Problem 1 (25 pts) Use a triple integral to show that the volume of a "napkin ring" — the region between the cylinder $x^2 + y^2 = B^2$ and the sphere $x^2 + y^2 + z^2 = C^2$ — is $V = \frac{4}{3}\pi A^2$, where $2A$ is the height of the interior cylindrical wall.

To receive full credit for this problem, you need to:

- *Indicate which coordinate system (cylindrical or spherical) you are using.*
- *Provide a sketch of the region of integration, with coordinate axes and surfaces labeled.*
- *Set up and evaluate a triple integral in either cylindrical or spherical coordinates to find the volume of the region.*

  Evaluation should be done by hand, showing all steps, with the exception of functions that do not appear on the course integral table. For example, if you choose to work in spherical coordinates, and need to find the antiderivative of $\csc^2 \varphi$, you can use an app, calculator, or look it up online — just make a note of how you find it.
- *Make sure your final answer is written in terms of A (half the height of the interior cylinder), with no B's (radius of the interior cylinder) or C's (radius of the sphere).*

This is a pretty cool result. It means that the volume of a "napkin ring" depends only on the height of the interior cylinder, and not on the radius of the sphere. So, if you make one "napkin ring" out of an orange, and another out of the earth, and they have the same height, they will have the same volume.

There is a video by Vsauce (`https://www.youtube.com/watch?v=J51ncHP_BrY`) that does this using only trig and geometry. It's a lot easier (IMO) using triple integrals!