

EXERCISE 1

An experimental team identified the following factors to be important with regard to the shrinkage of parts from an injection molding process:

- A. Mold temperature
- B. Pressure
- C. Cycle Time
- D. Screw Speed

Find a two-level design which will accommodate these factors and which will allow the following interactions to be investigated:

AxB

BxD

EXERCISE 2

Find a two-level design which will accommodate

Factors A, B, C, D, E, F, G, H and I, and
Interactions AxB, AxC, BxC, BxD

EXERCISE 3

Find a two-level design which will accommodate

Factors A, B, C, D, E, F, G, H and I, and
Interactions AxB, AxC, Ax E, Ax F, BxD

THREE-LEVEL DESIGNS

If we wish to vary all the factors in our experiment over three levels, we can use one of the three-level designs found in Appendix A.

EXAMPLE 2

Consider again the four factors in Example 1:

- A. Raw Materials
- B. Annealing Methods
- C. Temperature
- D. Treating Times

Suppose the experimenters wanted to test these factors over three levels (instead of two) and no interactions were expected. Then an L_9 orthogonal array (shown in Table 3) could be used.

TABLE 3
 L_9 ORTHOGONAL ARRAY

No	Column				
		1	2	3	4
1		1	1	1	1
2		1	2	2	2
3		1	3	3	3
4		2	1	2	3
5		2	2	3	1
6		2	3	1	2
7		3	1	3	2
8		3	2	1	3
9		3	3	2	1
Groups		1		2	

The L_9 array may also be used for an experiment involving two three-level factors and their interaction. Figure 3 contains the only linear graph for the L_9 array. Notice that the line segment is labeled with two columns: 3 and 4. This tells us that we need to use columns 3 and 4 for estimating the interaction between the two factors, say A and B, which were put in columns 1 and 2 (see Table 4).

Why are two columns needed to estimate the $A \times B$ interaction? It has to do with the required degrees of freedom. The degrees of freedom for a factor is one less than the number of levels. The degrees of freedom (df) for $A \times B$ is

$$\text{df for A} \times \text{df for B}.$$

Since A and B are three-level factors, they each have two degrees of freedom. So,

$$\text{df for A} \times \text{B} = 2 \times 2 = 4.$$

Thus, 4 degrees of freedom are needed to estimate $A \times B$. Each column of the L_9 array has 2 degrees of freedom associated with it (since they are three-level columns). Two columns gives 4 df, which is the required amount for the specified interaction.

FIGURE 3
LINEAR GRAPH FOR L_9 ARRAY

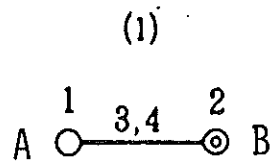


TABLE 4
 L_9 ARRAY FOR TWO FACTORS AND THEIR INTERACTION

No.	Column				
		A	R	A $\times$ R	
1		1	1	1	1
2		1	2	2	2
3		1	3	3	3
4		2	1	2	3
5		2	2	3	1
6		2	3	1	2
7		3	1	3	2
8		3	2	1	3
9		3	3	2	1
Groups		1		2	

EXERCISE 4

Find a three-level design which will accommodate the following factors and interactions:

A, B, C, D, E, F,

AxB, AxC, BxC.

EXERCISE 5

Find a three-level design which will accommodate the following factors and interactions:

A, B, C, D, E, F, G

BxC, BxD, BxE.