PART I
INVESTMENT OPPORTUNITY SET

1. Collect monthly data for three securities

Collect the price data ("adjusted closing prices" if you use Yahoo.) for the three stocks, (in this case MSFT, XOM, DIS), where they are Microsoft, Exxon Mobile and Walt Disney, respectively. We want data that are sufficient in frequency and duration. Let's use the **monthly data only over recent 5 years** for this project. This data can be easily collected from Yahoo Finance. Use the **"adjusted close"** price.

2. Calculating past mean returns

	A	B	C	D	E	F	G
1	Date	Price_MSFT	Price_XOM	Price_DIS	R_XOM	R_MSFT	R_DIS
2	1/2/09	14.74	66.11	19.35			
3	2/2/09	14.02	58.99	15.69	-0.114	-0.050	-0.210
4	3/2/09	15.94	59.16	16.99	0.003	0.128	0.080
5	4/1/09	17.58	57.92	20.49	-0.021	0.098	0.187
6	5/1/09	18.25	60.61	22.66	0.045	0.037	0.101
7	6/1/09	20.76	61.1	21.83	0.008	0.129	-0.037
8	7/1/09	20.54	61.52	23.5	0.007	-0.011	0.074
9	8/3/09	21.65	60.8	24.36	-0.012	0.053	0.036
10	9/1/09	22.59	60.33	25.69	-0.008	0.043	0.053
11	10/1/09	24.36	63.02	25.61	0.044	0.075	-0.003
12	11/2/09	25.95	66.39	28.28	0.052	0.063	0.099
13	12/1/09	26.89	60.31	30.52	-0.096	0.036	0.076
14	1/4/10	24.86	56.98	27.97	-0.057	-0.078	-0.087

Copy the price data into three columns in Excel and calculate the return between periods (in this example, between months) as follows:

$R_t = LN(P_t/P_{t-1})$

Make sure that is like LN("new"/"old").

Do this for each successive pair of returns for each of the three stocks.

Calculate the mean return of each stock using the MEAN function and create three excess returns columns for each stock by subtracting the mean return from each of the individual period returns.

$ER_t = R_t - R_{mean}$

	Std dev	MEAN RETURN
XOM	0.04725	0.00611
MSFT	0.06227	0.01677
DIS	0.06752	0.02296

E	F	G	H	I	J	K
R_XOM	R_MSFT	R_DIS		ER_xom	ER_MSFT	ER_DIS
-0.114	-0.050	-0.210		-0.120	-0.067	-0.233
0.003	0.128	0.080		-0.003	0.112	0.057
-0.021	0.098	0.187		-0.027	0.081	0.164
0.045	0.037	0.101		0.039	0.021	0.078
0.008	0.129	-0.037		0.002	0.112	-0.060
0.007	-0.011	0.074		0.001	-0.027	0.051
-0.012	0.053	0.036		-0.018	0.036	0.013
-0.008	0.043	0.053		-0.014	0.026	0.030
0.044	0.075	-0.003		0.038	0.059	-0.026
0.052	0.063	0.099		0.046	0.046	0.076
-0.096	0.036	0.076		-0.102	0.019	0.053
-0.057	-0.078	-0.087		-0.063	-0.095	-0.110
0.015	0.022	0.056		0.009	0.005	0.033

Concept: We are going to use the past mean returns as the expected returns (←Backward-looking approach to expected returns) assuming that past mean returns will remain unchanged in near future. Recall that we used an IRR approach to expected returns in the case analysis "GMO-The of Value Versus Growth Dilemma."

Notice: we are going to learn the CAPM and the Fama-French three factor model as the most formal approaches to determining the expected returns in chapter 7.

3. Calculating standard deviations

We are going to now calculate the variance-covariance matrix for these returns using the matrix operations built into Excel.

Caution: even if you have used the IRR approach to expected returns when calculating the expected mean returns, you must use the following method to determine the volatility (as a measure of risk).

Create a 3x3 matrix and label the rows and columns.

◇	L	M	N	O	P
2					
3		VARIANCE-COVARIANCE MATRIX			
4			R_XOM	R_MSFT	R_DIS
5		R_XOM			
6		R_MSFT			
7		R_DIS			

Highlighted

Highlighted

◇	H	I	J	K
1		ER_xom	ER_MSFT	ER_DIS
2				
3		-0.120	-0.067	-0.233
4		-0.003	0.112	0.057
5		-0.027	0.081	0.164
6		0.039	0.021	0.078
7		0.002	0.112	-0.060

⋮

◇	H	I	J	K
65		0.041	-0.031	-0.032
66		-0.018	0.004	0.034
67		-0.005	0.001	-0.003
68		-0.023	0.018	-0.021
69		0.007	0.039	0.025

Highlight the cells of the table and, with the highlighted cells, use the following command to matrix-multiply the excess returns:

=MMULT(TRANSPOSE(I3:K69),I3:K69)/(COUNT(I3:I69)-1)
***Caution: The "Count" function should use only the same column. In this example, it is the Column I.**

Then press either Ctrl+Shft+Enter or Cmd+Enter depending on whether you use Mac or PC. The variance-covariance matrix should fill in completely.

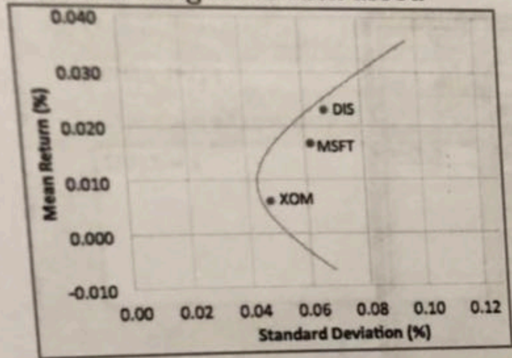
VARIANCE-COVARIANCE MATRIX

	R_XOM	R_MSFT	R_DIS
R_XOM	0.00223	0.00100	0.00184
R_MSFT	0.00100	0.00388	0.00196
R_DIS	0.00184	0.00196	0.00456

The entries along the main diagonal of the matrix represent the variance of each stock return, while the off-diagonal entries are the covariances of the respective corresponding stocks. Use the square roots of the variances to find the standard deviation of each stock.
For example:
Cov(R_{XOM}, R_{DIS}) = 0.00184
Var(MSFT) or $\sigma^2_{MSFT} = 0.00388$ then $\sigma_{MSFT} = \sqrt{0.00388} = 0.06229$

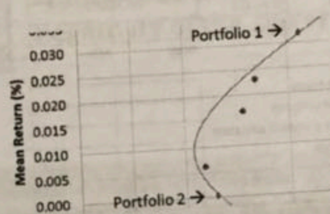
4. Concept of Investment Opportunity Set

This part explains only concepts on how to determine the frontier coordinates (or, investment opportunity set). You can plot the three coordinates of the return versus the risk of each company: see the three dots below in the figure. Now imagine the frontier created by the combination of different weights of each asset.

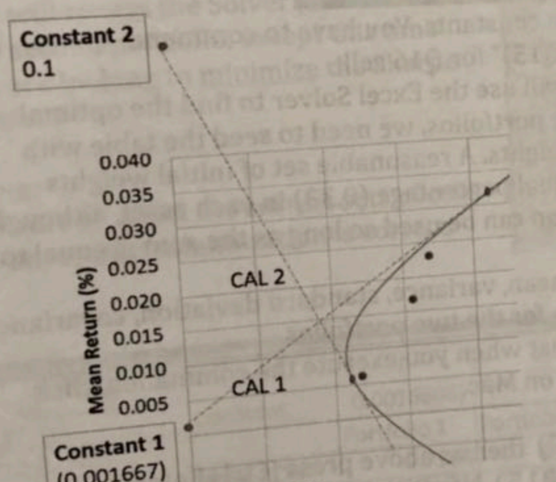


<u>You don't know how to determine the frontier set (investment opportunity set) now. You will learn how to determine the frontier in the following parts in this document.</u>

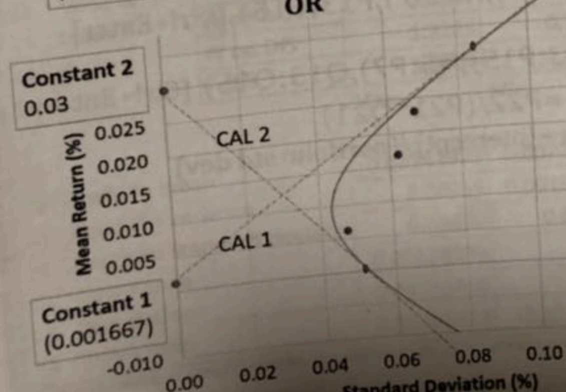
At least, you know there do exist this frontier by imagination.



Now also imagine two portfolios 1 and 2 on the frontier, which we will combine later to create the efficient frontier for all three assets.



OR



There must be the lines (capital allocation lines: "CAL") that are tangent to the portfolios 1 and 2. It is not difficult to know that there are respective constants on the y-axis.

Use 0.001667 for the first constant (as the monthly risk-free rate is around 0.1667%) and a high constant (like 0.1).
Notice that when we use risk-free rate for the constant 1, then the "Portfolio 1" will be the optimal risky portfolio sine it is determined by maximizing the Sharpe ratio (see below.)
Try to understand the fact that the two constants determine the portfolio 1 and 2 on which the CALs touch.

OR

<u>If the above constants give rise to (1) extremely high or low optimized weights) or (2) the correlation between between portfolio 1 and 2 is very closed to 1,</u> then change the value of Constant 2 as follows:
The highest mean of monthly return + 0.01 (or 0.005) as the value of Constant 2. For the example, Disney had the highest mean of 0.02296. Hence, the alternative value of Constant 2 is around 0.03.

If you recall, the Sharpe ratio is the slope of the capital allocation line (CAL). Keeping that in mind, we can see that, in the first portfolio, the Sharpe ratio is maximized. The Sharpe ratio is minimized with the Portfolio 2: the slope is maximum negative with the Portfolio 2.

To help us create these portfolios 1 and 2 and the frontier curve, we will use the "**Solver**" in Excel. Use the following diagram as reference for the next steps.

	M	N	O	P	Q	R	S	T	U
3	VARIANCE-COVARIANCE MATRIX					Monthly		Annual	
4		R_XOM	R_MSFT	R_DIS		Std dev	MEAN RETURN	Std dev	MEAN RETURN
5	R_XOM	0.00223	0.00100	0.00184		0.04725	0.00611	0.16366	0.07326
6	R_MSFT	0.00100	0.00388	0.00196		0.06227	0.01677	0.21570	0.20121
7	R_DIS	0.00184	0.00196	0.00456		0.06752	0.02296	0.23389	0.27548
8									
9									
10									
11			Constant	0.00166667	0.1	<-- Constants to optimize Sharp ratio			
12				Portfolio 1	Portfolio 2				
13		Asset 1	W on XOM						
14		Asset 2	W on MSFT						
15		Asset 3	W on DIS						
16			Total						
17				Max	Min	<--For Solver, select for each case			
18									
19		Porfolio mean							
20		Portfolio variance							
21		Portfolio standard deviaiton							
22		Covariance							
23		Correlation							
24		Sharp ratio							

Set up the table as shown in N10 – Q16 with the given constants. You have to command "=sum(P13:P15)" for P16 cell. Similarly, "=sum(Q13:Q15)" for Q16 cell.

	O	P	Q
11	Constant	0.00166667	0.1
12		Portfolio 1	Portfolio 2
13	W on XOM	0.3333	0.3333
14	W on MSFT	0.3333	0.3333
15	W on DIS	0.3333	0.3333
16	Total	1.0000	1.0000

Although we will use the Excel Solver to find the optimal weights on the portfolios, we need to seed the table with some initial weights. A reasonable set of initial weights might be an equal percentage (0.33) in each asset, although any combination can be used so long as the sum is equal to 1.

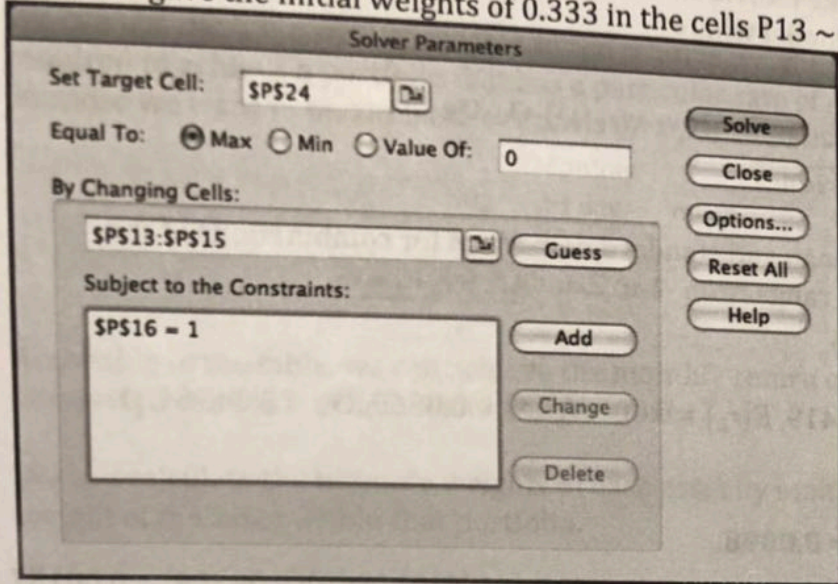
Calculate the mean, variance, standard deviation, covariance, and correlation for the two portfolios.

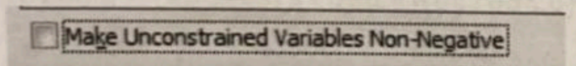
We will fill in the cells in P19 ~ Q24 as follows. Note that when you execute the commands, click Ctrl+Shft+Enter (or Ctrl+Enter) on PC and Cmd+Enter on Mac.

Mean: =MMULT(TRANSPOSE(P13:P15),S5:S7) then as above press [Ctrl+Enter]
Variance: =MMULT(MMULT(TRANSPOSE(P13:P15),N5:P7),P13:P15) [Ctrl+Enter]
Standard Deviation: =SQRT(P20)
Covariance: =MMULT(MMULT(TRANSPOSE(P13:P15),N5:P7),Q13:Q15) [Ctrl+Enter]
Correlation coefficient (ρ) = (Covariance)/($\sigma_1\sigma_2$): =P22/(P21*Q21)
The Sharpe ratio is calculated as the (portfolio mean – intercept)/(portfolio std dev)
Sharpe Ratio: =(P19-P11)/P21

	N	O	P	Q
19	Porfolio mean		0.01527642	0.01527642
20	Portfolio variance		0.00225	0.00225113
21	Portfolio standard deviaiton		0.04745	0.04745
22	Covariance		0.00225113	
23	Correlation		1	
24	Sharp ratio		0.28684652	-1.78568067

Recall we gave the initial weights of 0.333 in the cells P13 ~ Q15. We now use the Solver to find the actual optimal portfolio weights.



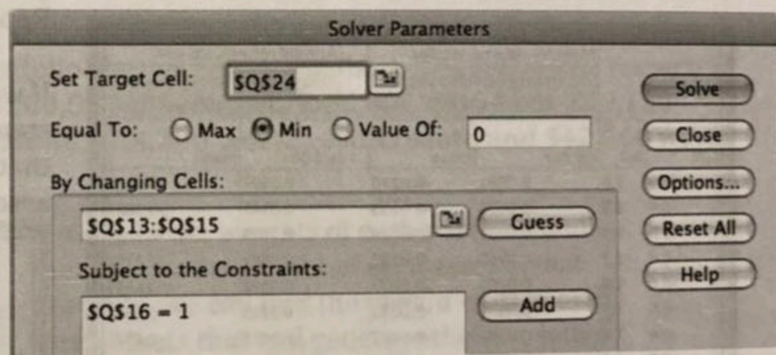


For the first portfolio, we are looking to <u>maximize</u> the Sharpe Ratio.

The target cell is the Sharpe ratio, which the Solver will try to maximize by changing the values within the cells that contain the portfolio weights, subject to the constraint that the weights add up to 1. Once you set up the parameters, click "Solve."
***<u>caution</u>: uncheck the non-negativity of the solver.**

We will repeat the Solver procedure for the second portfolio, except this time we are looking to <u>minimize</u> the Sharpe Ratio.

Note:
Be sure to select the correct sets of cells to ensure the minimization is correct.



	N	O	P	Q
11		Constant	0.00166667	0.1
12			Portfolio 1	Portfolio 2
13	Asset 1	W on XOM	-0.7584	0.8248
14	Asset 2	W on MSFT	0.5361	0.2807
15	Asset 3	W on DIS	1.2223	-0.1055
16		Total	1.0000	1.0000
17			Max	Min
18				
19	Porfolio mean		0.03241979	0.00732053
20	Portfolio variance		0.00755	0.00190403
21	Portfolio standard deviaitor		0.08690	0.04364
22	Covariance		0.00138838	
23	Correlation		0.36613865	
24	Sharp ratio		0.35388618	-2.1239642

We now end up with two portfolios whose asset weights have been optimized. Recall the Portfolio 1 and 2 are those on the frontier curve.

<u>**Caution**</u>: If the optimized weights for portfolios 1 and 2 are too big in size, like -5.2 or 12,890.45, then use different values for "constants." The optimized weights should be less than or at most around 1 to -1. Also check with the graph of investment opportunity set as shown in the next page. If the shape is not like the shape of "C", then redo determining the weights.

We can now treat these two portfolios like individual assets and combine them in different weights as we have been doing in the two-asset model where:

Portfolio mean (μ_P) = $w_1 E[r_1] + w_2 E[r_2]$

Portfolio variance (σ_P^2) = $(w_1 \cdot \sigma_1)^2 + (w_2 \cdot \sigma_2)^2 + 2 \cdot w_1 \cdot w_2 \cdot (\rho) \cdot \sigma_1 \cdot \sigma_2$

Portfolio standard deviation (σ_P) = $\sqrt{Variance}$

Create a table calculating the portfolio mean and standard deviations for combinations of the various weights of each portfolio. Let w_1 range from -1 to 2 and let $w_2 = 1 - w_1$

For example:

If we use the values above, $E[r_1] = 0.032419$, $E[r_2] = 0.00732$, $\sigma_1 = 0.0869$, $\sigma_2 = 0.04364$, $\rho = 0.366138$.

Then for $w_1 = 0.1$ and $w_2 = 0.9$,

$\mu_P = (0.1)(0.032419) + (0.9)(0.00732) = \mathbf{0.0098}$.

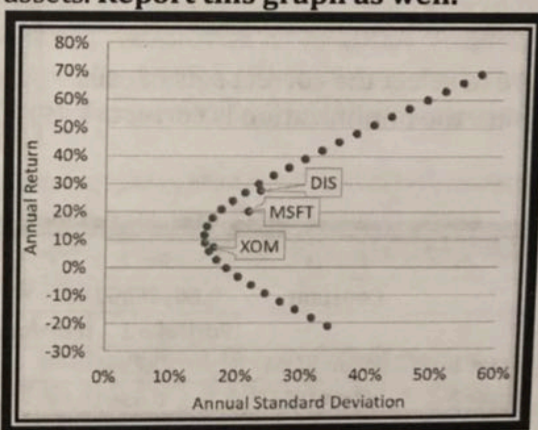
$\sigma_P^2 = [(0.1)(0.0869)]^2 + [(0.9)(0.04364)]^2 + 2(0.1)(0.9)(0.366138)(0.0869)(0.04364) = 0.00187$

$\sigma_P = \sqrt{0.00187} = \mathbf{0.0432}$.

		Monthly returns, std dev.		Annual returns, std dev.	
		Investment opportunity set with XOM, MSFT, DIS		Investment opportunity set with XOM, MSFT, DIS	
W1	W2	std dev	mean	std dev	mean
-1.0	2.0	0.0981	-0.0178	0.3397	-0.2133
-0.9	1.9	0.0908	-0.0153	0.3145	-0.1832
-0.8	1.8	0.0837	-0.0128	0.2899	-0.1531
-0.7	1.7	0.0768	-0.0102	0.2661	-0.1230
-0.6	1.6	0.0702	-0.0077	0.2432	-0.0929
-0.5	1.5	0.0639	-0.0052	0.2215	-0.0627
-0.4	1.4	0.0582	-0.0027	0.2015	-0.0326
-0.3	1.3	0.0531	-0.0002	0.1838	-0.0025
-0.2	1.2	0.0488	0.0023	0.1689	0.0276
-0.1	1.1	0.0455	0.0048	0.1578	0.0577
0.0	1.0	0.0436	0.0073	0.1512	0.0878
0.1	0.9	0.0432	0.0098	0.1497	0.1180
0.2	0.8	0.0443	0.0123	0.1536	0.1481
0.3	0.7	0.0469	0.0149	0.1623	0.1782
0.4	0.6	0.0506	0.0174	0.1753	0.2083
0.5	0.5	0.0553	0.0199	0.1916	0.2384
0.6	0.4	0.0607	0.0224	0.2104	0.2686
0.7	0.3	0.0667	0.0249	0.2312	0.2987
0.8	0.2	0.0732	0.0274	0.2535	0.3288
0.8	0.2	0.0732	0.0274	0.2535	0.3288
0.9	0.1	0.0799	0.0299	0.2768	0.3589
1.0	0.0	0.0869	0.0324	0.3010	0.3890
1.1	-0.1	0.0941	0.0349	0.3259	0.4192
1.2	-0.2	0.1014	0.0374	0.3513	0.4493
1.3	-0.3	0.1089	0.0399	0.3771	0.4794
1.4	-0.4	0.1164	0.0425	0.4033	0.5095
1.5	-0.5	0.1240	0.0450	0.4297	0.5396
1.6	-0.6	0.1317	0.0475	0.4563	0.5698
1.7	-0.7	0.1395	0.0500	0.4832	0.5999
1.8	-0.8	0.1473	0.0525	0.5102	0.6300
1.9	-0.9	0.1551	0.0550	0.5373	0.6601
2.0	-1.0	0.1630	0.0575	0.5645	0.6902

Repeat the calculations for other weights.

If we plot the **ANNUAL** mean versus standard deviation, we end with a curve that illustrates the frontier for the three assets. **Report this graph as well.**



To convert to **annualized values**, do:

$\sigma_P = \sqrt{12} \times (monthly\ \sigma_P)$

$E[r_P] = 12 \times (monthly\ E[r_P])$

(where, 12 represents the number of months.)

Example,

$0.1497\ (14.97\%) = \sqrt{12} \times 0.0432$

$0.118\ (11.8\%) = 12 \times 0.0098$

5. Investing in only risky assets (without risk-free assets)

We can use the table we just created to the relative weights of XOM, MSFT, and DIS stock that is required to achieve a portfolio that has a particular rate of return and standard deviation. Suppose we want to maximize the returns on a portfolio that has a risk of 5% (monthly std. dev.).

		Monthly		Annual	
W1	W2	std dev	mean	std dev	mean
...	...	...	...	...	...
0.4	0.6	0.0506	0.0174	0.1753	0.2083
...	...	...	...	...	...

According to the table, we can achieve the monthly return of 1.74% from a portfolio that is comprised of 40% Portfolio 1 and 60% Portfolio 2.

We can calculate the ultimate weights of each asset by multiplying the weight of portfolio by the weight of the asset within that portfolio.

	Portfolio 1	Portfolio 2
W on XOM	-0.7584	0.8248
W on MSFT	0.5361	0.2807
W on DIS	1.2223	-0.1055

Weight on XOM = (0.4)(-0.07584) + (0.6)(0.8248) = 0.1915
Weight on MSFT = (0.4)(0.5361) + (0.6)(0.2807) = 0.3829
Weight on DIS = (0.4)(1.2223) + (0.6)(-0.1055) = 0.4256

To replicate the risk and return of this portfolio, we would buy (or short, if negative) the respective proportion of each asset. **So if we had a $100,000 investment fund, we would buy $19,150 ($= \$100,000 \times 0.1915$) worth of ExxonMobil, $38,290 worth of Microsoft, and $42,560 worth of Disney stock.**

W1	W2	W on XOM	W on MSFT	W on DIS	Total
-1.0	2.0	2.4080	0.0254	-1.4333	1.000
-0.9	1.9	2.2496	0.0509	-1.3005	1.000
-0.8	1.8	2.0913	0.0764	-1.1678	1.000
-0.7	1.7	1.9330	0.1020	-1.0350	1.000
-0.6	1.6	1.7747	0.1275	-0.9022	1.000
-0.5	1.5	1.6164	0.1530	-0.7694	1.000
-0.4	1.4	1.4580	0.1786	-0.6366	1.000
-0.3	1.3	1.2997	0.2041	-0.5038	1.000
-0.2	1.2	1.1414	0.2297	-0.3711	1.000
-0.1	1.1	0.9831	0.2552	-0.2383	1.000
0.0	1.0	0.8248	0.2807	-0.1055	1.000
0.1	0.9	0.6664	0.3063	0.0273	1.000
0.2	0.8	0.5081	0.3318	0.1601	1.000
0.3	0.7	0.3498	0.3573	0.2929	1.000
0.4	0.6	0.1915	0.3829	0.4256	1.000
0.5	0.5	0.0332	0.4084	0.5584	1.000
0.6	0.4	-0.1252	0.4340	0.6912	1.000
0.7	0.3	-0.2835	0.4595	0.8240	1.000
0.8	0.2	-0.4418	0.4850	0.9568	1.000
0.9	0.1	-0.6001	0.5106	1.0896	1.000
1.0	0.0	-0.7584	0.5361	1.2223	1.000
1.1	-0.1	-0.9168	0.5616	1.3551	1.000
1.2	-0.2	-1.0751	0.5872	1.4879	1.000
1.3	-0.3	-1.2334	0.6127	1.6207	1.000
1.4	-0.4	-1.3917	0.6383	1.7535	1.000
1.5	-0.5	-1.5500	0.6638	1.8863	1.000
1.6	-0.6	-1.7084	0.6893	2.0190	1.000
1.7	-0.7	-1.8667	0.7149	2.1518	1.000
1.8	-0.8	-2.0250	0.7404	2.2846	1.000
1.9	-0.9	-2.1833	0.7659	2.4174	1.000
2.0	-1.0	-2.3416	0.7915	2.5502	1.000

If we take the weights of each portfolio and multiply by the weight of each asset in that portfolio, we can find the weight of each of the three assets that will generate the respective mean and standard deviation.

Thus we have created an efficient frontier for three assets. If we want to create an efficient frontier for more assets, we would include more columns of returns, but otherwise the process is identical.

In conclusion,
If you have $100,000, how much do you have to spend on each stock to achieve a certain expected rate of return ("mean") and the risk ("std dev")? The following nicely summarizes the plan. Notice that the negative values stand for the amount of short sales.

		Optimal weights			Annual returns, std dev. Investment opportunity set with XOM, MSFT, DIS		How much to spend on each stock? With total Investment of $100,000		
W1	W2	W on XOM	W on MSFT	W on DIS	std dev	mean	$ XOM	$ MSFT	$ DIS
-1	2	2.4080	0.0254	-1.4333	0.3397	-0.2133	$240,796	$2,536	-$143,332
-0.9	1.9	2.2496	0.0509	-1.3005	0.3145	-0.1832	$224,964	$5,090	-$130,054
-0.8	1.8	2.0913	0.0764	-1.1678	0.2899	-0.1531	$209,132	$7,644	-$116,776
-0.7	1.7	1.9330	0.1020	-1.0350	0.2661	-0.1230	$193,300	$10,198	-$103,497
-0.6	1.6	1.7747	0.1275	-0.9022	0.2432	-0.0929	$177,468	$12,751	-$90,219
-0.5	1.5	1.6164	0.1530	-0.7694	0.2215	-0.0627	$161,636	$15,305	-$76,941
-0.4	1.4	1.4580	0.1786	-0.6366	0.2015	-0.0326	$145,804	$17,859	-$63,663
-0.3	1.3	1.2997	0.2041	-0.5038	0.1838	-0.0025	$129,972	$20,412	-$50,384
-0.2	1.2	1.1414	0.2297	-0.3711	0.1689	0.0276	$114,140	$22,966	-$37,106
-0.1	1.1	0.9831	0.2552	-0.2383	0.1578	0.0577	$98,308	$25,520	-$23,828
0	1	0.8248	0.2807	-0.1055	0.1512	0.0878	$82,476	$28,073	-$10,549
0.1	0.9	0.6664	0.3063	0.0273	0.1497	0.1180	$66,644	$30,627	$2,729
0.2	0.8	0.5081	0.3318	0.1601	0.1536	0.1481	$50,812	$33,181	$16,007
0.3	0.7	0.3498	0.3573	0.2929	0.1623	0.1782	$34,980	$35,735	$29,286
0.4	0.6	0.1915	0.3829	0.4256	0.1753	0.2083	$19,148	$38,288	$42,564
0.5	0.5	0.0332	0.4084	0.5584	0.1916	0.2384	$3,316	$40,842	$55,842
0.6	0.4	-0.1252	0.4340	0.6912	0.2104	0.2686	-$12,516	$43,396	$69,120
0.7	0.3	-0.2835	0.4595	0.8240	0.2312	0.2987	-$28,348	$45,949	$82,399
0.8	0.2	-0.4418	0.4850	0.9568	0.2535	0.3288	-$44,180	$48,503	$95,677
0.9	0.1	-0.6001	0.5106	1.0896	0.2768	0.3589	-$60,012	$51,057	$108,955
1	0	-0.7584	0.5361	1.2223	0.3010	0.3890	-$75,844	$53,610	$122,234
1.1	-0.1	-0.9168	0.5616	1.3551	0.3259	0.4192	-$91,676	$56,164	$135,512
1.2	-0.2	-1.0751	0.5872	1.4879	0.3513	0.4493	-$107,508	$58,718	$148,790
1.3	-0.3	-1.2334	0.6127	1.6207	0.3771	0.4794	-$123,340	$61,272	$162,068
1.4	-0.4	-1.3917	0.6383	1.7535	0.4033	0.5095	-$139,172	$63,825	$175,347
1.5	-0.5	-1.5500	0.6638	1.8863	0.4297	0.5396	-$155,004	$66,379	$188,625
1.6	-0.6	-1.7084	0.6893	2.0190	0.4563	0.5698	-$170,836	$68,933	$201,903
1.7	-0.7	-1.8667	0.7149	2.1518	0.4832	0.5999	-$186,668	$71,486	$215,182
1.8	-0.8	-2.0250	0.7404	2.2846	0.5102	0.6300	-$202,500	$74,040	$228,460
1.9	-0.9	-2.1833	0.7659	2.4174	0.5373	0.6601	-$218,332	$76,594	$241,738
2	-1	-2.3416	0.7915	2.5502	0.5645	0.6902	-$234,164	$79,148	$255,017

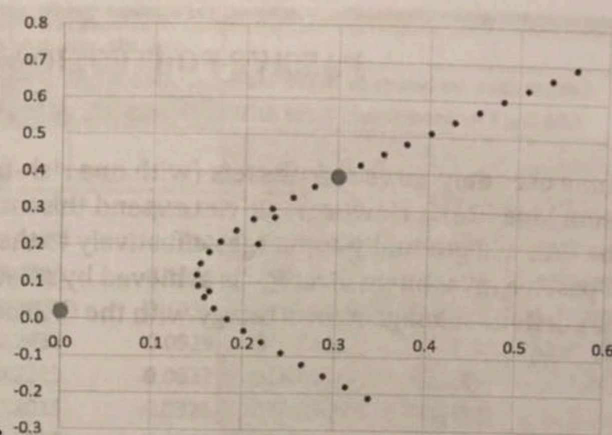
PART II

CAPITAL ALLOCATION LINE (CAL)
AND
PASSIVE PORTFOLIO MANANGEMENT

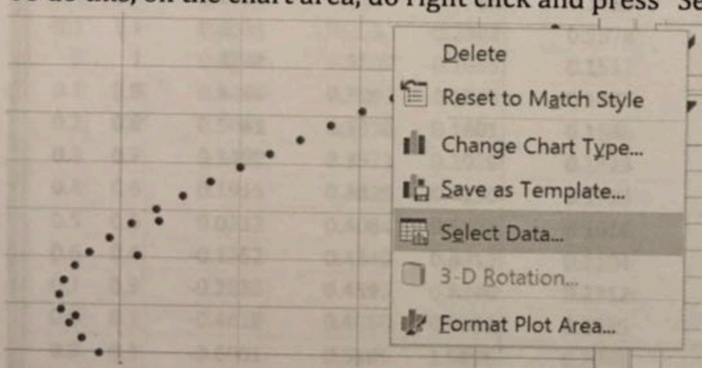
This guide uses only three risky assets (with one risk-free assets) and then determine the Capital Allocation Line (CAL). However, we can expend this analysis easily by adding more risky assets, then the CAL will gradually converges effectively to the Capital Market Line (CML). Recall, the formal passive investment strategy is achieved by selecting a portfolio on the CML. We practice the passive portfolio management strategy with the CAL on behalf of the CML.

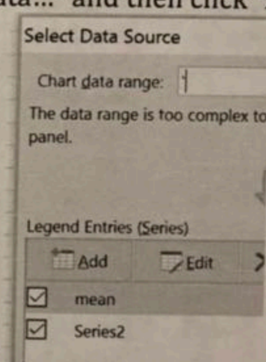
Construct the following table. Currently the cash-equivalent yield on less than 10-year maturities is around 2% (For example, 5-yr CD rate is around 2% as of June 2017). Let's use 2% as an approximate annual risk free yield, especially for around 5 year holding investments. The standard deviation of yield is not actually 0% but let's use 0% since there is less uncertainty in its value. And then you want to put the two coordinates into the investment opportunity set as shown below.

	Annualized	
	Std dev	Mean return
Risk-free asset	0	0.02
Portfolio 1	0.3010	0.3890

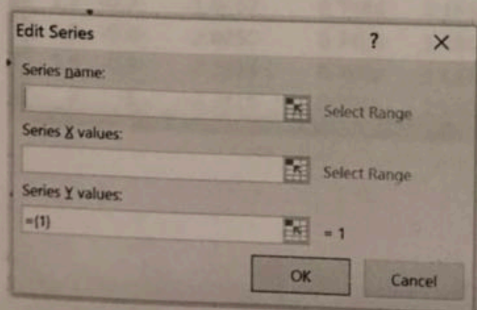


To do this, on the chart area, do right click and press "Select Data..." and then click "Add."

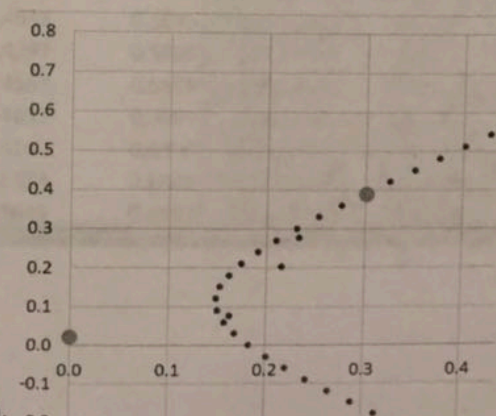




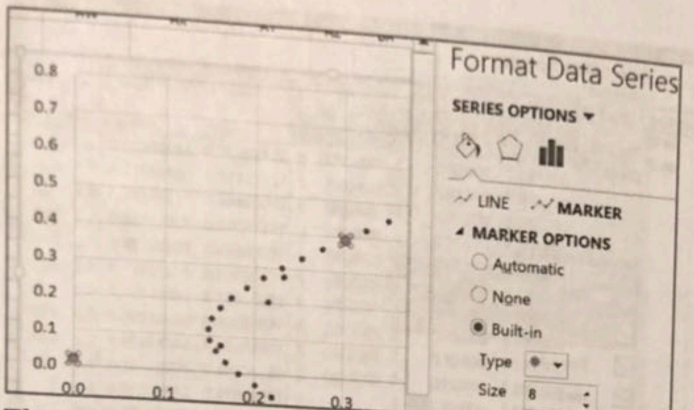
Then click the "Series X values" button and select only the values of the standard deviation (like 0 and 0.3010 without heading). Similarly, click the "Series Y values" button and select only the values of the Mean return (like 0.02 and 0.3890 without heading). Then, click "OK."

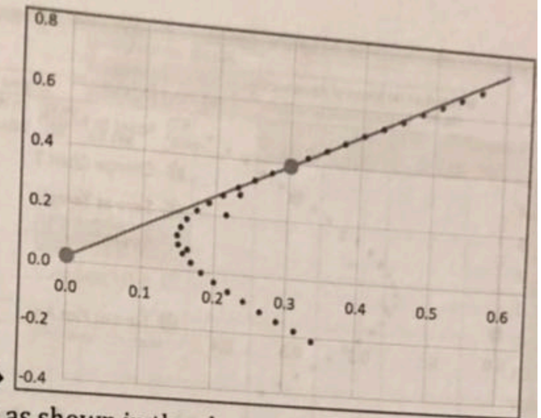


→Then,



You may want to make the two coordinates stand out by right click and change the size of the markers as shown below.





Then, you will draw the Capital Allocation Line (CAL) as shown in the above right hand side. (currently, you cannot. See below to know how to do it.)
To do this, we have to create the set of coordinates of standard deviation and mean returns of portfolios combining the coordinates of risk free asset and Portfolio 1. See below.

fx =BA28*BB24

AZ	BA	BB	BC
		Annualized	
		Std dev	Mean r
Risk-free asset		0	
Portfolio 1		0.3010	0
		Annualized CML	
W1	W2	std dev	mean
-1	2	0.60206901	0.758
-0.9	1.9	0.57196556	0.721
-0.8	1.8	0.54186211	0.684
-0.7	1.7	0.51175866	0.647
-0.6	1.6	0.48165521	0.610
-0.5	1.5	0.45155176	0.573
-0.4	1.4	0.42144831	0.536
-0.3	1.3	0.39134486	0.499
-0.2	1.2	0.36124141	0.462
-0.1	1.1	0.33113796	0.425
0	1	0.30103451	0.389
0.1	0.9	0.27093106	0.352
0.2	0.8	0.24082761	0.315
0.3	0.7	0.21072415	0.278
0.4	0.6	0.1806207	0.241
0.5	0.5	0.15051725	0.204
0.6	0.4	0.1204138	0.16
0.7	0.3	0.09031035	0.130
0.8	0.2	0.0602069	0.09
0.9	0.1	0.03010345	0.056
1	0	0	
1.1	-0.1	-0.0301035	-0.01
1.2	-0.2	-0.0602069	-0.05
1.3	-0.3	-0.0903104	-0.09
1.4	-0.4	-0.1204138	-0.1
1.5	-0.5	-0.1505173	-0.16
1.6	-0.6	-0.1806207	-0.20

→

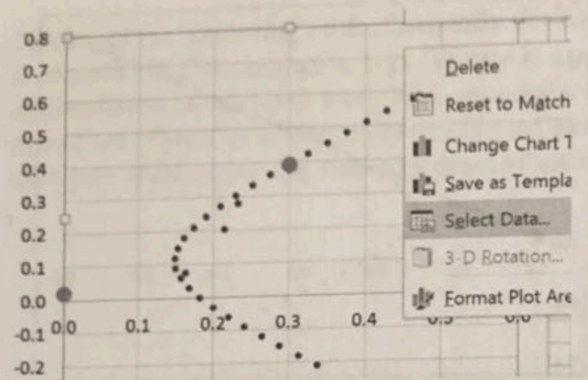
fx =AZ28*BC23+BA28*BC24

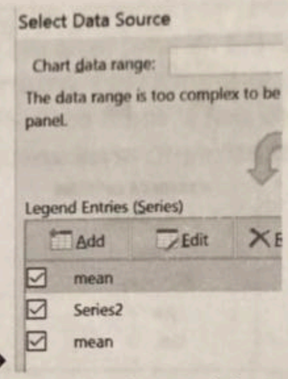
AZ	BA	BB	BC
		Annualized	
		Std dev	Mean return
Risk-free asset		0	0.02
Portfolio 1		0.3010	0.3890
		Annualized CML	
W1	W2	std dev	mean
-1	2	0.60206901	0.75807502
-0.9	1.9	0.57196556	0.72117127
-0.8	1.8	0.54186211	0.68426752
-0.7	1.7	0.51175866	0.64736377
-0.6	1.6	0.48165521	0.61046002
-0.5	1.5	0.45155176	0.57355626
-0.4	1.4	0.42144831	0.53665251
-0.3	1.3	0.39134486	0.49974876
-0.2	1.2	0.36124141	0.46284501
-0.1	1.1	0.33113796	0.42594126
0	1	0.30103451	0.38903751
0.1	0.9	0.27093106	0.35213376
0.2	0.8	0.24082761	0.31523001
0.3	0.7	0.21072415	0.27832626
0.4	0.6	0.1806207	0.24142251
0.5	0.5	0.15051725	0.20451875
0.6	0.4	0.1204138	0.167615
0.7	0.3	0.09031035	0.13071125
0.8	0.2	0.0602069	0.0938075
0.9	0.1	0.03010345	0.05690375
1	0	0	0.02
1.1	-0.1	-0.0301035	-0.0169038
1.2	-0.2	-0.0602069	-0.0538075
1.3	-0.3	-0.0903104	-0.0907113
1.4	-0.4	-0.1204138	-0.127615
1.5	-0.5	-0.1505173	-0.1645188
1.6	-0.6	-0.1806207	-0.2014225

→

		Annualized CML	
W1	W2	std dev	mean
-1	2	0.60206901	0.75807502
-0.9	1.9	0.57196556	0.72117127
-0.8	1.8	0.54186211	0.68426752
-0.7	1.7	0.51175866	0.64736377
-0.6	1.6	0.48165521	0.61046002
-0.5	1.5	0.45155176	0.57355626
-0.4	1.4	0.42144831	0.53665251
-0.3	1.3	0.39134486	0.49974876
-0.2	1.2	0.36124141	0.46284501
-0.1	1.1	0.33113796	0.42594126
0	1	0.30103451	0.38903751
0.1	0.9	0.27093106	0.35213376
0.2	0.8	0.24082761	0.31523001
0.3	0.7	0.21072415	0.27832626
0.4	0.6	0.1806207	0.24142251
0.5	0.5	0.15051725	0.20451875
0.6	0.4	0.1204138	0.167615
0.7	0.3	0.09031035	0.13071125
0.8	0.2	0.0602069	0.0938075
0.9	0.1	0.03010345	0.05690375
1	0	0	0.02

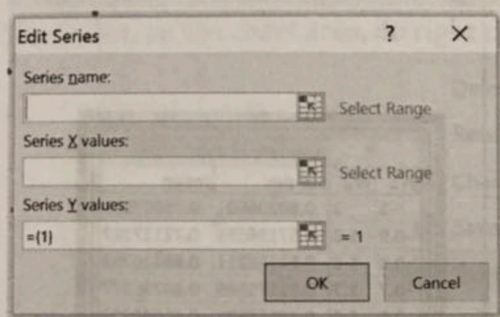
Since there are negative mean returns (as values in red), remove those parts as shown in the above, right most side. Then, plot the CAL onto the investment opportunity set with two coordinates of risk free asset and Portfolio 1. To do this, on the chart area, do right click and press "Select Data…" and then click "Add."



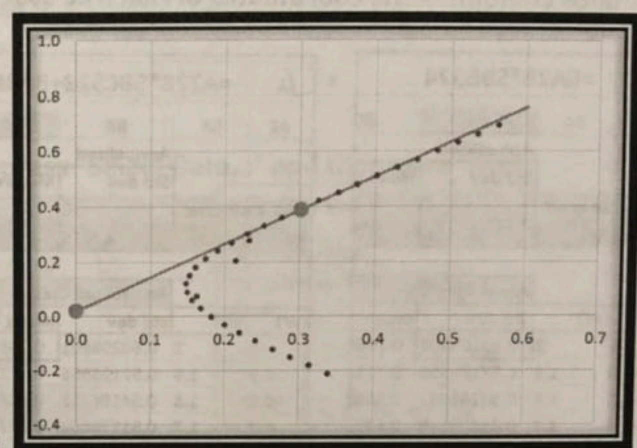


→

Then click the "Series X values" button and select only the values of the standard deviation of the CAL. Similarly, click the "Series Y values" button and select only the values of the Mean return of the CAL. Then, click "OK." Now, you have the CAL.



→Then,



EXPECTED RETURN AND RISK ON PASSIVE STRATEGY. Now, we have to determine how much to spend on risk-free assets and on the optimal risk portfolio ("Portfolio 1"). It depends on the degree of your risk aversion. Suppose that you want to put 30% on risk free assets and 70% on the optimal risky portfolio. With the total initial investment of $100,000, the amount of the optimal risky portfolio should be $70,000. Then, what are the expected return and standard deviation? This example shows that 0.2783 (27.83%) and 0.2107 (21.07%), respectively. See below.

Description on the expected return and risk of the passive portfolio (← **Report this table**):

	My Passive Portfolio
Expected return	27.83%
Standard deviation	21.07%
Sharpe Ratio (with risk free rate of 2%)	(27.83%-2%)/21.07% = 1.23

The recent data shows that the Sharpe ratio of the market portfolio with the risk-free rate using the annualized 3 month T-bills is around 0.4. Hence, the risk-adjusted return on this passive portfolio is very good.

		Annualized CML		Total initial invesment =		$ 100,000	Portfolio "1"	"2"	Optimal weights			Annual returns, std dev. Investment opportunity set with XOM, MSFT, DIS		How much to spend on each stock? With total Investment of $70,000			
W1	W2	std dev	mean	$Risk-free asset	$Optimal Risky Portfolio	Total	W1	W2	W on XOM	W on MSFT	W on DIS	std dev	mean	$ XOM	$ MSFT	$ DIS	$Optimal Risky
-1	2	0.60206901	0.75807502	$ (100,000)	$ 200,000	$ 100,000											
-0.9	1.9	0.57196556	0.72117127	$ (90,000)	$ 190,000	$ 100,000											
-0.8	1.8	0.54186211	0.68426752	$ (80,000)	$ 180,000	$ 100,000											
-0.7	1.7	0.51175866	0.64736377	$ (70,000)	$ 170,000	$ 100,000											
-0.6	1.6	0.48165521	0.61046002	$ (60,000)	$ 160,000	$ 100,000											
-0.5	1.5	0.45155176	0.57355626	$ (50,000)	$ 150,000	$ 100,000											
-0.4	1.4	0.42144831	0.53665251	$ (40,000)	$ 140,000	$ 100,000											
-0.3	1.3	0.39134486	0.49974876	$ (30,000)	$ 130,000	$ 100,000											
-0.2	1.2	0.36124141	0.46284501	$ (20,000)	$ 120,000	$ 100,000											
-0.1	1.1	0.33113796	0.42594126	$ (10,000)	$ 110,000	$ 100,000											
0	1	0.30103451	0.38903751	$ -	$ 100,000	$ 100,000											
0.1	0.9	0.27093106	0.35213376	$ 10,000	$ 90,000	$ 100,000											
0.2	0.8	0.24082761	0.31523001	$ 20,000	$ 80,000	$ 100,000											
0.3	0.7	0.21072415	0.27832626	$ 30,000	$ 70,000	$ 100,000	1	0	-0.758	0.536	1.222	0.3010	0.3890	-$53,091	$37,527	$85,564	$70,000
0.4	0.6	0.1806207	0.24142251	$ 40,000	$ 60,000	$ 100,000											
0.5	0.5	0.15051725	0.20451875	$ 50,000	$ 50,000	$ 100,000											
0.6	0.4	0.1204138	0.167615	$ 60,000	$ 40,000	$ 100,000											
0.7	0.3	0.09031035	0.13071125	$ 70,000	$ 30,000	$ 100,000											
0.8	0.2	0.0602069	0.0938075	$ 80,000	$ 20,000	$ 100,000											
0.9	0.1	0.03010345	0.05690375	$ 90,000	$ 10,000	$ 100,000											
1	0	0	0.02	$ 100,000	$ -	$ 100,000											

To conduct the passive strategy, how much do we have to spend on each of XOM, MSFT and DIS? Since the optimal risky portfolio was determined early by "Portfolio 1," (by maximizing the Sharpe ration with the constant 1 (same as the value of risk free rate)), find out the weights on the stocks with weights 1 and 0 on "Portfolio 1" and "Portfolio 2," respectively. Those were, -0.758, 0.536 and 1.222, respectively. With the amount of $70,000 on the optimal risky portfolio, the amounts of purchase are determined by multiplying the weights and $70,000. Then, the amounts are -$53,000 (shorting), $37,527 and $85,564 for XOM, MSFT and DIS, respectively.

[End of this guide]