Engineering Numerical Methods
Spring 2018

Applications of Solving Roots & Linear Algebraic Equations

Question 1: [15 points]

Figure 1 below shows three reservoirs connected by circular pipes. The pipes, which are made of asphalt-dipped cast iron (roughness $\varepsilon = 0.0012$ m), have characteristics as summarized in Table 1. You are interested in determining the elevation in Reservoir B and the flows in pipes 1 and 3.

Table 1. Characteristics of the asphalt-dipped cast iron pipes that connect three reservoirs.

Pipe	1	2	3
Length [m]	1750	500	1350
Diameter [m]	0.35	0.25	0.21
Flow [m^3/s]	?	0.1	?

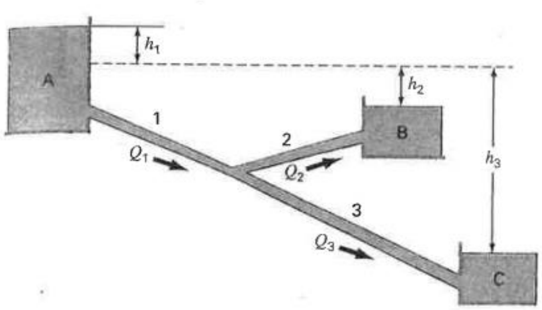

Figure 1. Three reservoirs connected by circular pipes.

(a) To answer the above problem, you are first required to determine a robust method for computing the friction factors in the pipes. Note that the pressure drop in a section of pipe can be calculated as $\Delta p = 0.5 f L \rho V^2 / D$ where Δp denotes the pressure drop [Pa], ρ is the fluid density [kg/m^3], f represents the friction factor, V is the velocity [ms^{-1}] while L and D correspond to the length [m] and diameter [m] of the pipe, respectively. For a turbulent flow, the Colebrook equation provides a method to calculate the friction factor in a pipe:

$$\frac{1}{\sqrt{f}} = -2.0\log_{10}\left(\frac{\varepsilon}{3.7D} + \frac{2.51}{\mathrm{Re}\sqrt{f}}\right),$$

where ε denotes the roughness [m], $\mathrm{Re} = VD/\mu$ corresponds to the Reynolds number and μ is the is the kinematic viscosity [m^2/s] (i.e., the kinematic viscosity of water is 10^{-6} m^2/s). It is also important to recognize that the flow Q in a circular pipe is given by $Q = (\pi D^2/4)V$ [m^3/s].

(a) **[7 points]** Verify your numerical calculations of the friction factor for turbulent flow in pipe 2 using at least three distinct algorithms for root location and a graphical method. One of the three algorithms may include any root location functions available in mathematical software packages.

A good initial guess can be obtained using the Blasius formula $f = 0.316/\mathrm{Re}^{0.25}$. Comment on any difficulties that you have encountered while solving the *Colebrook equation* for computing the friction factor in pipe 2.

(b) **[8 points]** If the water surface elevations in Reservoirs A and C 250 m and 170 m, respectively, compute the flows in pipes 1 and 3 and the elevation in Reservoir B.

Note that the flows in all the connected pipes must balance and the sum of the friction losses in pipes 1 and 3 must balance the elevation drop in Reservoirs A and C, where the friction loss is given by $\Delta p/(\rho g)$ and $g = 9.80665$ ms^{-1}.

Question 2: [10 points]

[S. Chapra & R. Canale, 6th Edition] Problem 8.46

Question 3: [18 points]

Suppose that you are studying the ventilation system (indoor air pollution) for a truck-stop restaurant located adjacent to a six-lane freeway. As shown in Figure 2 below, the restaurant serving area consists of two square rooms for smokers and kids and one elongated room. Room 1 and Section 3 have sources of carbon monoxide from smokers and a faulty grill, respectively. In addition, Room 1 and Room 2 gain carbon monoxide from air intakes that unfortunately are positioned alongside the freeway. Note that the one-way arrows in Figure 2 represent volumetric airflows, whereas the two-way arrows represent diffusive mixing. Although the smoker and grill loads add carbon monoxide mass to the system, their airflows are negligible. Steady-state mass balances can be written for each room. For example, for the smoking section (Room 1), the mass balance can be written as

$$\underset{\text{(load)}}{0 = W_{smoker}} \quad \underset{\text{(inflow)}}{+Q_a c_a} \quad \underset{\text{(outflow)}}{-Q_a c_1} \quad \underset{\text{(mixing)}}{+E_{13}(c_3 - c_1)}$$

or as $230c_1 - 30c_3 = 1400$ after substituting the parameters. Similar mass balances can be written for other rooms/sections.

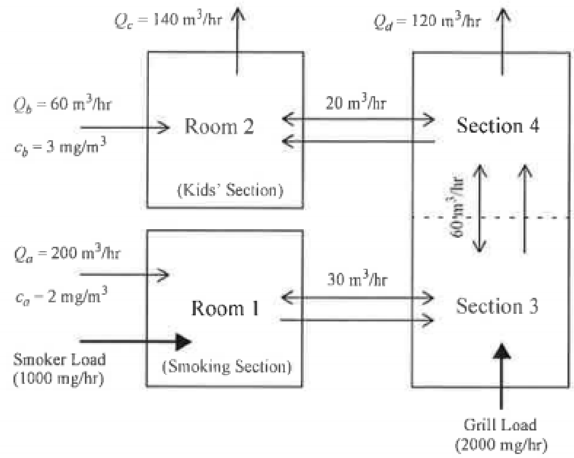

Figure 2. Overhead view of rooms in a restaurant to study indoor air pollution.

(a) **[6 points]** Solve for the steady-state concentrations of carbon monoxide in each section using: (i) Cramer's rule approach; (ii) Gauss elimination with partial pivoting technique. You may utilize any mathematical software tools (e.g., MATLAB, Excel, etc.) for computing the determinant of any square matrix.

(b) **[4 points]** Find the inverse of the system matrix via LU decomposition technique and then use the matrix inverse to find the steady-state concentrations of carbon monoxide in each section. To avoid tedious calculations using a calculator, you are strongly recommended to clearly highlight the decomposition of the system matrix $[A] = [L][U]$. In fact, you can easily deduce the $[L]$ and $[U]$ matrices from part (a). Next you may define an auxiliary matrix $[B] = [I]_{4x4}$ and then solve the equation $[D] = [L]^{-1}[B]$ using a mathematical software package to find another auxiliary matrix $[D]$. Finally, determine the inverse of matrix $[A]$ as $[A]^{-1} = [U]^{-1}[D]$.

(c) **[2 points]** Determine what percent of the carbon monoxide in the kids' section (Room 2) is due to: (i) the smokers; (ii) the grill; and (iii) the air in the intake vents.

(d) **[5 points]**

i. Compute the improvement in the kids' section concentration if the carbon monoxide load is decreased by banning smoking and fixing the grill.

ii. If the smoker and grill loads are increased to 2000 and 3000 mg/hr, respectively, determine the increase in concentration in the kids' section.

iii. How does the concentration in the kids' section change if a screen is constructed so that the mixing between Section 2 and Section 4 is decreased to 10 m^3/hr?

(e) **[1 point]** Is the above system ill-conditioned? Provide at least two reasons to justify your answer.

Question 4: [7 points]

Solve the following complex system of equations via:

(a) Cramer's rule with complex variables.

(b) Converting the complex system of n equations into its equivalent real system of $2n$ equations. You may then use the built-in functions of any chosen mathematical software package (e.g., MATLAB, Excel, etc.) to solve this problem. Compare your results obtained in part (a) and part (b).

$$\begin{bmatrix} 5+2i & 3 & 2-i \\ 2 & -3i & 2+i \\ 7i & 1+2i & 4+2i \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1-5i \\ 3+2i \\ 6i \end{bmatrix}$$

8.46 The space shuttle, at lift-off from the launch pad, has four forces acting on it, which are shown on the free-body diagram (Fig. P8.46). The combined weight of the two solid rocket boosters and external fuel tank is $W_B = 1.663 \times 10^6$ lb. The weight of the orbiter with a full payload is $W_S = 0.23 \times 10^6$ lb. The combined thrust of the two solid rocket boosters is $T_B = 5.30 \times 10^6$ lb. The combined thrust of the three liquid fuel orbiter engines is $T_S = 1.125 \times 10^6$ lb.

At liftoff, the orbiter engine thrust is directed at angle θ to make the resultant moment acting on the entire craft assembly (external tank, solid rocket boosters, and orbiter) equal to zero. With the resultant moment equal to zero, the craft will not rotate about its mass center G at liftoff. With these forces, the craft will have a resultant force with components in both the vertical and horizontal direction. The vertical resultant force component is what allows the craft to lift off from the launch pad and fly vertically.

The horizontal resultant force component causes the craft to fly horizontally. The resultant moment acting on the craft will be zero when θ is adjusted to the proper value. If this angle is not adjusted properly, and there is some resultant moment acting on the craft, the craft will tend to rotate about it mass center.

(a) Resolve the orbiter thrust T_S into horizontal and vertical components, and then sum moments about point G, the craft mass center. Set the resulting moment equation equal to zero. This equation can now be solved for the value of θ required for liftoff.

(b) Derive an equation for the resultant moment acting on the craft in terms of the angle θ. Plot the resultant moment as a function of the angle θ over a range of -5 radians to $+5$ radians.

(c) Write a computer program to solve for the angle θ using Newton's method to find the root of the resultant moment equation. Make an initial first guess at the root of interest using the plot. Terminate your iterations when the value of θ has better than five significant figures.

(d) Repeat the program for the minimum payload weight of the orbiter of $W_S = 195{,}000$ lb.

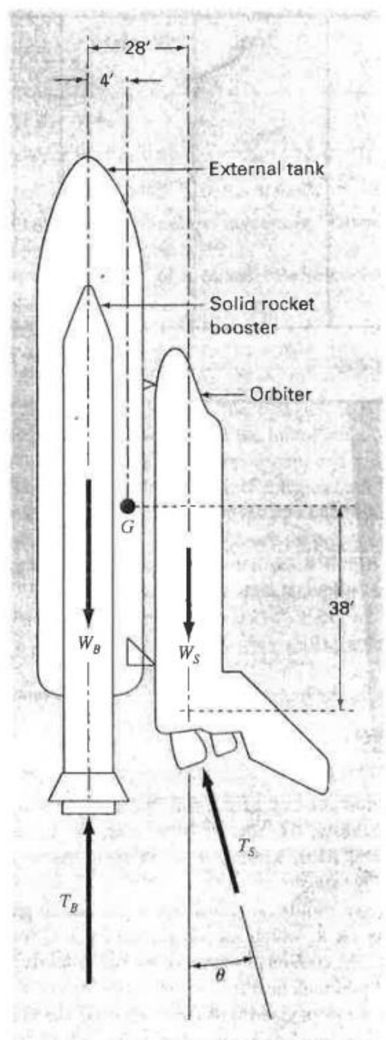

Figure P8.46