Case Study

THE IRIS CENTER

Mathematics: Identifying and Addressing Student Errors
Level B • Case 2

Background

Student: Elías

Age: 7

Grade: 2nd

Scenario

A special education teacher at Bordeaux Elementary School, Mrs. Gustafson has been providing intensive intervention to Elías, who has a learning disability, and collecting progress monitoring data for the past six weeks. His data indicate that he is not making adequate progress to meet his end-of-year goals. Mrs. Gustafson has decided that she needs to conduct a diagnostic assessment to identify areas of difficulty and to determine specific instructional needs. As part of the diagnostic assessment, Mrs. Gustafson conducts an error analysis using Elías' progress monitoring data.

Possible Activities

- Collecting Data
- Identifying Error Patterns
- Determining Reasons for Errors

Assignment

1. Read the *Introduction*.
2. Read the STAR Sheets for the possible strategies listed above.
3. Score Elías' progress monitoring probe below by marking each incorrect digit.
4. When Mrs. Gustafson scores the probe, she finds two possible explanations. One is that Elías is making a conceptual error, and the other is that he doesn't understand or is not applying the correct procedure.
   a. Assume that his error pattern is procedural. Describe Elías' possible procedural error pattern.
   b. Assume that his error pattern is conceptual. Describe Elías' possible conceptual error pattern.
5. Because the instructional adaptations Mrs. Gustafson will make will depend on Elías' error pattern, she must be sure of the reasons for his errors. Explain at least one strategy Mrs. Gustafson could use to determine Elías' error type.

Answer Key

1) 40	2) 87	3) 45	4) 22	5) 42
6) 34	7) 5	8) 122	9) 5	10) 80

Name: Elias Date: 2/17/xx

For illustrative purposes, only 10 of the 25 problems are shown.

1. $\begin{array}{r} 18 \\ +\ 22 \\ \hline 310 \end{array}$ **2.** $\begin{array}{r} 74 \\ +\ 13 \\ \hline 87 \end{array}$ **3.** $\begin{array}{r} 66 \\ -\ 21 \\ \hline 45 \end{array}$ **4.** $\begin{array}{r} 99 \\ -\ 77 \\ \hline 22 \end{array}$ **5.** $\begin{array}{r} 13 \\ +\ 29 \\ \hline 312 \end{array}$

6. $\begin{array}{r} 96 \\ -\ 62 \\ \hline 34 \end{array}$ **7.** $\begin{array}{r} 57 \\ -\ 52 \\ \hline 5 \end{array}$ **8.** $\begin{array}{r} 83 \\ +\ 39 \\ \hline 1112 \end{array}$ **9.** $\begin{array}{r} 20 \\ -\ 15 \\ \hline 5 \end{array}$ **10.** $\begin{array}{r} 61 \\ +\ 19 \\ \hline 710 \end{array}$

STAR Sheet ★

THE IRIS CENTER

Mathematics: Identifying and Addressing Student Errors
Determining Reasons for Errors

★ What a STAR Sheet is…

A STAR (STrategies And Resources) Sheet provides you with a description of a well-researched strategy that can help you solve the case studies in this unit.

What It Is…

Determining the reason for errors is the process through which teachers determine why the student is making a particular type of error.

What the Research and Resources Say…

- To help them to improve their mathematical performance, teachers must first identify and understand why students make particular errors (Radatz, 1979; Yetkin, 2003).
- Typically, a student's errors are not random; instead, they are often based on incorrect algorithms or procedures applied systematically (Cox, 1975; Ben-Zeev, 1998).
- Knowing what a student is thinking when she is solving a problem can be a rich source of information about what she does and does not understand (Hunt & Little, 2014; Baldwin & Yun, 2012).

Helpful Strategies

Determining exactly why a student is making a particular error is important in that it informs the teacher's instructional response. Though it is sometimes obvious why a student is making a certain type of errors, at other times determining a reason proves more difficult. In these latter instances, the teacher can use one or more of the following strategies.

Interview the student—It is sometimes unclear why a student is making a particular type of error. For example, it can be difficult for a teacher to distinguish between procedural or conceptual errors. For this reason, it can be beneficial to ask a student to talk through his or her process for solving the problem. Teachers can ask general questions such as "How did you come up with that answer?" or prompt the student with statements such as "Show me how you got that answer." Another reason teachers might want to interview the student is to make sure the student has the prerequisite skills to solve the problem.

Observe the student—A student might also reveal information through nonverbal means. This can include gestures, pauses, signs of frustration, and self-talk. The teacher can use information of this type to identify at what point in the problem-solving task that the student experiences difficulty or frustration. It can also help the teacher determine which procedure or set of rules a student is applying and why.

Look for exceptions to an error pattern—In addition to looking for error patterns, a teacher should note instances when the student does not make the same error on the same type of problem. This, too, can be informative because it might indicate that the student has partial or basic understanding of the concept in question. For example, Cammy completed a worksheet on multiplying whole numbers by fractions. She seemed to get most of them wrong; however, she correctly answered the problems in which the fraction was $\frac{1}{2}$. This seems to indicate that, though Cammy conceptually understands what $\frac{1}{2}$ of a whole is, she most likely does not know the process for multiplying whole numbers by fractions.

Considerations for Students with Learning Disabilities

Approximately 5–8% of students exhibit mathematics learning disabilities. Therefore, it is important to understand that their unique learning differences might impact their ability to learn and correctly choose and apply solution strategies to solve mathematics problems. A few characteristics that teachers might notice with students with learning disabilities is that these students often:

- Have difficulty mastering basic number facts
- Make computational errors even though they might have a strong conceptual understanding
- Have difficulty making the connection between concrete objects and semiabstract (visual representations) or abstract knowledge or mathematical symbols
- Struggle with mathematical terminology and written language
- Have visual-spatial deficits, which result in difficulty visualizing mathematical concepts (although this is quite rare)

References

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Mathematics: Identifying and Addressing Student Errors
Collecting Data

★ What a STAR Sheet is…

A STAR (STrategies And Resources) Sheet provides you with a description of a well-researched strategy that can help you solve the case studies in this unit.

What It Is…

Collecting data involves asking a student to complete a worksheet, test, or progress monitoring measure containing a number of problems of the same type.

What the Research and Resources Say…

- Error analysis data can be collected using formal (e.g., chapter test, standardized test) or informal (e.g., homework, in-class worksheet) measures (Riccomini, 2014).
- Error analysis is one form of diagnostic assessment. The data collected can help teachers understand *why* students are struggling to make progress on certain tasks and align instruction with the student's specific needs (National Center on Intensive Intervention, n.d.; Kingsdorf & Krawec, 2014).
- To help determine an error pattern, the data collection measure must contain at a minimum three to five problems of the same type (Special Connections, n.d.).

Identifying Data Sources

To conduct an error analysis for mathematics, the teacher must first collect data. She can do so by using a number of materials completed by the student (i.e., student product). These include worksheets, progress monitoring measures, assignments, quizzes, and chapter tests. Homework can also be used, assuming the teacher is confident that the student completed the assignment independently. Regardless of the type of student product used, it should contain at a minimum **three to five** problems of the same type. This allows a sufficient number of items with which to determine error patterns.

Scoring

To better understand why students are struggling, the teacher should mark *each incorrect digit* in a student's answer, as opposed to simply marking the entire answer incorrect. Evaluating each digit in the answer allows the teacher to more quickly and clearly identify the student's error and to determine whether the student is consistently making this error across a number of problems. For example, take a moment to examine the worksheet below. By marking the incorrect digits, the teacher can determine that, although the student seems to understand basic math facts, he is not regrouping the "1" to the ten's column in his addition problems.

Note: Marking each incorrect digit might not always reveal the error pattern. Review the STAR Sheets *Identifying Error Patterns, Word Problems: Additional Error Patterns, and Determining Reasons for Errors* to learn about identifying the different types of errors students make.

Mathematics: Identifying and Addressing Student Errors
Identifying Error Patterns

★ What a STAR Sheet is…

A STAR (STrategies And Resources) Sheet provides you with a description of a well-researched strategy that can help you solve the case studies in this unit.

What It Is…

Identifying error patterns refers to determining the type(s) of errors made by a student when he or she is solving mathematical problems.

What the Research and Resources Say…

- Three to five errors on a particular type of problem constitute an error pattern (Howell, Fox, & Morehead, 1993; Radatz, 1979).
- Typically, student mathematical errors fall into three broad categories: factual, procedural, and conceptual. Each of these errors is related either to a student's lack of knowledge or a misunderstanding (Fisher & Frey, 2012; Riccomini, 2014).
- Not every error is the result of a lack of knowledge or skill. Sometimes, a student will make a mistake simply because he was fatigued or distracted (i.e., careless errors) (Fisher & Frey, 2012).
- Procedural errors are the most common type of error (Riccomini, 2014).
- Because conceptual and procedural knowledge often overlap, it is difficult to distinguish conceptual errors from procedural errors (Rittle-Johnson, Siegler, & Alibali, 2001; Riccomini, 2014).

Types of Errors

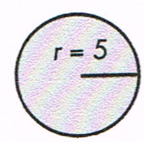
1. **Factual errors** are errors due to a lack of factual information (e.g., vocabulary, digit identification, place value identification).
2. **Procedural errors** are errors due to the incorrect performance of steps in a mathematical process (e.g., regrouping, decimal placement).
3. **Conceptual errors** are errors due to misconceptions or a faulty understanding of the underlying principles and ideas connected to the mathematical problem (e.g., relationship among numbers, characteristics, and properties of shapes).

FYI

Another type of error that a student might make is a careless error. The student fails to correctly solve a given mathematical problem despite having the necessary skills or knowledge. This might happen because the student is tired or distracted by activity elsewhere in the classroom. Although teachers can note the occurrence of such errors, doing so will do nothing to identify a student's skill deficits. For many students, simply pointing out the error is all that is needed to correct it. However, it is important to note that students with learning disabilities often make careless errors.

Common Factual Errors

Factual errors occur when students lack factual information (e.g., vocabulary, digit identification, place value identification). Review the table below to learn about some of the common factual errors committed by students.

Factual Error	Examples
Has not mastered basic number facts: The student does not know basic mathematics facts and makes errors when adding, subtracting, multiplying, or dividing single-digit numbers.	$3 + 2 = 7$ $7 - 4 = 2$ $2 \times 3 = 7$ $8 \div 4 = 3$
Misidentifies signs	$2 \times 3 = 5$ (The student identifies the multiplication sign as an addition sign.) $8 \div 4 = 4$ (The student identifies the division sign as a minus sign.)
Misidentifies digits	The student identifies a 5 as a 2.
Makes counting errors	1, 2, 3, 4, 5, 7, 8, 9 (The student skips 6.)
Does not know mathematical terms (vocabulary)	The student does not understand the meaning of terms such as *numerator, denominator, greatest common factor, least common multiple,* or *circumference.*
Does not know mathematical formulas	The student does not know the formula for calculating the area of a circle. 

Common Procedural Errors

Procedural knowledge is an understanding of *what* steps or procedures are required to solve a problem. *Procedural errors* occur when a student incorrectly applies a rule or an algorithm (i.e., the formula or step-by-step procedure for solving a problem). Review the table below to learn more about some common procedural errors.

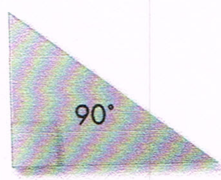
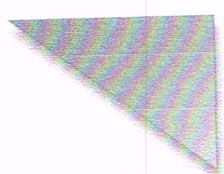
Procedural Error	Examples	
Regrouping Errors		
Forgetting to regroup: The student forgets to regroup (carry) when adding, multiplying, or subtracting.	$\begin{array}{r} 77 \\ +\ 54 \\ \hline 121 \end{array}$	The student added 7 + 4 correctly but didn't regroup one group of 10 to the tens column.
	$\begin{array}{r} 123 \\ -\ 76 \\ \hline 53 \end{array}$	The student does not regroup one group of 10 from the tens column, but instead subtracted the number that is less (3) from the greater number (6) in the ones column.
	$\begin{array}{r} 56 \\ \times\ 2 \\ \hline 102 \end{array}$	After multiplying 2 × 6, the student fails to regroup one group of 10 from the tens column.
Regrouping across a zero: When a problem contains one or more 0's in the minuend (top number), the student is unsure of what to do.	$\begin{array}{r} 304 \\ -\ 21 \\ \hline 323 \end{array}$	The student subtracted the 0 from the 2 instead of regrouping.
Performing incorrect operation: Although able to correctly identify the signs (e.g., addition, minus), students often subtract when they are suppose to add, or vice versa. However, students might also perform other incorrect operations, such as multiplying instead of adding.	$\begin{array}{r} 234 \\ -\ 45 \\ \hline 279 \end{array}$	The student added instead of subtracting.
	$\begin{array}{r} 3 \\ +\ 2 \\ \hline 6 \end{array}$	The student multiplied instead of adding.
Fraction Errors		
Failure to find common denominator when adding and subtracting fractions	$\frac{3}{4} + \frac{1}{3} = \frac{4}{7}$	The student added the numerators and then the denominators without finding the common denominator.
Failure to invert and then multiply when dividing fractions	$\frac{1}{2} \div 2 = \frac{1}{2} \times \frac{2}{1} = \frac{2}{2} = 1$	The student did not invert the 2 to $\frac{1}{2}$ before multiplying to get the correct answer of $\frac{1}{4}$.
Failure to change the denominator in multiplying fractions	$\frac{2}{8} \times \frac{5}{8} = \frac{10}{8}$	The student did not multiply the denominators to get the correct answer.
Incorrectly converting a mixed number to an improper fraction	$1\frac{1}{2} = \frac{4}{2}$	To find the numerator, the student added 2 + 1 + 1 to get 4, instead of following the correct procedure (2 × 1 + 1 = 3).

Procedural Error cont.	Examples cont.	
Decimal Errors		
Not aligning decimal points when adding or subtracting: The student aligns the numbers without regard to where the decimal is located.	120.4 + 63.21 75.25	The student did not align the decimal points to show digits in like places. In this case, .4 and .2 are in the tenths place and should be aligned.
Not placing decimal in appropriate place when multiplying or dividing: The student does not count and add the number of decimal places in each factor to determine the number of decimal places in the product. *Note: This could also be a conceptual error related to place value.*	3.4 × .2 6.8	As with adding or subtracting, the student aligns the decimal point in the product with the decimal points in the factors. The student did not count and add the number of decimal places in each factor to determine the number of decimal places in the product.

Common Conceptual Errors

Conceptual knowledge is an understanding of underlying ideas and principles and a recognition of when to apply them. It also involves understanding the relationships among ideas and principles. *Conceptual errors* occur when a student holds misconceptions or lacks understanding of the underlying principles and ideas related to a given mathematical problem (e.g., the relationship between numbers, the characteristics and properties of shapes). Examine the table below to learn more about some common conceptual errors.

Conceptual Error	Examples	
Misunderstanding of place value: The student doesn't understand place value and records the answer so that the numbers are not in the appropriate place value position.	67 + 4 17	The student added all the numbers together (6 + 7 + 4 = 17), not understanding the values of the ones and tens columns.
	10 + 9 91	The student recorded the answer with the numbers reversed, disregarding the appropriate place value position of the numbers or digits.
	Write the following as a number: a) seventy-six b) nine hundred seventy-four c) six thousand, six hundred twenty-four Student answer: a) 76 b) 90074 c) 600060024	When expressing a number beyond two digits, the student does not have a conceptual understanding of the place value position.

Conceptual Error cont.	Examples cont.	
Overgeneralization: Because of lack of conceptual understanding, the student incorrectly applies rules or knowledge to novel situations.	$\begin{array}{r} 321 \\ -\ 245 \\ \hline 124 \end{array}$	Regardless of whether the greater number is in the minuend (top number) or subtrahend (bottom number), the student always subtracts the number that is less from the greater number, as is done with single-digit subtraction.
	Put the following fractions in order from smallest to largest. $\frac{77}{486}$ $\frac{1}{351}$ $\frac{12}{200}$	The student puts fractions in the order $\frac{12}{200}$, $\frac{1}{351}$, $\frac{77}{486}$, because he doesn't understand the relation between the numerator and its denominator; that is, larger denominators mean smaller fractional parts.
Overspecialization: Because of lack of conceptual understanding, the student develops an overly narrow definition of a given concept or of when to apply a rule or algorithm.	Which of the triangles below are right triangles? a)  b)  c) both Student answer: a	The student chooses *a* because she only associates a right triangle with those with the same orientation as *a*.