

Proposition 1 *Suppose that all traders' valuations and costs are distinct, i.e. for all $i, i' \in \{1, \dots, M\}$ with $i' \neq i$ and all $j, j' \in \{1, \dots, N\}$ with $j' \neq j$ we have $v_i \neq c_j$, $v_i \neq v_{i'}$ and $c_j \neq c_{j'}$. Then in any Pareto optimum in which there are n trades, the good must go to the first n highest valuation buyers, and must originate from the first n lowest cost sellers.*

1. Consider an exchange economy, in which there are two goods. The first good is indivisible, i.e. its quantity is described by an integer. Preferences are such that each buyer is willing to purchase at most one unit of the indivisible good, and endowments are such that each seller is able to supply at most one unit of this good. There is also a perfectly divisible medium of exchange between traders called money. Preferences are additively separable in money and the indivisible good. Assume that each buyer's initial money holdings are large enough that (s)he is always able to afford a unit, if (s)he so desires. There are M potential buyers and N potential sellers.

(a) Formally, i.e. mathematically, describe an initial allocation for this economy. Be very precise, and make sure your answer provides a complete mathematical description of the allocation. A verbal answer does not suffice. How did you take into account the affordability restriction?

(b) Formally, i.e. mathematically, describe a feasible allocation for this economy. (HINT: Note that the definition of feasibility takes into account the initial allocation, through imposition of the individual rationality constraint).

(c) Describe the transfers implied by the feasible allocations in part (b), above. Use this to provide an alternative characterization of feasible allocations in terms of transfers.

2. Consider the exchange economy as described in Question 1, above.

- (a) Consider Proposition 1 from the lecture notes on Gains from Trade. Does the Proposition continue to hold if we remove the restriction that there are no ties in the valuations or costs of the traders? Substantiate your answer either with a proof, or with a counterexample.
- (b) Formulate a variant of Proposition 1 that holds regardless of the configuration of buyer valuations and seller costs.

3. continue to hold if you remove the restriction $v_i \neq c$

- (a) Suppose that there are 9 potential buyers, whose respective valuations are 8,7,7,6,5,5,4,3,3. Suppose also that there are 8 potential sellers, whose respective costs are 1,2,3,3,4,6,6,7. Is the Pareto optimal volume of trade uniquely determined? Is the identity of the set of individuals who trade in a Pareto optimum uniquely determined? Is it uniquely determined who should trade with whom in a Pareto optimum? Are the monetary transfers uniquely determined? (If the answer to any of the questions is non-uniqueness, be sure to illustrate the non-uniqueness).
- (b) Suppose instead that the sellers' costs are 1,2,3,3,4,5,6,7, but the buyers' valuations remain as in part (a). Answer the same questions as in part (a) above.