression analysis makes use of basic principles of statistics, some of which are discussed in this chapter, to analyze comparable sales and determine line-item adjustments. A single regression analysis can consist of millions of individual calculations. Until a few years ago, the rigorous computations required to make use of this technique made it impractical for most fee appraising. Such calculations were simply not feasible, given the limited capability of even the state-of-the-art personal computers of 10 years ago. With today's equipment and software, however, they are a relatively trivial matter. Software with built-in regression analysis can enter recommended adjustment values automatically as the appraiser completes the URAR form. The result is not only to make the appraisal process more efficient but also to give the appraiser's value conclusions a high degree of accuracy, even in difficult rural markets where good comparable sales data can be hard to find.

With the aid of regression analysis, appraisers can make subtle predictions about the market that would be impossible otherwise. Perhaps more important is the

URE 2.11
ll-shaped Curve

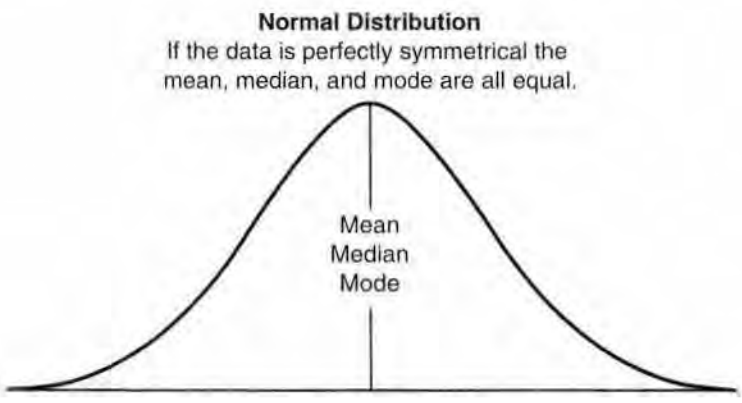
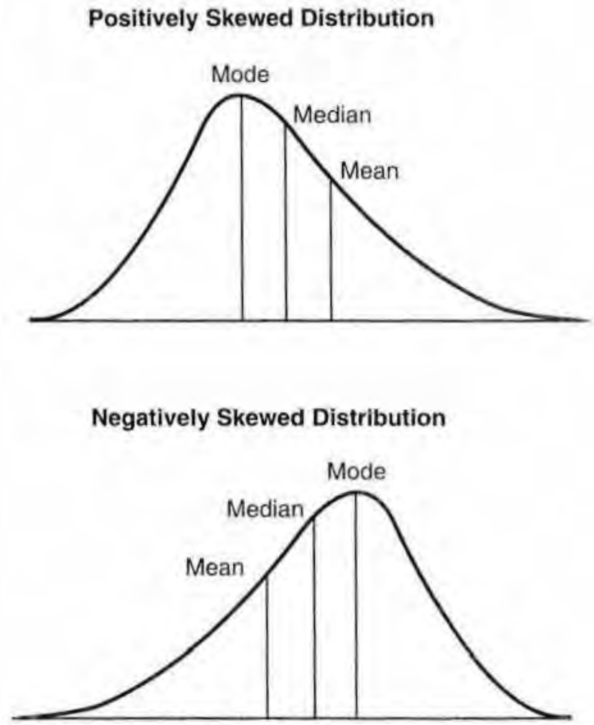


FIGURE 2.12
Positively and Negatively Skewed Distributions



level of accuracy possible because the technique provides detailed statistical justification for the values derived.

A thorough treatment of the use of regression analysis in appraising is beyond the scope of this text, but books, courses, and other materials explaining how to use this technique are available.

■ SUMMARY

Many mathematical concepts are involved in the study of appraisal, ranging from simple arithmetic to sophisticated statistical techniques.

Calculators and computers are great time-savers, but you still need to have a good basic knowledge of math so you can handle problems involving fractions, decimals, and percentages, or be able to compute the area and volume of buildings and sites.

The use of tables can help the investor or appraiser determine what is necessary to achieve a stated financial goal.

Knowing the terminology of statistics can help the appraiser understand the significance of the data collected and how they affect property value. Proponents of the statistical approach claim that it offers an objective method of analysis, and that any appraiser can master the technique by taking a course in elementary statistics.

In statistical analysis, three common measures of central tendency are used—the mean, the median, and the mode. The arithmetic mean is calculated by summing the data and dividing it by the number of items or variates. The mean is influenced by extremes or outliers; that is, exceptionally high or low values will inordinately affect the mean. If the data is skewed or strung out further to the left or right, the mean will be shifted in the same direction. The median is the middle value in an array of numbers. It is not affected by extremes, but it is affected by the data being skewed to the low or the high sides. The mode is the number which occurs most frequently in a series. The data tends to cluster about the mode.

A measure of dispersion describes differences in a population. This measure may be the range, average deviation, or standard deviation. The standard deviation is the most accurate measure of dispersion, assuming a normal distribution. In statistical analysis, normal distribution is represented by a bell-shaped curve.

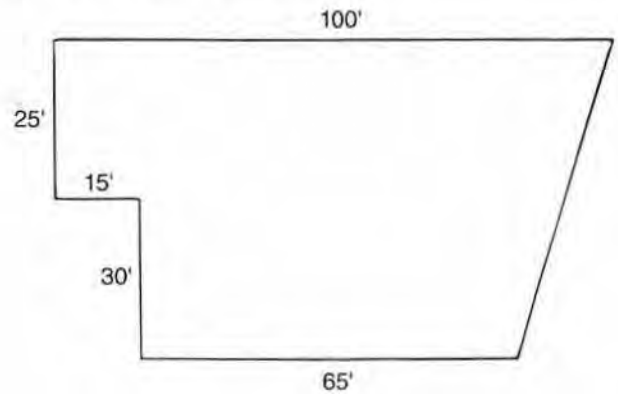
Random sampling implies that any one item or variate selected from a population has the same chance of being selected as any other.

■ Review Questions

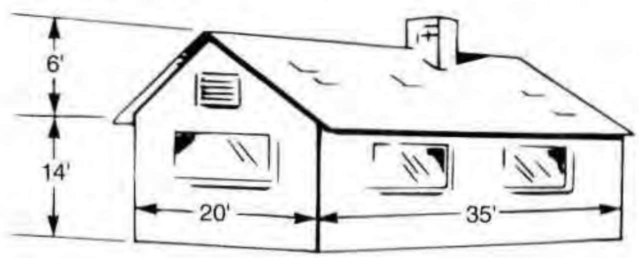
1. A property has an assessed value of $45,000. If the assessment is 36% of market value, what is the market value?

2. A property valued at $200,000 produces a net operating income of $24,000 per year. What percentage of value (rate) does this property earn?

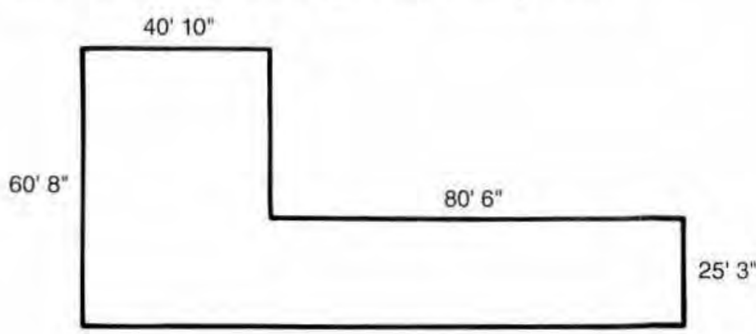
3. Find the total area of the figure below.



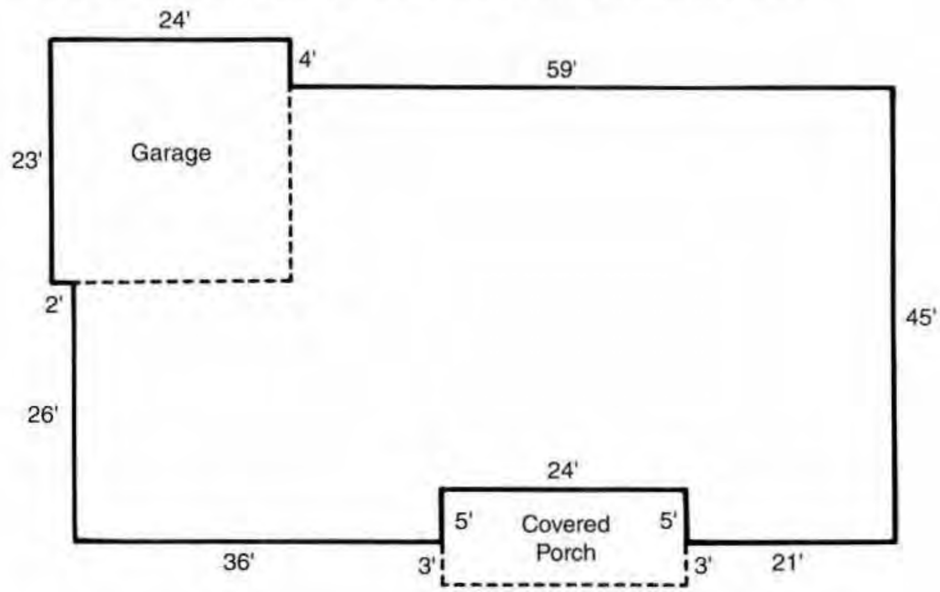
4. The house below would cost $2.75 per cubic foot to build. What would be the total cost, at that price?



5. What is the total area of the figure below in square feet?



6. What is the living area of the house shown in the sketch below?



7. The average of all variates is the
   a. mean.
   b. mode.
   c. median.
   d. range.

8. The center of all variates is the
   a. mean.
   b. mode.
   c. median.
   d. range.

9. The difference between the highest and lowest variates is the
   a. mean.
   b. mode.
   c. median
   d. range.

10. The mean of five house sales prices of \$100,000, \$75,000, \$175,000, \$200,000, and \$150,000 is
    a. \$140,000.
    b. \$150,000.
    c. \$700,000.
    d. \$175,000.

11. The median of the house sales prices in question 10 is
    a. \$140,000.
    b. \$150,000.
    c. \$700,000.
    d. \$175,000.

12. The aggregate of the house sales prices in question 10 is
    a. \$140,000.
    b. \$150,000.
    c. \$700,000.
    d. \$175,000

13. Using Figure 2.1, calculate the value in five years of an investment of \$7,500 at 10% interest compounded daily.
    a. \$20,387
    b. \$12,365
    c. \$12,340
    d. \$12,290

14. To determine the value in eight years of an investment of \$10,000 at 10% interest compounded annually, the applicable factor is
    a. 2.143589.
    b. 2.357948.
    c. 1.948717.
    d. 2.182875.

15. Using Figure 2.2, calculate the monthly payment required to amortize a loan of \$270,000 at 9½% interest for a term of 40 years.

    a. \$221.40

    b. \$2,241

    c. \$2,403

    d. \$2,187

16. The factor used to find the monthly payment required to amortize a loan of \$147,000 at 6% interest over 30 years is

    a. 0.0193.

    b. 0.0060.

    c. 0.0059.

    d. 0.0080.

17. In a positively skewed distribution, the mean will be

    a. less than the mode.

    b. less than the median.

    c. greater than the median.

    d. the same as the mode.

18. In a negatively skewed distribution, the mode will be

    a. greater than the mean.

    b. less than the median.

    c. less than the mean.

    d. the same as the median.

19. Extreme values in a data set are referred to as

    a. measures of variability.

    b. outliers.

    c. weighted averages.

    d. deviations.

20. The mean, median, and mode are all equal
    a. if the data is negatively skewed.
    b. in a bimodal distribution.
    c. if the data is positively skewed.
    d. if the data follows a normal distribution.

Check your answers against those in the answer key at the back of the book.