

1. (15 points) There are four cards, labeled 1 to 4 in a drawer. Cards are drawn at random one by one, without replacement, until the sum of the numbers on the cards drawn exceeds 3. The sequence of the cards drawn is recorded.
  - (a) (5 points) Please write down the sample space for this experiment.
  - (b) (5 points) Let  $A$  be the event "one of the cards drawn is 1", and  $B$  be the event "the final sum of numbers on the cards drawn is odd". Please give the set of outcomes corresponding to the event "both  $A$  and  $B$  occur".
  - (c) (5 points) Keeping the same definition for Event  $A$  and  $B$ , please give the set of outcomes corresponding to the event "neither  $A$  nor  $B$  occurs".
2. (8 points) Thomas performs 5 independent experiments, each of which results in any of the numbers 0, 1, or 2, with respective probabilities of 0.3, 0.5, and 0.2. Find the probability that both number 1 and number 2 occur at least once. (Hint: Please consider the complementary probability.)
3. (10 points) John has 12 colored balls, including 6 red, 4 blue, 1 green, and 1 yellow. Note that for the balls of the same color, they don't have any differences.
  - (a) (5 points) If John puts all the balls in a row, how many possible arrangements are there?
  - (b) (5 points) If one of the arrangements in part (a) is randomly selected, what is the probability that no two red balls are next to each other?
4. (10 points) A box contains four coins, two of which are fair, one double-headed (i.e., heads on both sides), and the third is biased in such a way that it comes up heads with probability  $1/4$ . A coin is drawn at random from the box and flipped twice. If both flips result in heads, what is the probability that the coin drawn was double-headed?
5. (15 points) There are 24 bottles of hand sanitizers on the shelf. Four bottles are sampled at random and without replacement. Suppose that only six of the bottles meet the quality requirements.
  - (a) (5 points) What is the probability that exactly one bottle from the sample meets the quality requirements?
  - (b) (5 points) What is the probability that *at least one* bottle in the sample meets the quality requirements?
  - (c) (5 points) In addition to the six bottles that meet the quality requirements, four bottles that do not meet the quality requirements also contain fragrance. What is the probability that exactly one bottle in the sample meets the quality requirements and exactly one bottle in the sample contains fragrance?

6. (8 points) Two balls, each equally likely to be colored either red or blue (we don't know their colors for sure), are put in an urn. At each stage one of the balls is randomly chosen, its color is noted, and it is then returned to the urn. If the first two balls chosen are colored red, what is the probability that
- (a) (4 points) both balls in the urn are colored red
  - (b) (4 points) the next ball chosen will be red?

Hint: the probability of the first two balls chosen are red given the total number of red balls (0, 1, and 2 red balls) in the urn is different.

7. (20 points) Three balls are selected from a bag that contains 3 blue, 2 green, and 5 yellow balls. Here, we define random variables as  $B$  = number of blue balls drawn,  $G$  = number of green balls drawn,  $Y$  = number of yellow balls drawn. Please list the following values:
- (b) (4 points) The joint probability mass function  $p_{G,B}(g, b)$ .
  - (c) (3 points) The marginal probability mass function  $p_G(g)$  and  $p_B(b)$ .
  - (d) (4 points) The joint distribution function  $F_{G,B}(g, b)$ .
  - (e) (3 points) The marginal distribution function  $F_G(g)$  and  $F_B(b)$ .
  - (f) (6 points) The conditional probability mass functions  $p_{G|B}$ . Are  $B$  and  $G$  independent?

Hint: express the results of (a) and (b) in one table and the same for (c) and (d).

8. (8 points) If the density function of  $X$  equals

$$f(x) = \begin{cases} ce^{-2x} & 0 \leq x < \infty \\ 0 & x < 0 \end{cases}$$

- (a) (4 points) For what value of  $c$  is this a density function?
- (b) (4 points) What is  $P\{X > 1\}$ ?

9. (10 points) A lot containing 7 components is sampled by a quality inspector; the lot contains 4 good components and 3 defective components. A sample of 3 is taken by the inspector. Find the expected value of the number of good components in this sample.