DC BRIDGES

1. Objectives

1) To use a balanced DC bridge and to investigate its sensitivity;
2) To calibrate an unbalanced bridge and to use it;
3) To become familiar with the effects of errors and tolerances.

2. Theory

2.1 The balanced DC bridge (Wheatstone bridge)

Historically, the balanced DC bridge (commonly referred to as the Wheatstone bridge) was widely used for accurate resistance measurements. Nowadays, with high impedance meters available, it is seldom used in electronics. However, it provides a good simple example of an electric measurement circuit suitable for demonstrating important principles and techniques.

The bridge circuit is shown in Fig. 1. It consists of two pairs of resistors, R_1-R_V and R_2-X, which form two sides, a meter M, which bridges between the sides at nodes 1 and 2, and a voltage source V. Here, R_1 and R_2 are called **ratio arm** resistors, R_V is a variable resistor and X is an unknown resistor. To balance the bridge, the variable resistor R_V is adjusted, while the ratio arm resistors R_1 and R_2 are kept constant. The bridge is said to be balanced when there is no voltage difference across the bridge between the nodes 1 and 2.

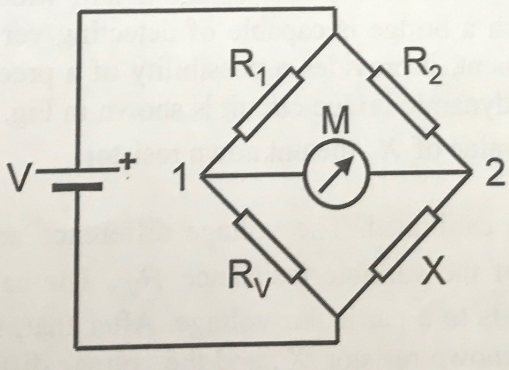

Fig. 1. The circuit of the balanced DC bridge.

If M is a high-impedance voltmeter with the internal resistance much higher than those of the other resistors in the bridge, the voltages at the bridge nodes 1 and 2 are

$$V_1 = \frac{R_V}{R_1 + R_V} V \quad \text{and} \quad V_2 = \frac{X}{R_2 + X} V \tag{1}$$

In this case, the voltage difference across the bridge between the nodes 1 and 2 is

$$V_{12} = V_2 - V_1 = \left(\frac{X}{R_2 + X} - \frac{R_V}{R_1 + R_V} \right) V \tag{2}$$

When the bridge is balanced, $V_{12} = 0$ and therefore

$$\frac{X}{R_2 + X} = \frac{R_V}{R_1 + R_V} \quad \text{hence} \quad \frac{X}{R_V} = \frac{R_2}{R_1} \tag{3}$$

If the ratio arm resistors are equal to each other, $R_1 = R_2$, then $X = R_V$.

2.2 *Bridge sensitivity*

Sensitivity is an important characteristic of any measuring instrument. It shows how strongly a small change in the quantity being measured changes the reading of the instrument. For a DC bridge, the sensitivity s shows how big the change in ΔV_{12} (the voltmeter reading) is due to a small change ΔR_V in the variable resistor or ΔX in the unknown resistor. It can be expressed as

$$s = \frac{\Delta V_{12}}{\Delta R_V} \quad \text{V}\,\Omega^{-1} \text{ (volts per ohm)} \tag{4}$$

Note that for **small** changes in any of the bridge resistances, when all other resistors are kept constant, the voltage V_{12} is a **linear** function of the resistance.

General rules for the bridge sensitivity are as follows:

1) The sensitivity of a bridge depends on the resistances in the ratio arms;
2) For fixed values of the ratio arms, a bridge is most sensitive at balance;
3) A bridge is most sensitive when **all** the resistances are equal.

2.3 *The dynamic DC bridge*

The principle of the unbalanced or dynamic bridge is still widely used for interfacing electronic sensors, as such a bridge is capable of detecting very small changes in the resistance. In our experiment, it provides a possibility of a precise measurement of an unknown resistance. The dynamic bridge circuit is shown in Fig. 2. The value of R_{ref} is to be chosen close to the value of X, the unknown resistor.

First of all, the bridge is calibrated. The voltage difference across the bridge V_{12} is measured as a function of the variable resistance R_V. The calibration graph tells us what resistance corresponds to a particular voltage. After that, the **variable resistance** R_V is replaced by the unknown resistor X, and the voltage difference V_{12} is measured again. The value of X is then obtained from the calibration graph.

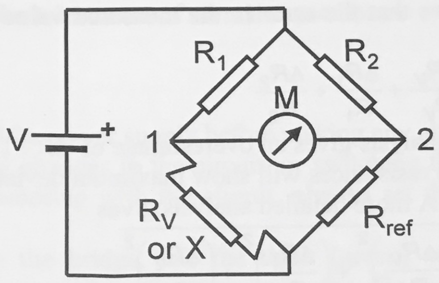

Fig. 2. The circuit of the dynamic DC bridge.

2.4 Effects of errors and tolerances.

No experimental value can be measured absolutely exactly: any measurement unavoidably results in an **experimental error**. (The only possible exception is quantities measured by integer numbers!) In general, for each measured quantity it is necessary to provide not only the value but also the error. This is typically given in the form

$$X = X_0 \pm \Delta X \qquad (5)$$

This means that the value of the quantity X (which was measured as X_0) really may be within the range from $X_0 - \Delta X$ to $X_0 + \Delta X$.

In the experiment with the balanced bridge, there are two primary sources of experimental errors. The first one is called **reading errors** and arises from how precisely the balance point can be found. The reading error can be determined in one of two ways dependent on whether the variable resistor R_V or the voltmeter M is more precise:

1) If the variable resistor is more precise, it is impossible to spot any movement of the voltmeter away from balance for a small change in R_V. In this case, the error ΔR_V is approximately a change is R_V which is required for a discernible movement of the voltmeter.
2) If the voltmeter is more precise, it is impossible to obtain an exact zero on the voltmeter. In this case, the error ΔR_V is approximately half of the smallest increment of R_V.

In both cases, the value of the variable resistor at the balance is determined as $R_V \pm \Delta R_V$.

Another source of experimental error is in **tolerances** of the resistances in the circuit. The tolerance shows how strongly the real resistance of a ratio arm resistor $R_1 \pm \Delta R_1$ may deviate from the specified value R_1. Tolerances are typically specified as relative values in percents, i.e., $\Delta R_1 / R_1 \times 100\%$.

From the Eq. (3),

$$X = R_V \frac{R_2}{R_1} \qquad (6)$$

A simple analysis shows that the error in the measured value of the unknown resistor $X \pm \Delta X$ is

$$\frac{\Delta X}{X} = \frac{\Delta R_V}{R_V} + \frac{\Delta R_1}{R_1} + \frac{\Delta R_2}{R_2} \tag{7}$$

However, such simple analysis gives an overestimate of ΔX. This happens because it is very improbable that **all** resistances will show maximum deviations from their specified values **simultaneously**. A more detailed analysis gives

$$\frac{\Delta X}{X} = \sqrt{\left(\frac{\Delta R_V}{R_V}\right)^2 + \left(\frac{\Delta R_1}{R_1}\right)^2 + \left(\frac{\Delta R_2}{R_2}\right)^2} \tag{8}$$

This value is smaller than that given by Eq. (7). Note that if the relative tolerance in one of the terms is just 2-3 times higher than that in other terms, this term becomes the main source of the experimental error. You are likely to find that the tolerances in the ratio arm resistors are the main source of the experimental error for the measurements with the balanced bridge.

A great advantage of the unbalanced bridge is that the tolerances in the ratio arm and reference resistors do not contribute to the experimental error. This occurs because their exact values, though unknown, are fixed. Therefore, their tolerances affect the voltage difference across the bridge in the same way when measuring both the calibration curve and the unknown resistor. The calibration curve effectively circumvents the effect of the tolerances, as well as that of most stray resistances in the connecting wires. In this case, the experimental error can be reduced to the reading error of the voltmeter.

More details on the experimental errors can be found in Ref. [1].

3. Equipment list.

1) Four-decade resistance boxes with × 10 kΩ top scale - 2
2) Four-decade resistance boxes with × 1 kΩ top scale - 1
3) Six- or seven-decade precision resistance box (range at least 0.1 Ω to 10 kΩ) - 1
4) Centre-zero wide-range voltmeter (range at least 1 mV to 10 V) - 1
5) Power supply unit with controlled voltage output (range at least to 15 V) - 1
6) Unknown resistor (in a small box) - 1

4. Procedures

4.1 *Equipment checks and precautions*

1) Check that you have all the equipment and that it satisfies the specifications in the list.
2) Check all mains plugs, leads and cable entry points, switches and knobs. If any malfunction, report to a technician, a demonstrator or a staff member.
3) Check equipment for hidden controls at the rear or underneath and make sure that their settings are reasonable. If in doubt, seek advice.
4) Short-circuit the input of the voltmeter and adjust the zero point. Note that this procedure may need to be repeated after the voltmeter has warmed up. Check the zero setting for all ranges used.

- Make sure to **record the tolerances** of all resistance boxes.

Precautions

1) **Always** switch off the power supply before making any alterations in the circuit.
2) Before making any changes in the circuit, or switching the power supply on or off, or when not working with the circuit, **always** set the voltmeter on a high scale.
3) For wiring within the bridge, use the thick (green) wire. Connecting wires should be kept short to reduce the stray resistance. Do not use banana plugs and sockets, as their contact resistances may be relatively high. Connect bare ends of wires into the screw terminals. Please, don't remove banana plugs from existing connecting leads!
4) Check the power-supply voltage frequently in case it drifts.

4.2 Preliminary estimation

Make a rough estimate of the unknown resistor. For this purpose, borrow an avometer and measure the resistance to the nearest single significant number. (For example, this may be 3 or 4 kΩ but not 3.4 or 3.7 kΩ.) We will call this the approximate resistor value and denote as R_{appr}.

4.3 Balanced bridge

Assemble the balanced bridge, Fig. 1. Use the four-decade resistance boxes with $\times 10$ kΩ top scale as ratio arms R_1 and R_2 and the precision resistance box as the variable resistor.

Set the ratio arm resistors and the variable resistor to R_{appr}.

Turn on the power supply and set any convenient voltage in the range from 8 to 15 V.

Balance the bridge by adjusting the variable resistor. Start balancing on a **high scale** of the voltmeter and then go successively to **more sensitive scales**. Record the **resistor value** at balance and the **reading errors** for each scale. (Make sure that you clearly understand what the reading error means for this particular measurement!)

Choose yourself at which scale to stop.

4.4 Bridge sensitivity

To determine the sensitivity of the bridge, you should change the resistance of the variable resistor and take the readings of the voltmeter. If you plot the voltage across the bridge V_{12} as a function of R_V, the slope gives the sensitivity s.

1) We assume that after the previous measurement, the ratio arms have been set to R_{appr} and the bridge was balanced.
Make **preliminary measurements** in order to choose the range of R_V to use and an appropriate scale of the voltmeter. Remember that the overall change in R_V **should be small**. After that, take about 10-12 measurements, changing the resistance through regular intervals. Note that you should have a reasonable number of points on both side of the balance. It is advisable to record the readings in a table and to make a plot **immediately** after that, in order to spot all possible irregularities.

2) Set the ratio arms to $4 \times R_{appr}$. (Remember about the precautions!) Balance the bridge. Make a note of the value of R_V at the balance and compare with that found in the section 4.3. Repeat the sensitivity measurements.

3) Repeat the same for the ratio arms set to $\frac{1}{4} \times R_{appr}$.

4.5 Dynamic bridge

First of all, you should obtain the calibration curve. Return the ratio arm resistors and the variable resistor to R_{appr}. Remove the **unknown resistor** and replace it with the **reference resistor** R_{ref}, Fig. 2. For this, use a four-decade resistance box with $\times 1$ kΩ top scale. Set it close to some average of the resistance of the unknown resistor obtained from previous measurements. Proceed as for sensitivity measurements, i.e., change the resistance of the variable resistor R_V through regular intervals and take the readings of the voltmeter. The resulting plot of the voltage V_{12} as a function of R_V will give you the calibration graph. **Use your own judgement** to estimate the **range** over which you are going to change R_V, and the **scale of the voltmeter.**

Now remove the **variable resistor** R_V and replace it with the **unknown resistor**. Take the reading of the voltmeter. Obtain the resistance of the unknown resistor X from the calibration graph. Note that to obtain the value of X, the **reading of the voltmeter should be within the range of calibration**, ideally in its central region. Be prepared not to achieve this from the first attempt. In this case, you need to repeat the calibration. You may also consider changing the value of the reference resistor before repeating measurements.

Observe all precautions when changing the circuit!

5. Tips for analysis and writing the report

- Start with re-reading "The Formal Report" section in the Laboratory Handbook.
- In the experiment with the balanced bridge, you should have measured the unknown resistor three times (with different resistances of ratio arm resistors). Each time,

determine **both the resistance value and the experimental error**. Compare obtained results and comment if they are in agreement or not.

- In your report, each result must be **ultimately presented in the form of Eq. (5)**, i.e., as $X \pm \Delta X$. To determine the experimental error ΔX, use Eq. (8). Comment on the main contribution to ΔX.

- For at least one of the measurements, **compare** the experimental errors which result from Eqs. (8) and (7). **Comment on the difference**.

- **Determine the sensitivity** of the balanced bridge for each of the values of the ratio arm resistors. Try to estimate the experimental error.

- **Provide graphs** of sensitivity plots and the calibration graph in your report, but **not** the tables with the raw data.

- **Compare the sensitivity** of the balanced bridge with different ratio arms.

- **Determine the unknown resistor** from the calibration graph for the unbalanced bridge.

- **Find the experimental error** and **compare** it with that in the measurements with the balanced bridge. (You may wish to think in advance whether you expect a larger or a smaller experimental error in this case.)

- **Comment in detail** on the difference between the experimental errors for measurements using the balanced or unbalanced bridges.

References

[1] L. Kirkup. Experimental Methods: an Introduction to the Analysis and Presentation of Data: Wiley, 1994.