Experiment 3

ELECTRIC FIELD PLOTTING

3.1 Purpose

To determine the equipotential lines for two-dimensional charge distribution and the corresponding electric field.

3.2 Apparatus

Cork board, thumb tacks, conductive paper, multi-meter, DC power supply, and ruler.

3.3 Theory

In this experiment two-dimensional electrical potential distributions are explored. Two electrodes are aluminized on a sheet of black paper and a potential difference is established between them by connecting them to opposite terminals of a DC power supply. Thus there will be an electric field $\vec{E}$ established in the area of the paper that lies between the electrodes. The electric field is a vector field representing the force per unit charge acting on a small positive q test charge placed in the electric field,

$$\vec{E}(\vec{r},t) \equiv \frac{\vec{F}_q(\vec{r},t)}{q} \quad . \tag{3.1}$$

Because the electric force is a conservative one, a simpler description of the physics can be found by examining quantities related to the energy. The *electric potential* at some point is defined as the *electrical potential energy per unit charge* of some small positive "test" charge placed at that point. The unit of electric potential is called a Volt, abbreviated V, and is equivalent to J/C (Joules per Coulomb), the standard units of energy divided by charge. The electric potential is often called simply "potential" or "voltage" and is often represented by the letter V, so

$$V(\vec{r},t) \equiv \frac{U(\vec{r},t)}{q} \quad . \tag{3.2}$$

Electric potential is usually measured relative to an arbitrary point, where the potential would therefore be 0. A common reference potential is *electrical ground*.

The relationship between electric potential and electric field can be derived as follows. The work done ΔW by a force $\vec{F}$ on an object moving from point a to point b along a path $d\vec{s}$ is given by

$$\Delta W = \int_a^b \vec{F} \cdot d\vec{s} \tag{3.3}$$

The force is related to the electric field through $\vec{F} = q\vec{E}$ (equation 3.1). Noting that the change in potential energy due to a force is the negative of the work done by that force this equation becomes

$$\Delta U = -\Delta W = -\int_a^b \vec{F} \cdot d\vec{s} = -\int_a^b q\vec{E} \cdot d\vec{s} \tag{3.4}$$

Since the result is proportional to q, we can divide it out and thereby get an expression for the potential difference,

$$\Delta V = V_b - V_a = \frac{\Delta U}{q} = -\int_a^b \vec{E} \cdot d\vec{s} \quad . \tag{3.5}$$

One can map the electric field within a region by first finding the *equipotential lines* in that region. Equipotential lines are lines along which the electric potential does not change. The electric field lines are then plotted beginning at the positive electrode (high potential) and ending at the negative electrode continually running perpendicular to the equipotential lines previously found.

To obtain the magnitude of the average electric field along a particular line with distance d between two equipotential lines the electric field is taken as being constant, we can rewrite equation 3.5 into

$$|\Delta V| = |V_b - V_a| = \left| -\int_a^b \vec{E} \cdot d\vec{s} \right| \equiv |\vec{E}|_{\text{av}} \left| \int_a^b ds \right| = |\vec{E}|_{\text{av}} \, |b - a|$$

Hence, with $d = b - a$,

$$|\vec{E}|_{\text{av}} = \left| \frac{V_b - V_a}{b - a} \right| = \left| \frac{\Delta V}{d} \right| \quad . \tag{3.6}$$

3.4 Procedure

The paper to be used in this experiment is slightly conductive. The equipotential lines are to be determined directly by using a transistorized voltmeter (*e.g.* Multi-meter MM) as shown in Fig. 3.1 below. The D.C. power supply serves to establish a potential difference between the silver electrodes. As long as the power supply is connected, a small current flows through the paper. At the start of the experiment the power supply is set to 6 V. Select 7 values in increments of 0.75 V of the potential fairly evenly distributed from 0 to 6 V. Once the power supply has been set to 6 V, do not change it. Now, using the probe, find points on the paper that have the first of the potentials, *i.e.* 0.75 V. Make sure you find enough points to be able to later connect them with a smooth curve. Then proceed to the other selected voltages in 0.75 V increments, making sure to keep track of which points correspond to which voltages.

3.5 Precautions

1. Handle the black paper only by the edges. Carefully avoid contaminating it.
2. Fix the black paper to the corkboard with tacks.
3. Connect all circuit elements with the power supply in the OFF position. In all electrical experiments the final connections to the power supply are made after the instructor has checked and approved the circuit. See Fig. 3.1 for an example.

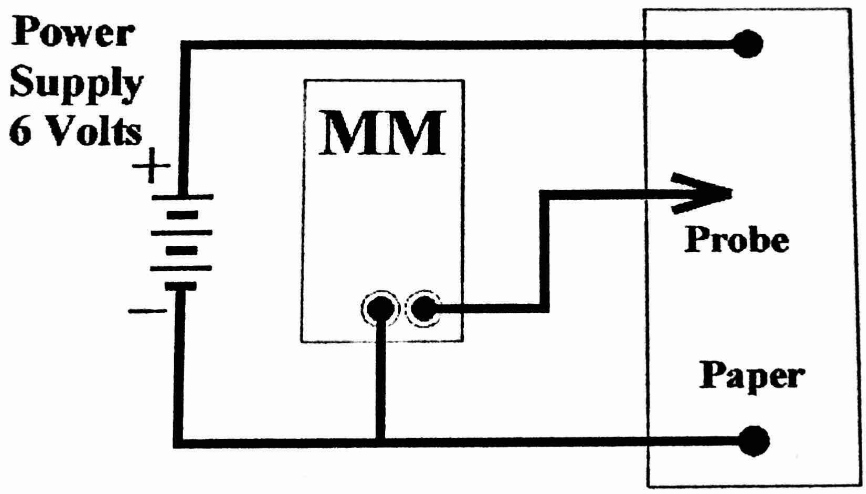

Figure 3.1: Field plotting using a multi-meter

4. Make sure the negative terminal of the voltmeter is connected to the negative terminal of the power supply and that the voltmeter is set to the correct range of voltages to be read.
5. Make sure your connections to your electrodes are good.
6. Do not make large points or holes in the paper, as those will alter the potential distribution. Do not connect the points with lines until after the sheet is finished.

3.6 Analysis and Calculations

1. For each selected voltage, connect the points corresponding to that potential with a smooth curve. These are the equipotential lines. Record the voltage of that line at one end of the line.
2. From the equipotential lines, produce the electric field lines. These lines will be smooth curves beginning at the positive electrode, crossing each equipotential line at right angles and ending at the negative electrode. You should draw one electric field line directly between the two electrodes and two other lines on either side of it, making sure to spread them out so that they are reasonably distinct.
3. Measure and record the distance between each of the equipotential lines along the center electric field line. (Be sure to include the electrodes as equipotential surfaces.)
4. Calculate the magnitude of the average electric field between each of the equipotential lines along the center electric field line. Is the electric field constant along this line? If not, where is it greatest?
5. Repeat steps 3 and 4 for one of the outermost electric field lines. Is the electric field constant along this line? If not, where is the electric field the greatest? Is the magnitude of the electric field along this line greater, less than, or equal to the magnitude along the center electric field line? Why?

3.7 Questions

1. Show from equation 3.5 that the electric field lines run from higher voltages to lower voltages.

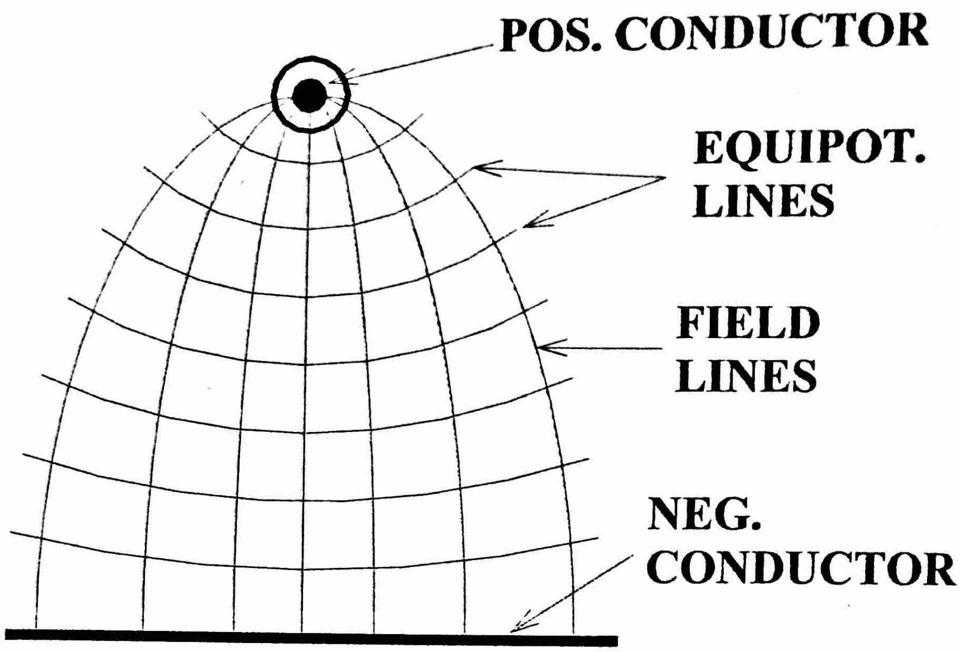

Figure 3.2: Electric field pattern for a plate.

2. Show from (the same) equation 3.5 that the electric field lines are perpendicular to equipotential lines.

3. What would change in this experiment if the power supply were set to 12 V instead of 6 V?

Write the answers on the Data Sheet.

Data

Return your sheet with equipotential points marked.

Measurements and Calculations

1. Record the positions where you find the selected potential values of 0.75, 1.50, . . . 5.25 V.
2. Draw equipotential lines; label with the potential.
3. Draw electric field lines.
4. Measure the distance along the center electric field line:

Between	0– –0.75	0.75– –1.50	1.50– –2.25	2.25– –3.00	3.00– –3.75	3.75– –4.50	4.50– –5.25	5.25– –6.00
cm	1.5	1.6	3.2	5.3	6.1	5.4	3.6	1.8

5. Calculate the magnitude of average electric field along the center line field with equation 3.6.

Between	0– –0.75	0.75– –1.50	1.50– –2.25	2.25– –3.00	3.00– –3.75	3.75– –4.50	4.50– –5.25	5.25– –6.00
V/m								

Is the electric field constant along this line? __________

If not, where is the magnitude of the average electric field the greatest?

____

6. Measure the distance along the outermost electric field line:

Between	0– –0.75	0.75– –1.50	1.50– –2.25	2.25– –3.00	3.00– –3.75	3.75– –4.50	4.50– –5.25	5.25– –6.00
cm	1.7	2.3	4.0	5.6	6.4	5.9	4.7	2.4

Calculate the magnitude of average electric field along the outmost field line

Between	0– –0.75	0.75– –1.50	1.50– –2.25	2.25– –3.00	3.00– –3.75	3.75– –4.50	4.50– –5.25	5.25– –6.00
V/m								

Is the electric field constant along this line? __________

If not, where is the magnitude of the average electric field the greatest?

____