

Problem 1 (20 pts) Find the component form of the cross product $\mathbf{v} \times \mathbf{w}$, based on the following information:

- (i) Both vectors lie in the tangent plane of the function $f(x, y) = \sqrt{x^2 + y^2}$ at the center $(a, b) = (3, 4)$.
- (ii) The area of the parallelogram spanned by $\mathbf{v}$ and $\mathbf{w}$ is 10 (units²).
- (iii) The direction of the cross product $\mathbf{v} \times \mathbf{w}$ relative to the graph of $f(x, y) = \sqrt{x^2 + y^2}$ is pointing in towards the z-axis.

To receive full credit for this problem, you need to:

- Sketch the graph of the function $f(x, y) = \sqrt{x^2 + y^2}$, the tangent plane at the point $(3, 4, f(3, 4))$, and a vector representing the direction of $\mathbf{v} \times \mathbf{w}$ (don't worry about the magnitude). Label the three coordinate axes.
- Explain and/or show work supporting how you determined the direction and magnitude of the cross product $\mathbf{v} \times \mathbf{w}$.
- give the correct component form of the cross product $\mathbf{v} \times \mathbf{w}$.

Problem 2 (15 pts) E-Roc the Super-Pigeon from Planet X is heading to a weekend retreat with their frienemy, Morg the Merciless Space-Bat. The directions to get to Morg's cabin in the Andromeda galaxy are as follows:

- (i) Fly along the curve $\mathbf{r}(t) = \langle t, t^2 - t - 2 \rangle$, beginning at the point P_0 (E-Roc's location on the curve when $t = 0$).
- (ii) The cabin is located at the point P on the curve where E-Roc's acceleration and velocity are orthogonal.

Find the coordinates of the point P where Morg's cabin is located, and find the distance along the curve traveled by E-Roc between starting point P_0 and the final destination P .

To receive full credit for this problem, you need to:

- Plot the starting and ending positions of E-Roc on the curve: label the starting point P_0 , and the end point P on the graph of the curve $\mathbf{r}(t)$ (graph is provided). Explain and/or show work supporting how you determined the coordinates of these two points.
- Sketch and label the velocity vector $\mathbf{v}$ and acceleration vector $\mathbf{a}$ at the point P on the graph of the curve $\mathbf{r}(t)$. These vectors need to have both correct magnitude and direction.
- Set up the integral that gives the distance traveled by E-Roc between the points P_0 and P . Evaluate this integral, rounding to 2 decimal places. You may use a calculator or app to evaluate this integral.