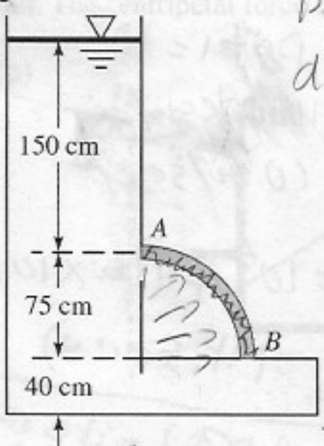
ME721 Fundamentals of Fluid Flow Name: [redacted]

**Please include all the intermediate steps in your answers. No score will be given to an answer without justification.**

1. (20 points) The tank in the figure below is 120 cm long into the paper. Determine the horizontal and vertical hydrostatic forces on the quarter-circle panel AB. Water density: 998 kg/m$^3$.




pressure on x direction

$dF_y = (P - P_a)\frac{d}{2}\, d\theta \Rightarrow \gamma(h + \frac{d}{2}\sin\theta)\, b\, d\theta$

$F_y = -\int_0^{\pi}(h + \frac{d}{2}\sin\theta)\frac{d}{2} b\cos\theta\, d\theta$

$= -\gamma\frac{d}{h} b(h\cos\theta + \frac{d}{2}\sin\theta\, d\theta$

$F_x = \int_0^{\pi} dF_y = -\int_0^{\pi}(h + \frac{d}{2}\sin\theta)\frac{d}{2} b\sin\theta\, d\theta$

$= -\gamma\frac{d}{2} d\int_0^{\pi} h\sin\theta + \frac{d}{2}\sin^2\theta\, d\theta$

$F_x = -\gamma\frac{d}{2} b\{-h\cos\theta \,|_0^{\pi} + \frac{d}{4}\pi\}$ $\frac{1}{2}(1-\cos 2\theta)$

$F_x = -\gamma\frac{d}{2} b(2h + \pi\frac{d}{4})$

what are d and θ?
when you introduce new variables, you should define them first.

5

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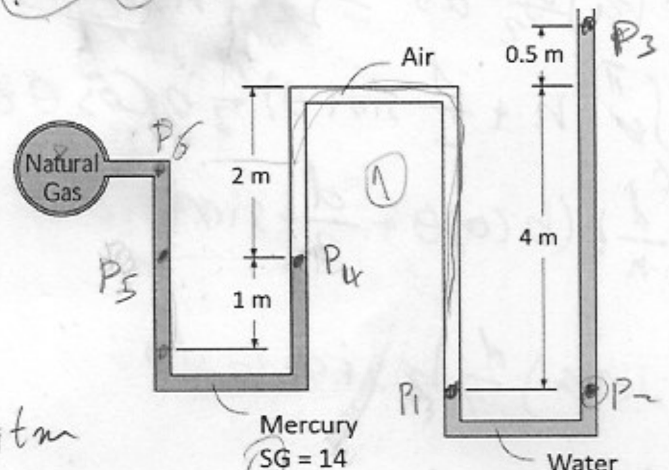
2. (20 points) The pressure in a natural gas pipeline is measured by the manometer shown in the figure below. One of the arms is open to the atmosphere air (1 atm). Determine the absolute pressure of natural gas in the pipeline using the data given below.

- acceleration of gravity: $\boxed{g = 10\ m/sec^2}$ ✓
- water density: $\rho = 1000\ kg/m^3$ ✓
- pressure variation in the gases in the manometer can be ignored
- 1 atm = $10^5$ Pa ✓




$P_3 = P_{atm} = 10^5$

$\rho = 1000\ kg/m^3$

$g = 10\ m/sec^2$

$P_2 = 10^5 + (1000 \times 10 \times 4.5)$

$P_2 = 145000$

$\boxed{P_1 = P_2 = 145000}$

$P_6 = P_{atm}$

$P_4 \neq P_5$

$P_5 = P_6 + \rho g L$ ? what is L ?

$P_{\approx} = P_{atm} + \rho g L$

$P \approx 10^5 + (1000 \times 10 \times 3)$

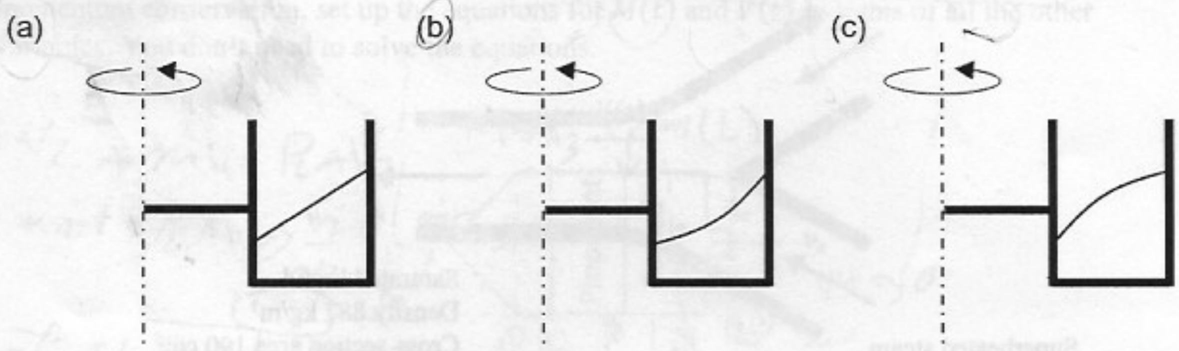
$\boxed{= 130000}$

10

ME721 Fundamentals of Fluid Flow Name:

3. (20 points) A centrifuge is a mechanical device that can subject an experimental sample to a sustained centrifugal force. Select the figure that best describes the free surface in the tube rotating in a centrifuge and explain the reason of your choice. No point will be given to the answer without justification.

Hint. The centripetal force is linearly proportional to the distance from the rotation axis.




$$\rho \vec{a} = -\nabla P + \rho g + \rho r w^2 \vec{e_r} = 0$$

$$r\text{-dir}: -\frac{\partial P}{\partial r} + \rho r w^2 = 0$$

$$\frac{\partial P}{\partial r} = \rho r w^2$$

$$P(r,z) = \frac{1}{2}\rho r^2 w^2 + f(z)$$

$$z\text{-dir}: -\frac{\partial P}{\partial z} - \rho g = 0 \;--\; \frac{\partial P}{\partial z} = -\rho g \;--\; f = -\rho g z$$

$$P(r,z) = \frac{1}{2}\rho r^2 w^2 - \rho r^2 w^2 - \rho g z + P_0$$

find $P(r,z) = P_{atm}$

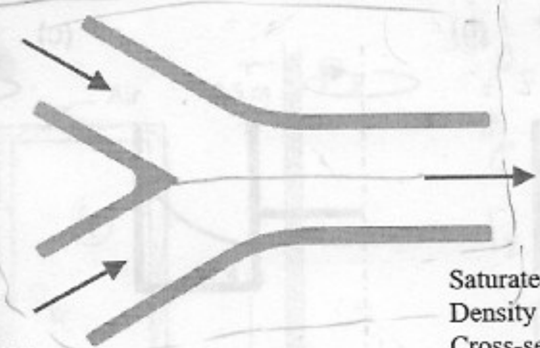
$$\frac{1}{2}\rho r^2 w^2 - \rho g z + f_0 = P_{atm}$$

$$\Rightarrow z = \frac{w^2}{2g} r^2 + z$$

20

4. (20 points) Compressed water is to be mixed with superheated steam in an open feedwater heater (shown in the figure below). What is the velocity at the outlet, given that the cross-section area of outlet is 100 cm$^2$?

Compressed water
Density: 990 kg/m$^3$
Volume flow rate: 10 liters/sec



Saturated liquid
Density 887 kg/m$^3$
Cross-section area 100 cm$^2$

Superheated steam
Density 4.9 kg/m$^3$
Volume flow rate: 0.55 m$^3$/sec

$$\frac{\partial}{\partial t}(\rho\vec{V}) + \nabla(\rho\vec{V}\vec{V}) + \nabla P$$

From Mass conservation

$$\int \frac{\partial \rho}{\partial t}\, d\forall + \int_{cs} \rho\vec{V}\cdot\vec{n}\, dA = 0$$

For ① => $\int_{cs} \vec{V}\cdot\vec{n}A = \rho(-V_0 k)(\hat{k})A_0 = -\rho V_0 A_0$

② and ③ $\int_{cs_{2,3}} \rho\vec{V}\cdot\vec{n}\, dA = \rho(V_r \hat{e}_r)\cdot(\hat{e})A_s = \rho V_r A_s$, $h = \frac{V_0 A_0}{2\pi r V_r}$

$$\int_{R_1,R_2} \frac{\partial \vec{V}}{\partial t}\cdot dx + (P+\tfrac{1}{2}\rho|\vec{V}|) + \rho g z)_{R_2} - (P+\tfrac{1}{2}\rho|\vec{V}|^2+\rho g z)^2_{R_2} = 0 \quad 5$$

① -- ②

$$\frac{P_1}{\rho} + \frac{1}{2}V_1^2 + gz_1 = \frac{P_2}{\rho} + \frac{1}{2}V_2^2 + gz_2 \quad \text{Put } P_1 = P_2 = P_{atm}$$

$V_1 = V_0,\ z = L,\ z_2 = h,\ V_2 = V_r$

$V_r^2 = V_0^2 + 2g(L-h)$ -- (7)

From (7) and (8)

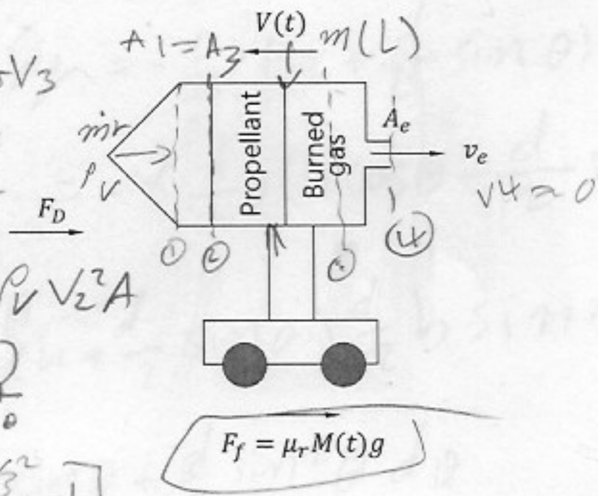
$$V_r = \sqrt{-(100\text{ cm}) + (2\times 9.81)\times(14.9)} = 14.005$$

There is no height difference between 1 and 2 or 1 and 3.

The Bernoulli equation cannot be used when the streamline is not defined. There is a mixing in the pipe. No way to define streamlines in such a case.

5. (20 points) A rocket using solid propellant is mounted on a car. The propellant is burned to produce the gas of density $\rho$, which is exhausted through the nozzle of area $A_e$ at the speed of $v_e$ (with respect to the rocket). The total mass of rocket car is $M(t)$, which decreases over time due to the fuel consumption and gas exhaust. The rocket car experiences drag force $F_D$ due to the air flow and the friction force $F_f$ with the ground. We assume that $F_D$ is constant, while $F_f = \mu_r M(t) g$, where $\mu_r$ is the rolling friction coefficient and $g$ is the gravity. Using the mass and momentum conservation, set up the equations for $M(t)$ and $V(t)$ in terms of all the other variables. You don't need to solve the equations.

$\rho_V A V_2 + \dot m l = \rho_L A V_3$




$\dot m r + \dot m l = \rho_L A V \Rightarrow \frac{V_3}{V_2}$

$\rho_2 A - \rho_3 A = \rho_L V_3^2 A - \rho_V V_2^2 A$

$P_2 - P_3 = \rho_L V_3^2 - \rho_V V_3^2$ ?

$\Rightarrow P_2 - P_3 = \rho_V V_2^2 \left[\frac{\rho L\, V_3^2}{\rho V\, V_2^2} - 1\right]$

$\Rightarrow P_2 - P_3 = \rho_V V_2^2 \left[\frac{\rho L}{\rho V}\left[\frac{\rho V}{\rho L} + \frac{\dot m l}{\dot m V}\,\frac{\rho_V}{\rho_L}\right]^2 - 1\right]$

$P_2 - P_3 = \rho_L V_2 \left[\frac{\rho_L}{\rho V}\left[\frac{\rho_V}{\rho_L} + \frac{\dot m L}{\dot m V}\,\frac{\rho_V}{\rho_L}\right]^2 - 1\right]$

$1 + \left[\frac{\dot m L}{\dot m r}\right]^2 < \sqrt{\frac{\rho_L}{\rho_V}}$

$\frac{\dot m L}{\dot m r} < \sqrt{\frac{\rho L}{\rho_V}} - 1$

5

45