1. (a) A cylindrical, homogeneous, isotropic rod has flat ends at $x = 0$ and $x = L$ and insulated sides. Initially the temperature distribution in the rod is a function of x only, and heat generation at points x in the rod takes place uniformly over the cross section at x. Apply an energy balance to a segment of the rod from a fixed point $x = a$ to an arbitrary value of x to show that the PDE governing temperature $U(x, t)$ in the rod is

$$\frac{\partial U}{\partial t} = k\frac{\partial^2 U}{\partial x^2} + \frac{k}{\kappa}g(x, t),$$

where $g(x, t)$ is the amount of heat per unit volume per unit time generated at position x and time t.

(b) What form do Robin boundary conditions take at $x = 0$ and $x = L$?

In Exercises 2–18, set up, but do not solve, an initial boundary value problem for the required temperature. Assume that the medium is isotropic and homogeneous.

2. A cylindrical rod has flat ends at $x = 0$ and $x = L$ and insulated sides. At time $t = 0$ its temperature is a function $f(x)$, $0 \leq x \leq L$, of x only. For $t > 0$, both ends are kept at 100°C.

* Answers

1. (b) $-L_1 U_x(0,t) + h_1 U(0,t) = f_1(t)$; $t > 0$

$L_2 U_x(L,t) + h_2 U(L,t) = f_2(t)$; $t > 0$

2. $$\frac{\partial U}{\partial t} = K \frac{\partial^2 U}{\partial x^2} \qquad 0 < x < L \; ; \; t > 0$$

$U(0,t) = 100$; $t > 0$

$U(L,t) = 100$; $t > 0$

$U(x,0) = f(x)$; $0 < x < L$