1. Using three iterations of the secant method, estimate the root of $f_{(x)} = e^x - 3x$ in the interval $0 \le x \le 1$. Underline your answer. **(25 points)**

Secant method: $x_{i+1} = x_i - \frac{f_i(x_{i-1} - x_i)}{f_{i-1} - f_i}$ $f x_i = f_i$

i	x_{i-1}	f_{i-1}	x_i	f_i	x_{i+1}	f_{i+1}
1	~~1~~ 0	0	~~0~~ 1	1	−1	3.367
2	0	1	1	−0.2817	0.7802	0.164
3	−0.5	2.11	0.5	0.15	0.58	0.046

$f(x) = e^x - 3x$ $0 \le x \le 1$ −15/10

when $x_i = 0$

$x_{i+1} = 0 - \frac{(1)(-1-0)}{0-1}$

$x_{i+1} = -1$

$f(-1) = e^{-1} - 3(-1)$

$= 0.367 + 3$

$f_{x+1} = 3.367$

when $x_i = 1$

$f_i = e^1 - 3(1)$

$= 2.7182 - 3$

$f_i = -0.2817$

$x_{i+1} = 1 - \frac{(-0.2817)(0-1)}{1 + 0.2817}$

$x_{i+1} = 0.7802$

$f_{i+1} = e^{.7802} - 3(.7802)$

$= -0.164$

$x_i = 0.5$

$f_i = e^{0.5} - 3(0.5)$

$= 0.148$

$f_{i-1} = e^{-0.5} - 3(0.5)$

$= 2.11$

$x_{i+1} = x_i - \frac{f_i(x_{i-1} - x_i)}{f_{i-1} - f_i}$

$= 0.5 - \frac{0.15(-0.5-.5)}{2.11 - 0.15}$

$x_{i+1} = 0.58$

$f_{i+1} =$

$e^{0.58} - 3(0.58)$

$= 0.046$ →

1. Use three iterations of the Modified Secant Method to estimate the root of $f(x) = e^{-x} - x$. Use $x_i = 1$ and $\delta = 0.01$. Show all your work. **(25 Points)**

$$x_{i+1} = x_i - \frac{\delta x_i f(x_i)}{f(x_i + \delta x_i) - f(x_i)} \quad \text{(Modified Secant Formula)}$$

i	x_i	$f(x_i)$	$x_i + \delta x_i$	$f(x_i + \delta x_i)$	x_{i+1}	$\Delta x = x_{i+1} - x_i$
1	1	-0.632	1.01	-0.6458	0.5386	0.461
2	0.5386	0.045	0.544	0.0364	0.5668	0.0282
3	0.5668	0.00054	0.5725	-0.0084	0.5671	0.00034

1. In this part of the examination, you are to assume that the following variables have been created as shown below:

$$A = \begin{bmatrix} 4 & 1 & 0 & 1 \\ 3 & 2 & 0 & 4 \end{bmatrix} \qquad B = \begin{bmatrix} 4 & -1 \\ 2 & 0 \\ 0 & -3 \\ 2 & 1 \end{bmatrix}$$

In each of the following 5 questions below, write the result of the following MATLAB commands. Each question is worth 3 points. **(15 Points)**

4B
9C
4D
11F

max 85
min 27
Avg 61.5

a) max(A) = 4 ✗ (-2) = [4 2 0 4]

b) A(:,3) = 0 ✗ (-2) = [0 0]

A(2,:) =

c) A.*B' = 2 ✗ (-2) = [16 2 0 2; -3 0 0 4]

d) A(1:2,2:3) = (1, 0) ✗ (-2) = [1 0; 2 0]

e) A'|B>1 = 2 ✗ (-2) [1 1; 1 1; 0 0; 1 1]

2. Determine LU factorization without pivoting to determine a lower triangular matrix L and an upper triangular matrix U such that $A=LU$. **(20 Points)**

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -4 & 6 \\ 3 & -9 & -3 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & -3 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & -1 & -4 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & 0 & +16 \end{bmatrix}$$

$\frac{1}{2}R_2 - R_1$, $\frac{1}{3}R_3 - R_1$; $R_3 - R_2$; $4R_3 + R_2$

$$U = \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & 0 & -16 \end{bmatrix}$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 1 & -2 & 3 \\ 1 & -3 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 6 \\ 4 & -11 & 0 \\ 1 & -3 & -1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 6 & 0 \\ 4 & -11 & 0 \\ 1 & -3 & -1 \end{bmatrix} = L$$

$\frac{1}{2}R_2$, $\frac{1}{3}R_3$; $R_1 + R_2$, $R_2 + 3R_3$

$$A = \begin{bmatrix} 1 & 0 & 0 \\ 4 & -11 & 0 \\ 1 & -3 & -1 \end{bmatrix} \begin{bmatrix} 1 & 2 & 3 \\ 0 & -4 & 0 \\ 0 & 0 & -16 \end{bmatrix}$$

-9

2. Use four iterations of the Gauss-Seidel method to solve the following nonlinear system of equations. Use initial guess of $x = 0,\ y = 3$. Show all your work. **(25 Points)**

$$x^2 + 3x - y = -2$$
$$-2x + y = 3$$

In Liebmann Form

$$x = \frac{-2 - x^2 + y}{3}$$
$$y = 3 + 2x$$

i	x	y
1	**0.33**	**3.67**
2	**0.52**	**4.04**
3	**0.59**	**4.18**
4	**0.61**	**4.22**

Alternatively:

$$x^2 + 3x - y = -2$$
$$-2x + y = 3$$

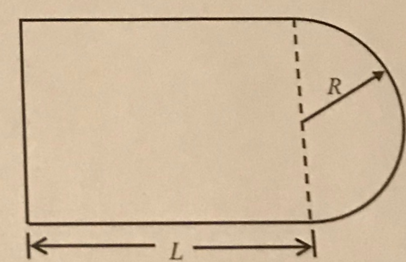
In Liebmann Form

$$y = x^2 + 3x + 2$$
$$x = \frac{y-3}{2}$$

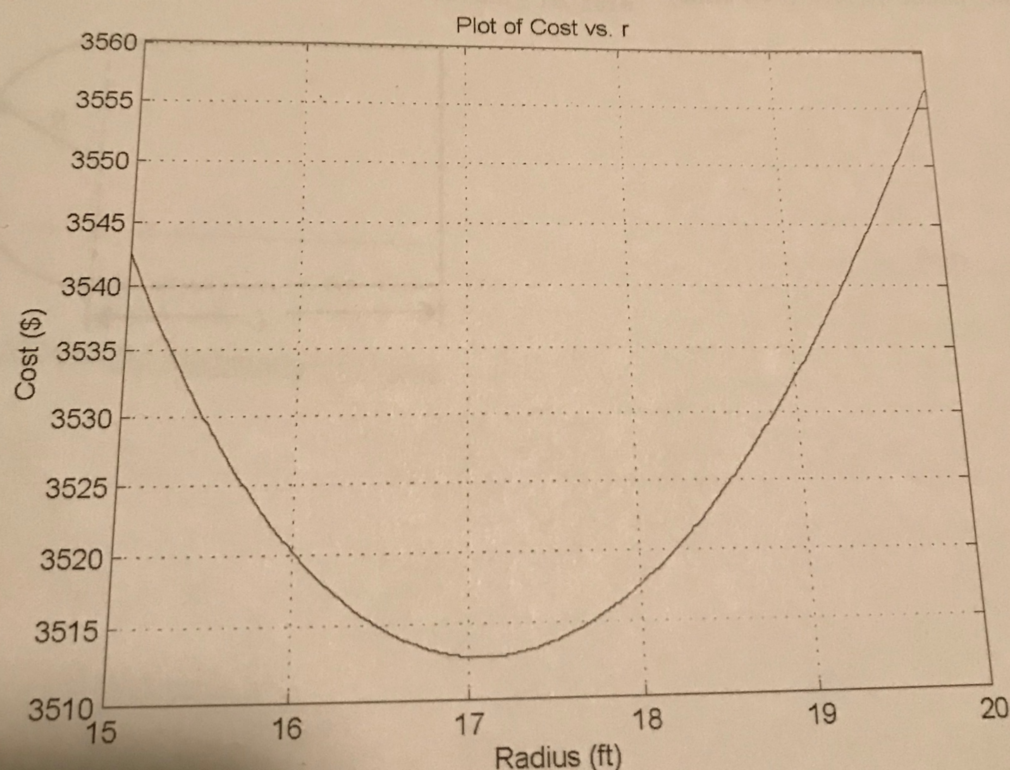
i	x	y
1	**0**	**2**
2	**-0.5**	**0.75**
3	**-1.125**	**-0.1093**
4	**-1.554**	**-0.247**

- ***This part of the exam is closed book and closed note. You may NOT use any of the MATLAB files on your PC or the course material on Blackboard. You may use the MATLAB help.***
- ***Submit your program as m.file and graph in pdf.***
- ***Your name should be displayed on your script file.***
- ***The graph should have labels and your name as the title.***
- ***Copy and paste your output from*** fprintf ***statement(s) into your script file as a comment.***

A fenced enclosure consists of a rectangle of length L and width $2R$, and a semicircle of radius R, as shown. The enclosure is to be built to have an area of *1500 ft²*. The cost of the fence is *$30* per foot for the curved portion and *$20* per foot for the straight sides. Plot the cost versus R for $15 \le R \le 20$ meter in increments of *0.01 ft*, and determine the radius that results in the least cost. Use `fprintf` to display the minimum cost and the corresponding radius. Note that $L = (1500 - 0.5\pi R^2)/(2R)$. **(50 Points)**



```
% This is a script file for ME-140-03 Exam 1, Part 2
%
%                   February 16, 2016
%Problem No. 1
disp('Solution to problem 1')
r=15:0.01:20;
L=(1500-0.5*pi*r.^2)./(2*r);
c=20*(2*r+2*L)+30*(pi*r);
plot(r,c);grid
xlabel('Radius (ft)')
ylabel('Cost ($)')
title('Plot of Cost vs. r')
pause
[c_min,k]=min(c);
r_min=r(k);
fprintf('The minimum cost is $%3.2f and the optimum radius is %3.2f
feet\n',c_min,r_min)
%The minimum cost is $3512.81 and the optimum radius is 17.08 feet
```



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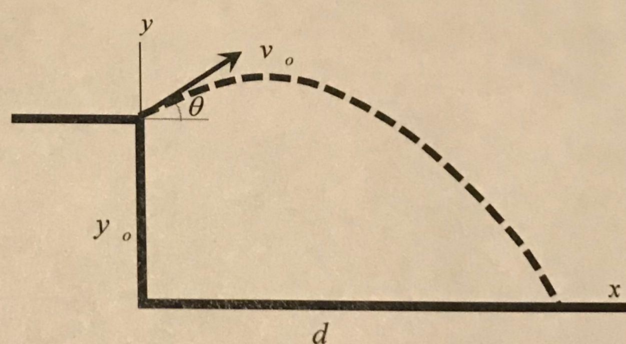
- ***Submit your program as an m.file and the graph in pdf.***
- ***Your name should be displayed on your script file.***
- ***The graph should have labels and title.***
- ***Copy and paste your output from*** `fprintf` ***statement into your script file as a comment.***

A ball is kicked from the roof of a 50 feet tall building with an initial velocity $v_o = 40$ ft/s at an angle $\theta = 30^0$ Write a script file that asks the user to input the initial velocity v_o and angle θ in degrees and determine the location where the ball hits the ground and the maximum height it reaches. Display your results in a single `fprintf` statement in decimal format with two decimals. Also plot the trajectory of ball. The kinematic equations for the projectile motion are given by: **(50 Points)**

$$x = x_o + v_{ox}t$$
$$y = y_o + v_{oy}t - 0.5gt^2$$



Hint: Solve for *t* by finding roots of the polynomial.

3. Using three iterations of the secant method to find the root of $f_{(x)} = \sin(\sqrt{x}) - x + 0.4$ in the interval $0 \le x \le 2$. Show the first two iterations. Note that x is in radians. **(15 points)**

Secant method: $x_{i+1} = x_i - \dfrac{f_i(x_{i-1} - x_i)}{f_{i-1} - f_i}$

i	x_{i-1}	f_{i-1}	x_i	f_i	x_{i+1}	f_{i+1}	$\Delta x = x_{i+1} - x_i$
1	0	0.4	2	-0.6122	0.7903	0.386134	-1.2097
2	0.7903	0.386134	2	-0.6122	1.25818	0.04265	-0.74182
3	1.25816	0.04265	2	-0.6122	1.30649	0.003399	-0.69351