

MAD 2104  
Exam II A  
Spring 2020

Name:  
PID:

#1 Use mathematical induction to prove  
(10 pts)

$$\frac{1}{(1)(4)} + \frac{1}{(4)(7)} + \dots + \frac{1}{(3n-2)(3n+1)} = \frac{n}{3n+1}$$

for  $n=1, 2, 3, \dots$

#2 Prove that  $7^{(n+2)} + 8^{(2n+1)}$  is divisible by 57  
(10 pts) for  $n=0, 1, 2, \dots$

#3 Define a sequence  $f_0 = 1 = f_1$ ,  $f_n = f_{n-1} + f_{n-2}$   
(10 pts) Prove  $f_n < \left(\frac{5}{3}\right)^n$  for  $n = 1, 2, 3, \dots$

#4 Use a factorial argument to prove  
(6 pts)  $C(2n, n+1) + C(2n, n) = \frac{1}{2} C(2n+2, n+1)$ .

#5 Use the Binomial theorem

(9 pts)  $(x+y)^n = \sum_{k=0}^n C(n,k) x^{n-k} \cdot y^k$

to find the coefficient of  $x^8$  in  $(2x^2-3)^{12}$ .

#6 Our panther ID is made up of 7 digits, (12 pts) with the first digit non zero,

a) How many Panther ID's are possible?

b) How many have exactly 3 zeros?

c) How many with no repeating digits?

d) How many with at least one repeating digit?

#7 Suppose a club has 9 men and 8 women.  
(9 pts)

a) How many groups of 6 have at most 3 women?

b) How many groups of 6 have at least 2 men?

c) Suppose 2 members of the club refuse to work together. How many groups of 6 can be chosen with this condition?

#8 How many ways can 16 people be seated...  
(12 pts)

a) In a row if 5 of the 16 do not want to sit next to one another?

b) In a row if 4 of the 16 must be seated next to one another?

c) In a circle if 4 of the 16 must be seated next to one another?

#9 Give a truth table for  
(6 pts)  $(\neg q \rightarrow (p \wedge r)) \oplus (r \rightarrow p)$

#10 Translate the following into symbolic  
(10 pts) language and use truth tables to determine  
if the following argument is valid.

If it does not rain or if it is not foggy,  
then the sailing race will not be held.

The sailing race will not be held or  
it is not foggy. Therefore it does  
not rain.

#11 Put negations next to the propositional  
(6 pts) functions.

a) write the negation (in words)

"Some students graduating from FIU  
go on to graduate school and don't  
take a full time job.

b) Symbolically, write the negation of

$$\exists z \forall x (\forall y \neg P(x, y, z) \vee \exists y Q(x, y, z)).$$