

Industrial Organization and Regulation

Assignment 2, 2019

due date: September 20th (Friday, before
4pm)

Please read the following instructions carefully:

- The assignment should be handed in on time. You can submit your answers to your tutors in the tutorials or to me at my office (E977 Menzies building). You can slide in the answer sheets if I am not in the office and confirm with me afterwards. **Important:** Please DO NOT put your submission in the assignment box on floor 9 of Menzies building as I do not have access to that!
- This assignment accounts for 20% of your final grades.
- The answer to each question should be concise. Do NOT submit all your workouts. Only results and some key steps should appear in the answer sheet.
- **Write down your name and student ID in the answer sheet.** I do not require a cover sheet though.
- The grades will be returned in the tutorials that you attend. So **write down the tutorial that you intend to attend.**
- **Very important:** If several of you work in group, I DO NOT expect to see group members hand in similar answers. Keep in mind you need to write/type your answers independently even if you work in group with others. I will give zero mark if I see two answer sheets that are very similar.

Q1. Hotelling Model with Collusion (10 points)

Two firms are located at the end points of a line of unit length $[0, 1]$. Firms interact repeatedly over an infinite time horizon $t = 0, 1, 2, \dots$ and face a common per-period discount factor $\delta \in (0, 1)$. Firms compete in prices in each period and do not change their location over time. Both firms incur zero marginal costs of production.

Consumers are uniformly distributed over the same interval with their location denoted by $x \in [0, 1]$. In every period, each consumer demands at most one unit of the product and derive utility $U_1 = V - x - p_1$ from buying from firm 1 (located at position 0), and utility $U_2 = V - (1 - x) - p_2$ from buying from firm 2 (located at position 1). x and $1 - x$ represent the linear disutility from having to travel to either firm's location to purchase the product. Consumers do not change their location over time and purchase the product if they get non-negative utility. Furthermore, we assume that $V > 2$, such that firms will always prefer to sell to the entire market.

We will begin by considering a one-shot interaction between firms. First, suppose that firms do not collude (think normal Hotelling price competition) and consider the equilibrium in a single-stage game.

- a. (0.5 pt) Find the indifferent consumer, $\hat{x}$, in terms of p_1 and p_2 .
- b. (0.5 pt) Using your answer from a), write out the profit maximisation problem for both firms 1 and 2.
- c. (1 pt) Hence find the non-collusive equilibrium prices $p_{1,NC}^*$ and $p_{2,NC}^*$ set by either firm in equilibrium. Furthermore, find the equilibrium profits earned by each firm, $\pi_{1,NC}^*$ and $\pi_{2,NC}^*$, and the industry profit Π_{NC}^* .

Now, consider when the firms collude. They may collude in two ways: either by concentrating all their production on one firm (located at position 0) and charging a single price, $p = p_{C,1}^*$, or by continuing to operate both firms such that they both charge a common price $p_1 = p_2 = p_{C,2}^*$ (while remaining at positions 0 and 1). You can imagine that firm 2 agrees to set a sky-high price in the former case so that all consumers will only buy from firm 1. In

both cases, firms will equally share the collusive profit. Assume that regardless of either case, firms are obliged to set their price such that the market is fully covered (all consumer must be willing to buy the product).

- d. (0.5 pt) Firstly, we will assume that firms concentrate all their production on one store. What will be the optimal price in equilibrium? [Hint: Consider the furthest positioned consumer - what price will entice him to purchase the product?]
- e. (1 pt) What will be the resulting industry profit $\Pi_{C,1}^*$? Is this higher or lower than Π_{NC}^* in c)?
- f. (0.5 pt) Now, we will assume that firms choose to continue producing at both stores and charging a common price. In this case, each firm sells to half the market. What will be the optimal price in equilibrium? [Hint: Consider the indifferent consumer's decision - what price will entice him to purchase the product?]
- g. (1 pt) What will be the resulting industry profit $\Pi_{C,2}^*$? Is this higher or lower than Π_{NC}^* in c)?
- h. (0.5 pt) Compare your answers from e) and g). Which of the two arrangements do the firms prefer? Justify your answer.

We will now move on to the infinite period game in which firms collude by choosing the latter collusive arrangement, that is, to continue operating at both stores and equally share $\Pi_{C,2}^*$. Firms will continue to collude until a firm deviates by setting a different p at time t . If deviation occurs, both firms will revert to playing the non-cooperative equilibrium for all subsequent periods.

- i. (0.5 pt) Consider the scenario that firm 1 deviates at time t . Given that $p_2 = p_{C,2}^*$, write out the profit maximisation problem for firm 1.
- j. (1 pt) Hence find the optimal deviation price, p_D^* charged by firm 1 in equilibrium and the associated deviation profit. Is it higher than under collusion?
- k. (0.5 pt) Find the position of the indifferent consumer in this case. How does it change with regards to V ?

We will now check firm 1's incentive to deviate. Remember that firm 1 accounts for both current and future profits.

- l. (0.5 pt) Suppose that firm 1 is deciding whether to deviate at period t .
Write out under what condition firm 1 chooses to deviate at period t .
- m. (1 pt) Find the values of δ such that firm 1 will decide to deviate.
- n. (1 pt) Does an increase in V facilitate collusion? Explain your answer.

Q2. To Enter or to Merge? (10 points)

Mitch (M) and Indra (I) are competing in quantities in the market for retail mobile services. The inverse demand function of the market is given by $P = 1 - Q$, where P represents the price set in the market and Q represents total market demand. M and I face a constant marginal cost of $c_M \in (0, \frac{1}{2})$ and $c_I \in (0, \frac{1}{2})$ respectively. We further assume that M is more efficient in production such that $c_M < c_I$. Neither firm faces any fixed costs.

An entrant, Chris (K), may either enter the market as a separate competitor or merge with I to form a new entity I' . K possesses advanced technology such that he faces a marginal cost of 0 if operating separately, or, if K merges, reduces the marginal cost of I' to 0. If K enters the market independently, he incurs an entry cost of $F > 0$. There are also no additional costs if K is allowed to merge with I .

The current mobile network regulator is unsure as to which option will provide the highest welfare. As such, the regulator has turned to you, a budding Economist, to help it decide whether it should allow K to enter separately or merge to form I' .

We will first begin by investigating the benchmark case such that K has not entered nor merged with I .

- a. (0.5 pt) Write down the profit maximisation problem for each of M and I .
- b. (0.5 pt) Write out the best response functions for each of M and I .
- c. (1 pt) Solve for each firm's production quantity and profits, the equilibrium market price and consumer surplus.

We will now assume that K enters as a separate competitor in the market (alongside M and I).

- d. (0.5 pt) Write down the profit maximisation problem for each of M , I and K .
- e. (0.5 pt) Write out the best response functions for each firm.
- f. (1 pt) Solve for each firm's production quantity and profits. Considering this situation in isolation, when will K decide to enter the market?

- g. (1 pt) Given that K enters the market, find the equilibrium market price and consumer surplus. Do consumers benefit from the entrance of K ?

Now assume that K instead merges with I to form I' . I' and M now compete in the market.

- h. (0.5 pt) Write down the profit maximisation problem for each of M and I' .
- i. (1 pt) Solve for each firm's production quantity and profits, the equilibrium market price and consumer surplus. Do consumers benefit from the merger between K and I ?

The regulator will now decide whether to allow the K to enter independently or to merge with incumbent I .

- j. (1 pt) Suppose that the regulator is concerned with maximising consumer welfare. Compare your answer in g.) to that of j.). When will the merger benefit consumer welfare more than K entering the market separately? [Hint: if comparing consumer surplus proves to be too difficult, consider comparing prices instead]. Explain the meaning/intuition behind your answer (max 3-4 sentences).

For the remainder of the question, we will assume that $c_M = \frac{1}{20}$ and $c_I = \frac{1}{10}$.

- k. (0.5 pt) Using your answer from j) or otherwise, will consumers prefer the merger or for K to enter independently?
- l. (1 pt) Now suppose that the regulator is concerned with maximising total welfare (that is, the sum of consumer welfare and firm profits). For what values of K will the merger benefit total welfare more than K entering the market separately? (you may round your **final** answer to three significant figures).
- m. (1 pt) Compare your answers from k) and l). Are the preferences of consumers and the regulator for the merger/independent entry always the same? If they are (or are not), explain why (max 3-4 sentences).