- Syllabus
- Laboratories (remote)
- Projects
- TA's { Stephen Barnes
  TBD

- Power Point Presentation
- 3 handouts on "Introduction to the Musculoskeletal System"

Biomechanics

- application of mechanics to Biological Systems
- Goes back to Leonardo Da Vinci

Biomechanics of Human Movement

Interdisciplinary :
- describes
- analyze
- assess human movement

Examples
- performance of an athlete
- lifting a load by a factory worker
- how a person with disabilities walk

Who might be interested

- orthopaedic surgeons
- athletic coaches
- rehab engineers
- therapists
- prosthetists
- orthotists
- sport equipment designers

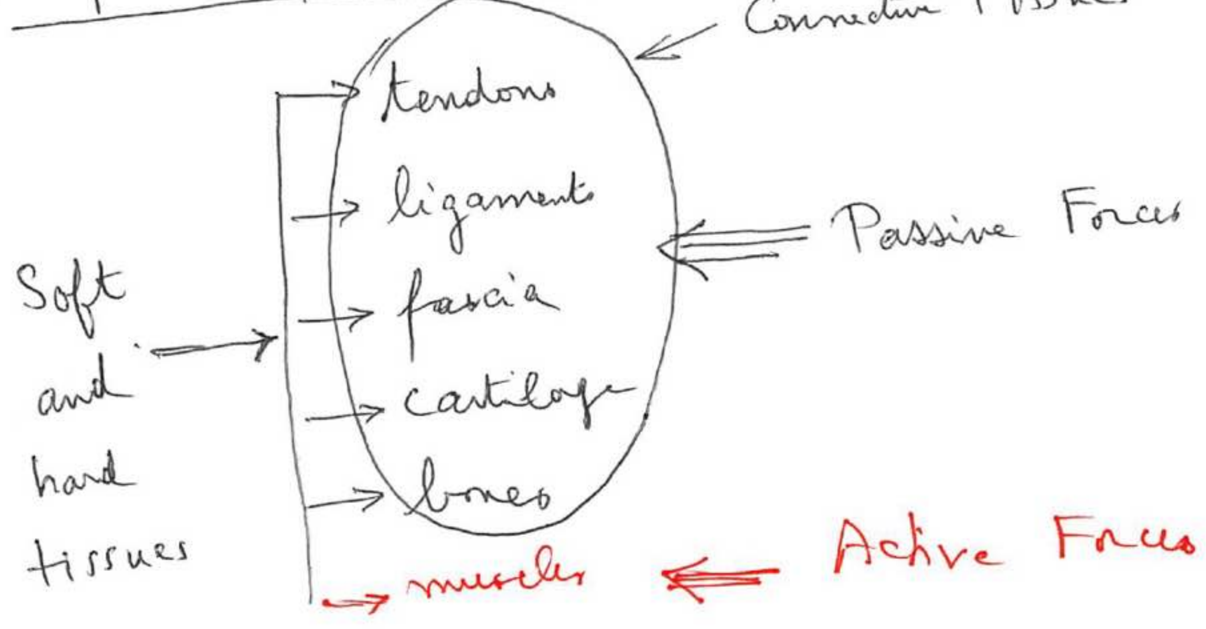
Biomechanics :
- Physics
- Mathematics
- Chemistry
- Physiology
- Anatomy ←

What is the function of the Musculoskeletal System?

1) to provide motion
2) to support the body and its components

Joints are the functional Units of the Musculoskeletal System.

Components of the Musculoskeletal System



Connective Tissues

- ① provide support
- ② transmit forces
- ③ maintain the integrity, structurally

Cells + extracellular matrix

fibers　　Ground substance

- In skins, tendons and ligaments
  Cells are called fibroblasts
- In cartilage
  Cells are called chondroblasts or chondrocytes
- In bones
  Cells are called 'Osteoblasts" and transformed to osteocytes"

Fibers

- Collagen fibers (12 different types of collagen fibers)
  - • high tensile strength
  - • resilient to deformation
- Elastin Fibers

→ Proportion of collagen/elastin fibers in a tissue has a major influence on its mechanical properties

Collagen Fibers			Elastin Fibers
Type I	Type II	Type III	
• tendons • ligaments • bones • skin	• cartilage	• walls of large blood vessels	→ low tensile strength → elastic
→ high tensile strength → resistant to deformation			

Ground Substances

① "Prateoglycans"

(sugar molecules linked to protein cores)

<u>In bone</u> few %ges of "proteoglycans"

<u>In cartilage</u> 25% of ground substance is "proteoglycans"

2) Water

3) Calcium (specially in bones)

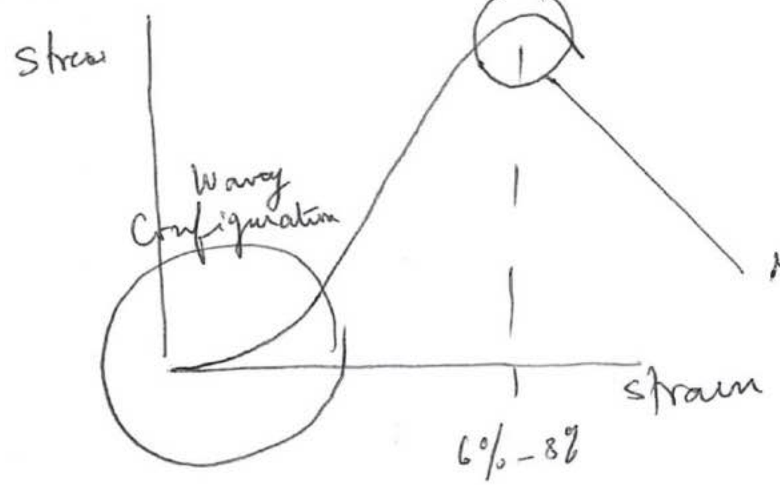
Components of the Musculoskeletal System

Ligaments	Tendons	Fascia
• connect "bone" to "bone" • Provide stability at joints. • 90% of fibers are collagen • some collagen fibers are non parallel	• attach muscles to bones - • transmit forces from muscles • collagen fibers are parallel	• Covers parts of organs and separate them from each other (separate muscles from each other) • Irregular collagen fibers • large proportion of elastin fibers

Note
- type of arrangement of collagen fibers
- proportion of collagen fibers to elastin fibers

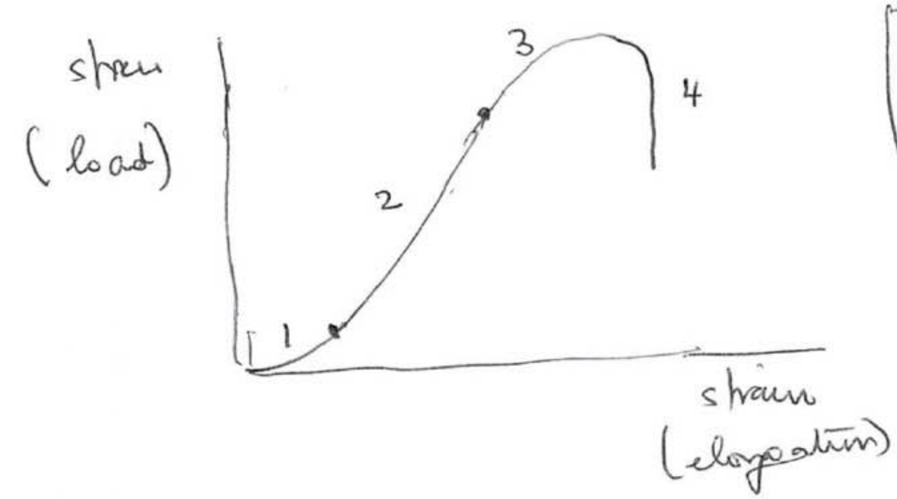
→ factors that determine the mechanical properties of soft tissues

STRESS - STRAIN DIAGRAMS

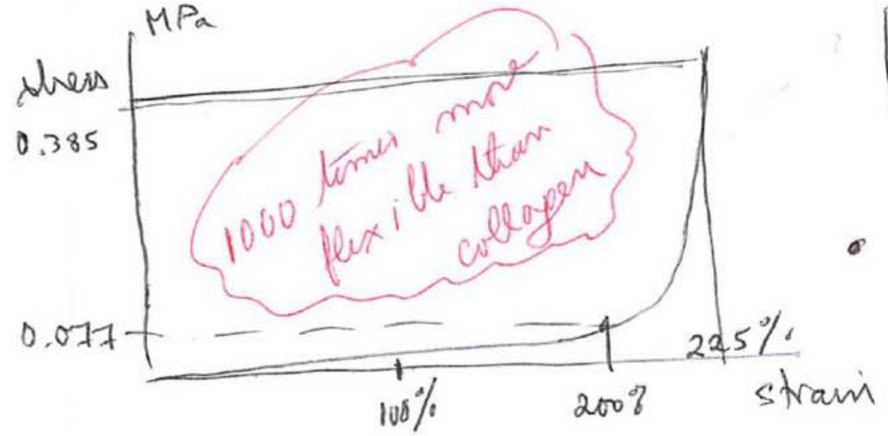


Collagen fibers

ultimate strength @ 50% of cortical bone in tension



Anterior Cruciate Ligament



1000 times more flexible than collagen

Elastin Fibers

- Weak
- [redacted]

examples
- pelvic connective tissue
- skin
- lungs

Cartilage

- hyaline (covers the articular surface of bones)
- elastic (greater resilience ; present when you have functional distortion ; external ear)
- fibrous or fibrocartilage (increased strength due to the presence of collagen fibers in its ground substance ; weight bearing intervertebral disks)

JOINTS

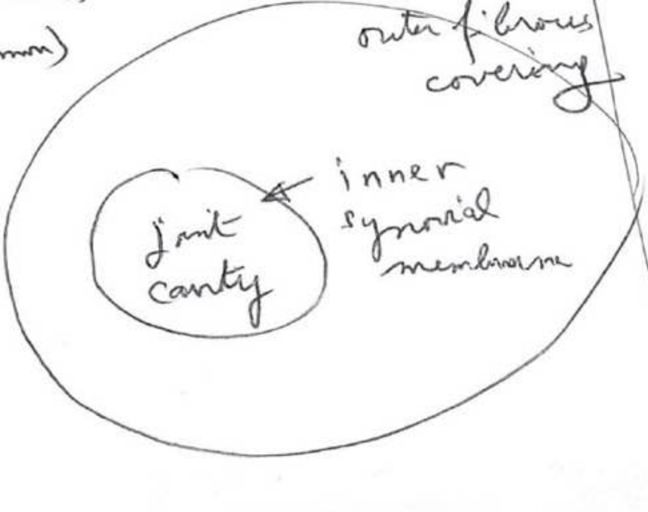
1) fibrous joints [little or no movement]

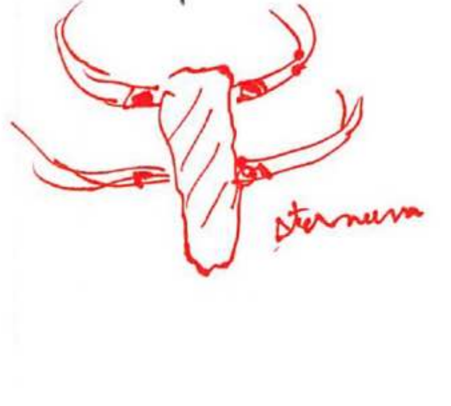
(tibia-fibular joint; gomphosis joint (teeth)

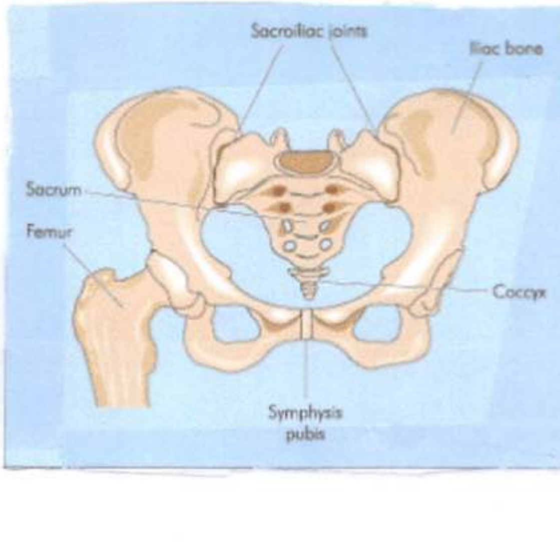
2) cartilagenous joints [slightly moveable]

- (symphyses pubis)
- (joints between the ribs and the sternum) (breast bone)

3) synovial joints (most common)



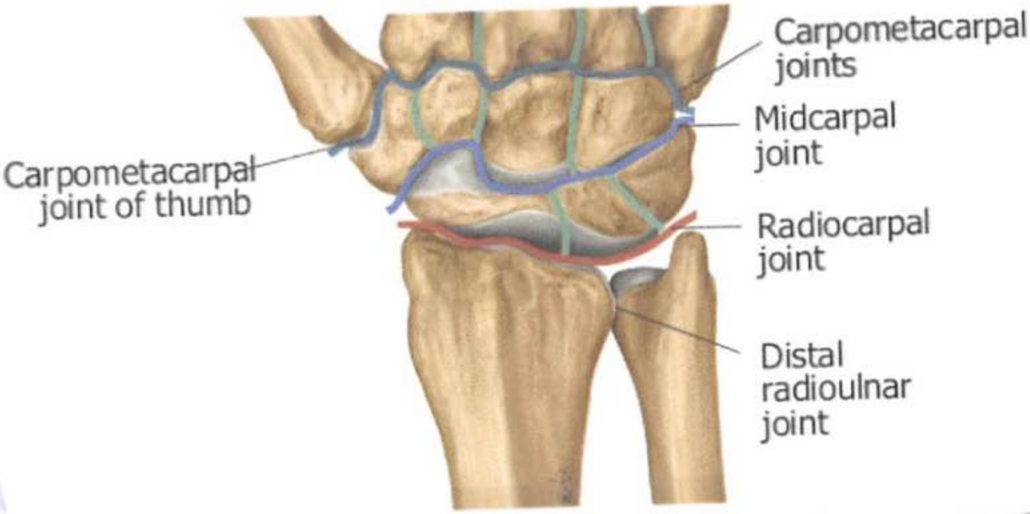




Synovial Joints

- most common
- several classification
- forms a cavity

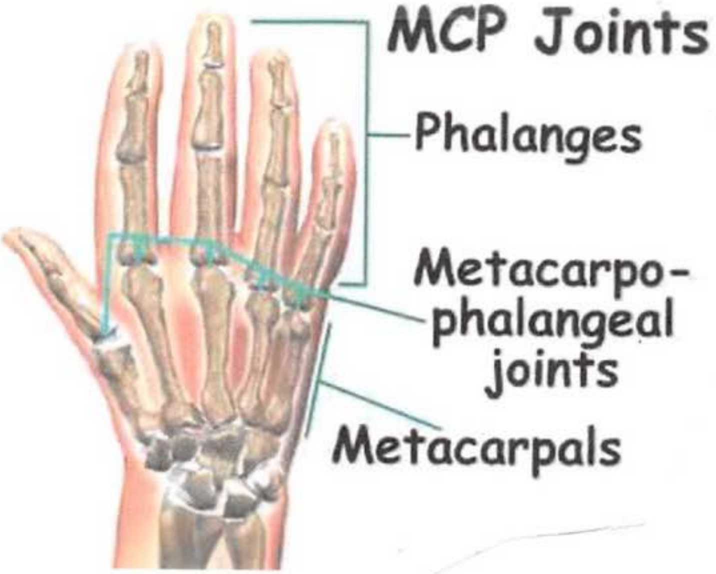
① Plane Joint
- a flat surfaces
- very limited amount of sliding
- carpal joints



② Spheroidal Joint
(ball and socket joints)
- shoulder and hip joints
- 3 degrees of rotations

③ Condylar Joint

- modified spheroid
- rotations are limited
- shallow ellipsoidal socket receives a ball that is not truly round
- metacarpophalangeal joint



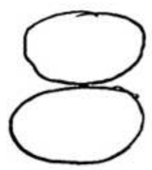
④ Ellipsoid Joints

- similar to condylar joints
- both bones are ellipsoidal shaped articulating surfaces
- radiocarpal articulation

(see picture on previous page)

⑤ Trochoid or Pivot Joint

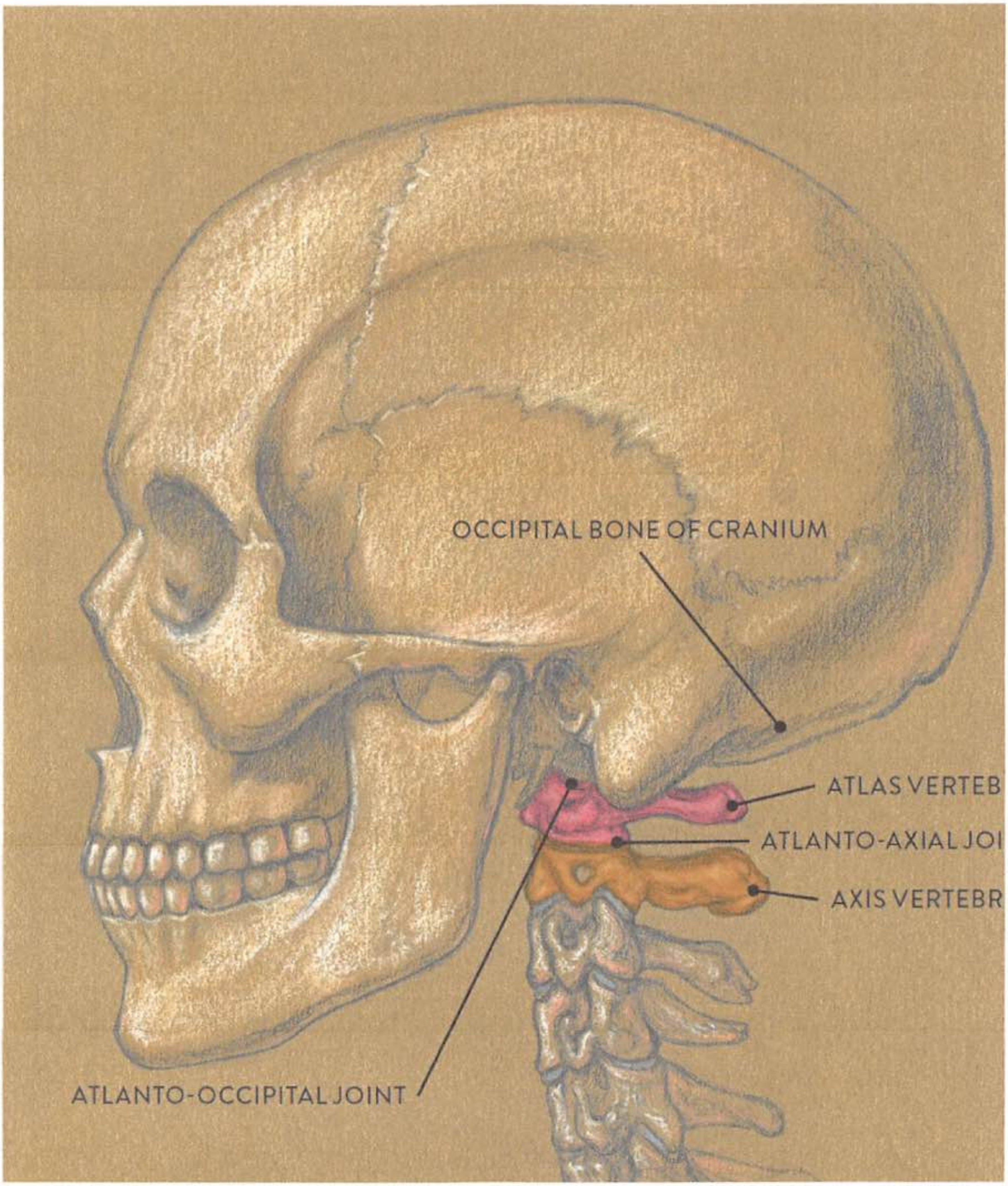
- one bone serving as a pin around which the other rotates.
- Atlantoaxial joint (Cervical Spine)
- ligaments form the circle enclosing the pin



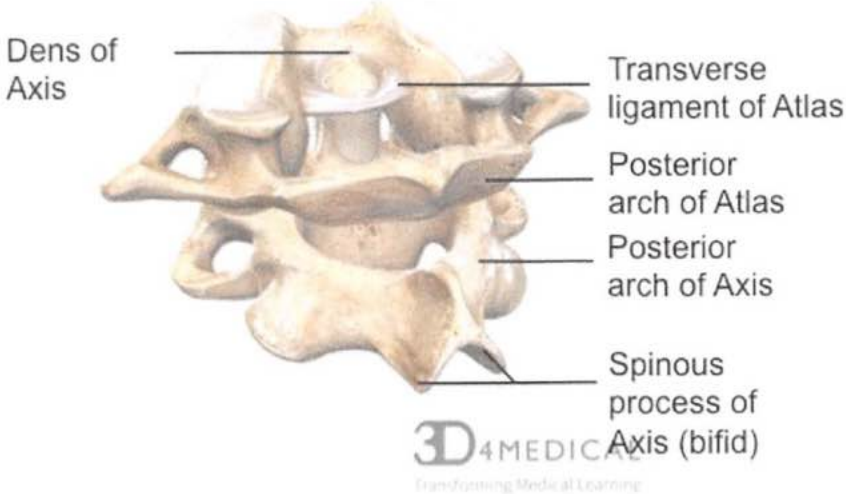
1st vertebrae - atlas

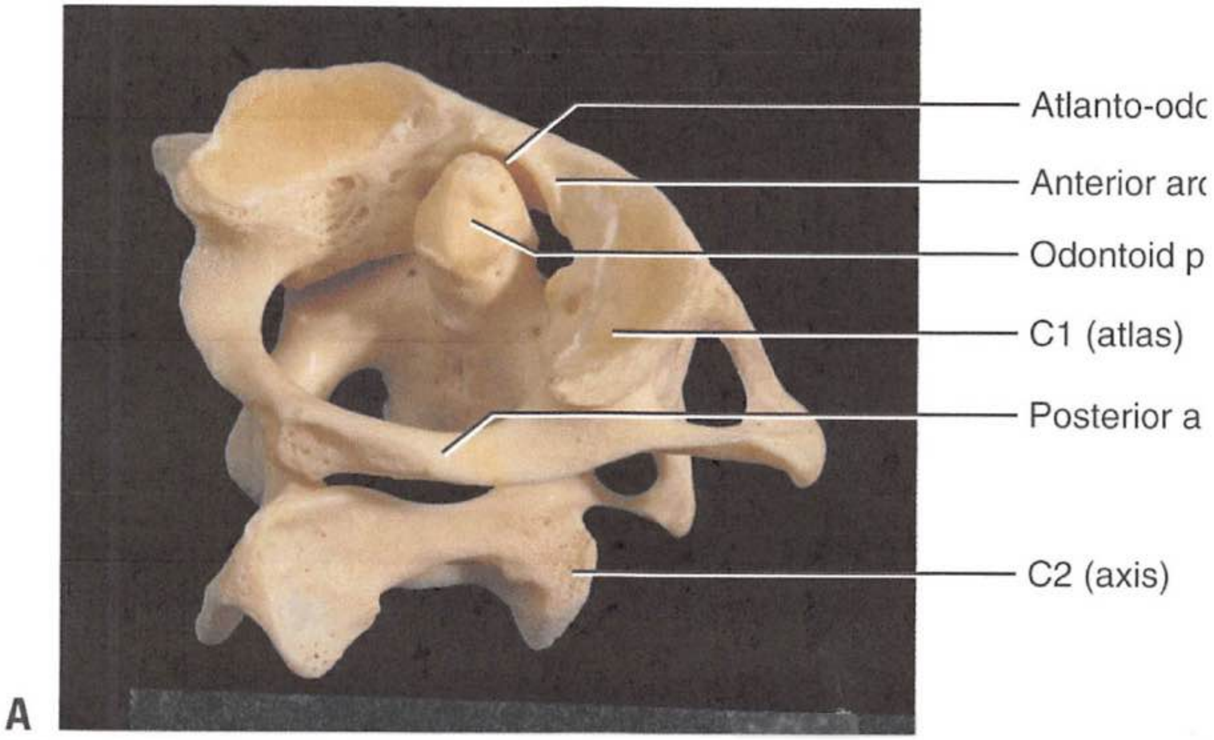
2nd vertebrae - axis [acts as a pin]

See pictures on next two pages →



https://schoolbag.info/biology/anatomy/anatomy.files/image093.jpg 8/18/2020


Dens of Axis
Transverse ligament of Atlas
Posterior arch of Atlas
Posterior arch of Axis
Spinous process of Axis (bifid)
3D4MEDICAL
Transforming Medical Learning


Atlanto-od
Anterior ar
Odontoid p
C1 (atlas)
Posterior a
C2 (axis)
A

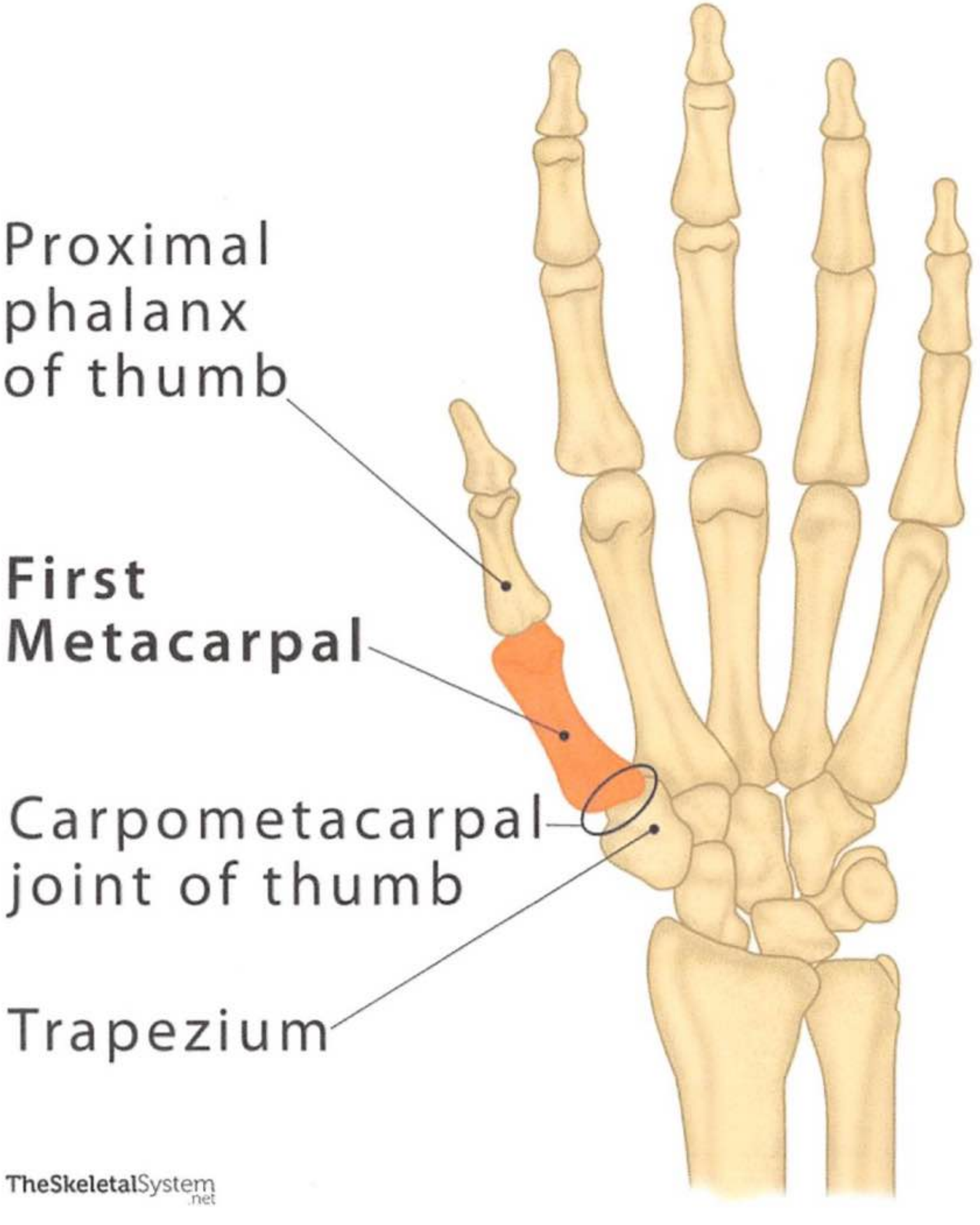
⑥ Sellar or Saddle Joint

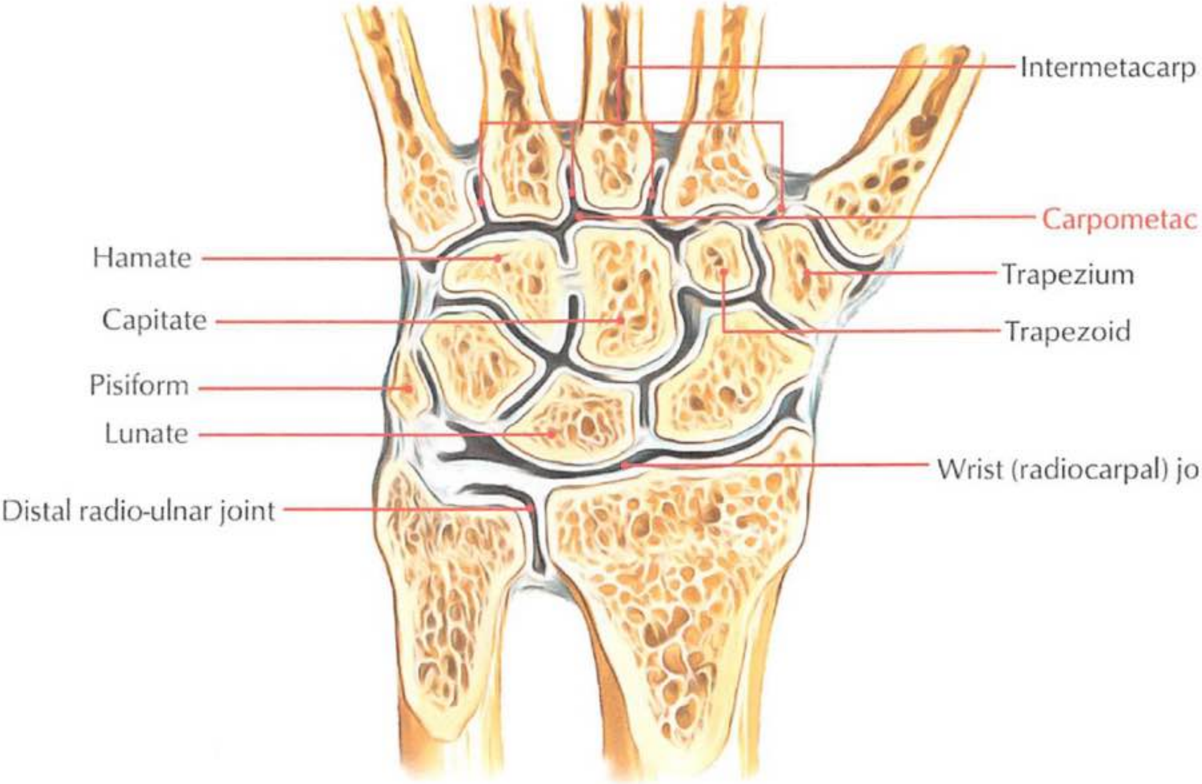
- Composed of complex curved surfaces resembling those of a saddle [Trapezium & 1st metacarpal]

- one bone is convex- concave
- the other bone is reciprocally curved so that the two fit together.

⟶ See pictures on the next two pages.

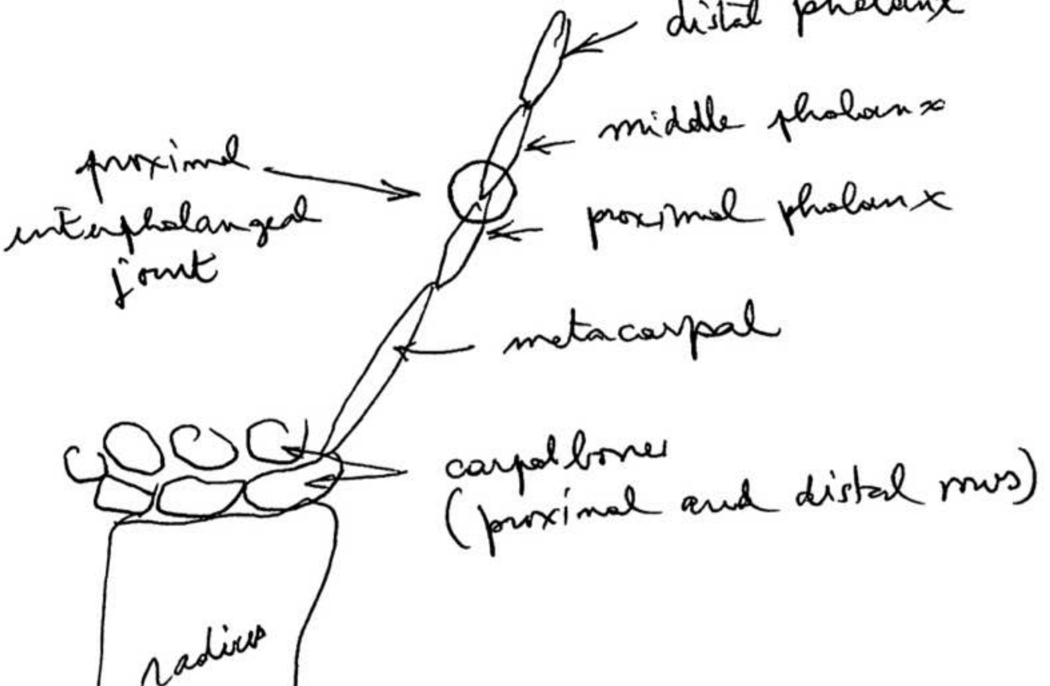
First Metacarpal




Intermetacarp
Carpometac
Hamate
Trapezium
Capitate
Trapezoid
Pisiform
Lunate
Wrist (radiocarpal) jo
Distal radio-ulnar joint

⑦ Hinge Joint

- Movement can take place around one axis: flexion and extension
- Elbow joint
- Interphalangeal joints



BONES

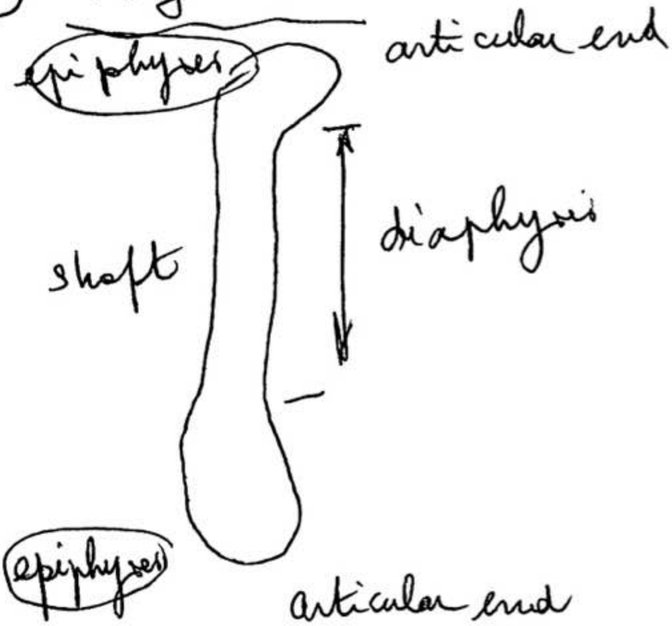
. around 206 bones

☆ Skeleton is divided into 2 parts :

- Axial Skeleton → bones in head, neck & trunk
- Appendicular skeleton → bones of the extremities

Bones are classified according to their shapes

① Long Bones



- Cortical bone
  (Compact bone)
- Cancellous bone
  (→ spongy bone
  → trabecular bone)

↓ thin plates in a loose mesh structure which is enclosed by the cortical bone

② Short bones

- $l_1 = l_2 = l_3$
- carpal bones

③ Sesamoid bones

- embedded within tendons or joint capsules
- Patella

④ flat bones

- bones of skull
- sternum
- scapula
- ribs

⑤ Irregular bones

- Pelvic bones

(pneumatic bones contain air-filled cavities or sinuses)

What is a bone?

- It is a composite material with various solid and fluid phases
- It consist of cells and an organic extracellular matrix of fibers and ground substances produced by the cells
- Inorganic substance in the form of mineral salts are combined with the ground substance.

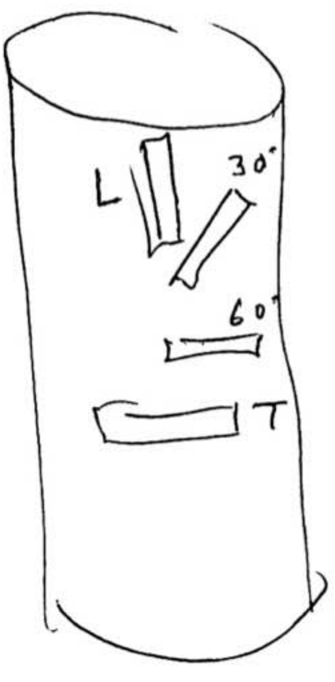
$$\begin{cases} \text{Inorganic component (mineral)} \rightarrow \text{hard \& relatively rigid} \\ \text{Organic component} \rightarrow \text{provides flexibility and resilience.} \end{cases}$$

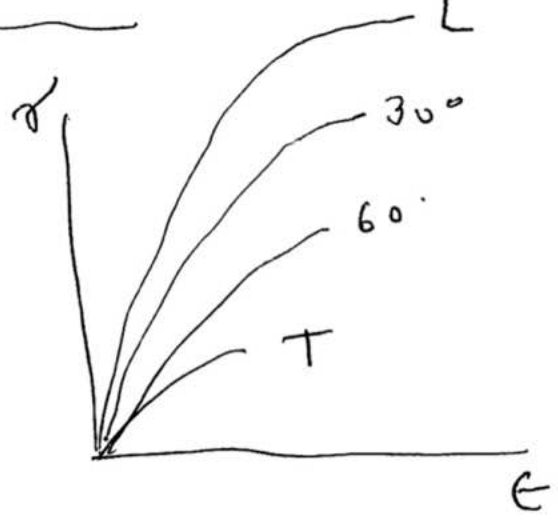
Bones are biphasic composite materials with the mineral as one phase and the collagen and ground substance as the other

1 • Bone is a non homogeneous material

Cancellous Bone	Cortical Bone
• Anisotropic	• Anisotropic
• fracture due to:	• fracture due to:
→ large muscle contraction	→ bending loads
→ applied external loads	→ torsion loads

$\boxed{2}$ Anisotropic Material

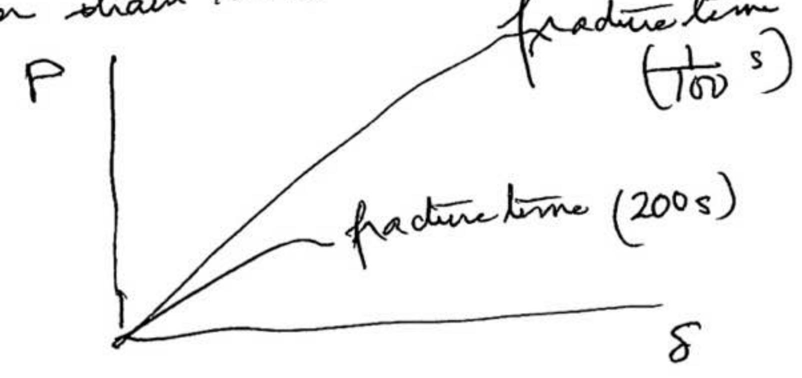




T transverse
L longitudinal

3 Viscoelastic Material (Rate Dependent)

It can resist rapidly applied loads much better than slowly applied loads. That is, it is stiffer and stronger at higher strain rates



Cancellous bone fracture at 75% strain

and Cortical bone fracture at 2% strain

Another Way to look at bones :-

- Chemical composition of both types of bones are similar
- Porosity is different where cancellous bone are more porous.

So both types of bones can be regarded as a single material of variable density.

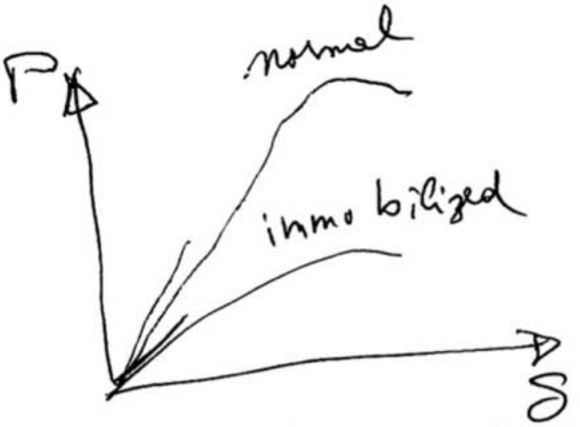
Bone Remodeling

Bone changes its shape, size and structure as a result of mechanical demands placed on it over time

Wolff's Law (1892)

Bone will be deposited when needed and resorbed when not needed

Effect of Immobilization on bone strength and stiffness ⟹



Why it is important to Occupational Biomechanics?

This is stress dependent

Effects of Ageing on bones

1. decrease in bone mineral content
2. decrease in cortical bone thickness
3. decrease in number of trabeculae in cancellous bone
4. increase in outer diameter of long bones

- Anatomical Terminology
- Structures comprising the Knee Joint
  - ↳ terminology

→ Handouts #1, #2 and #3.

①

- Anatomical Terminology
- Structures comprising the Knee Joint
  - ↳ terminology

→ Handouts #1, #2 and #3.

Kinetics : Forces and Moments

- Study of forces and moments that cause movements is called "KINETICS"
- how to measure forces in Biomechanics ? IN-VIVO
  - * in animals : we can insert transducers to measure muscle forces
  - * in human beings : we need to calculate and estimate these forces using "Link - Segment Modeling"

→

Assumptions made in analysis

① mass of a segment is considered as a concentrated load acting at the center of mass of the segment

② Joints are considered to be hinge joints.

What are the forces acting on the link-segment model?

① Gravitational forces

② Ground reaction forces or external loads

③ muscle and ligament forces

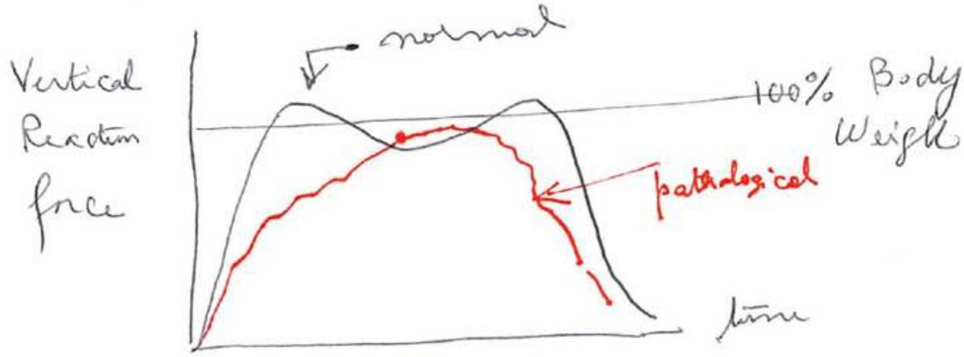
⇓

* net effect of muscle forces is calculated as net muscle moments at joints

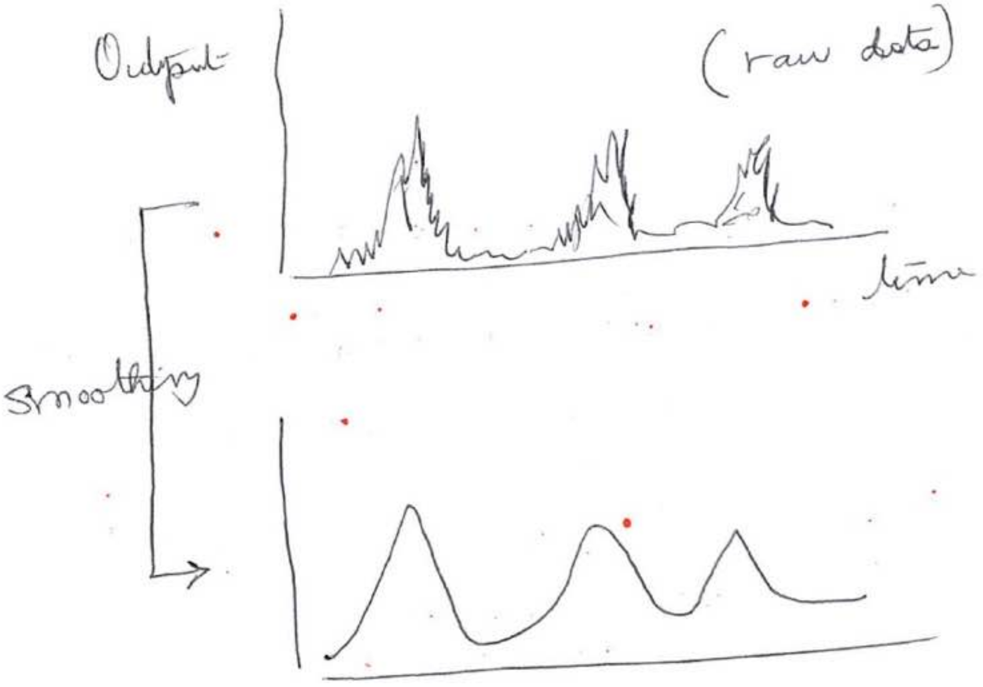
So:- If a contraction happens, what you predict is the net effect

Usually, it is the net effect of muscle and ligament and joint reaction forces.

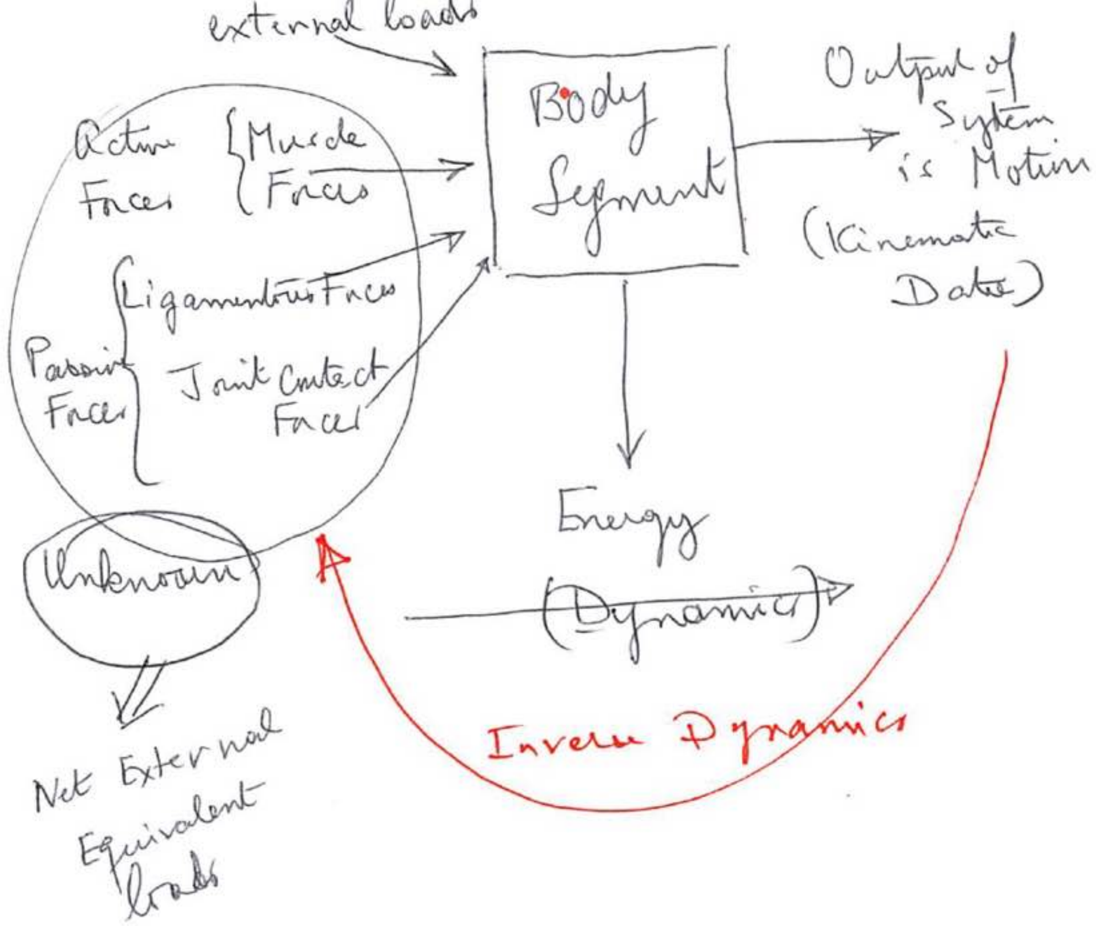
So on page 7 is ground reaction force example



On page 5, is an example of EMG data



$$\sum \bar{F} = M\bar{a}$$



Kinematic Data

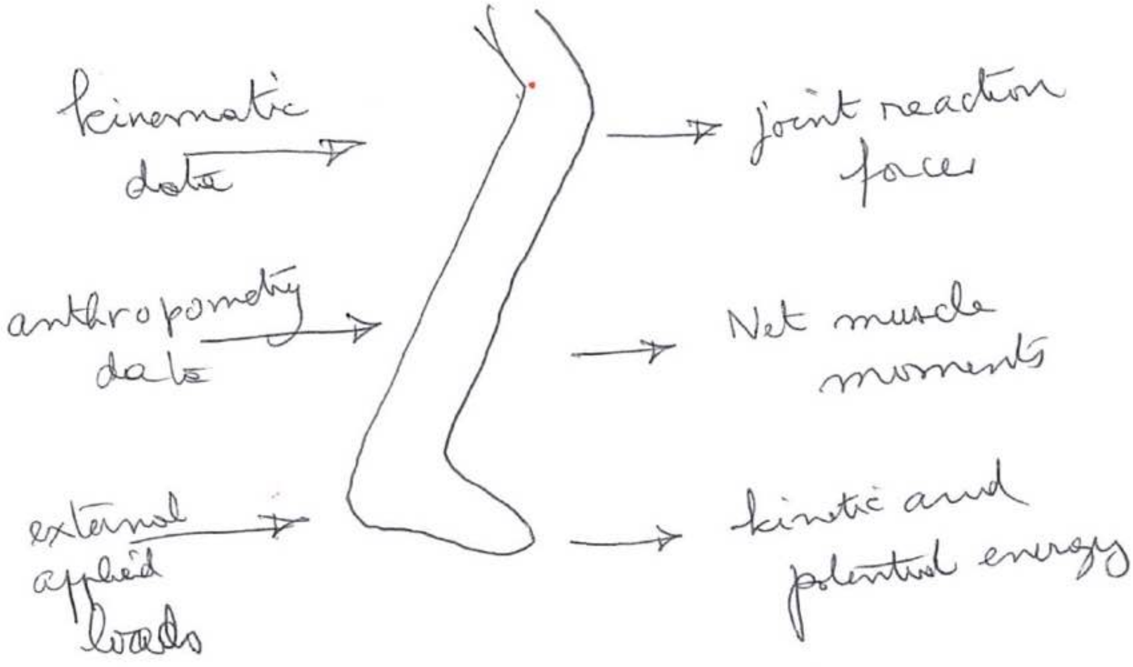
- Linear
- Angular

* displacements

* Velocities

* accelerations

"INVERSE SOLUTION"



This prediction is called "Inverse Solution"

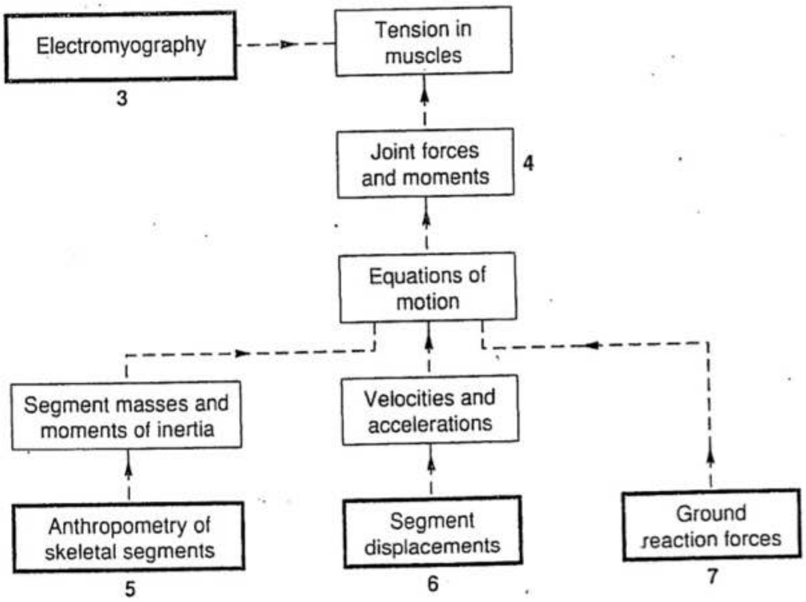


Figure 1.4 The inverse approach in rigid body dynamics expressed in words.

Synthesis of Human Movement (pp12)

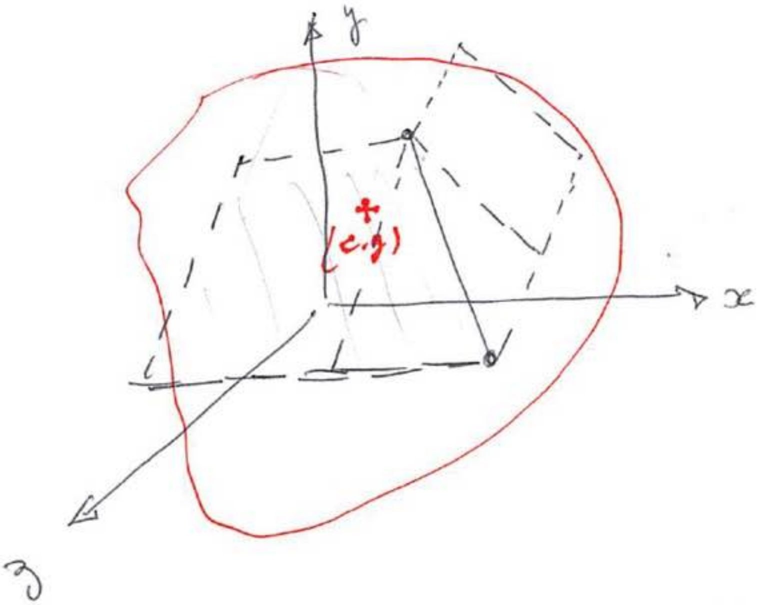
- Inverse solution → predict net moments, energy, power

- Reverse is called synthesis, use assumed moments as forcing functions, and ask the question:
  "What would happen if ?" to predict kinematics

- Poor and limited to very simple movements

(Chapter 8 – ???

Total description of a body segment in Space (PP 47-48)



Center of mass x, y, z

linear velocity of c.g $\dot{x}, \dot{y}, \dot{z}$

linear acceleration of c.g $\ddot{x}, \ddot{y}, \ddot{z}$

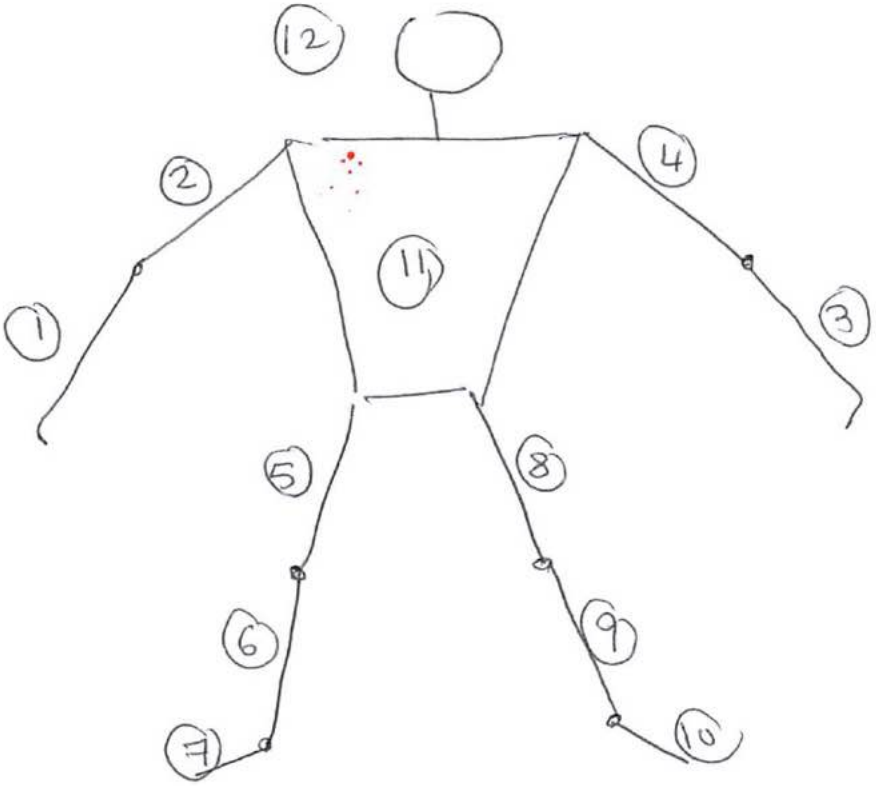
angle of segments in 2 planes θ_{xy}, θ_{yz}

angular velocity of segments in 2 planes $\dot{\theta}_{xy}, \dot{\theta}_{yz}$

angular acceleration of segments in 2 planes $\ddot{\theta}_{xy}, \ddot{\theta}_{yz}$

15 Variables

S. in 3D



We have 12 segments.
So at each instant in time, we need to have:

$12 \times 15 = 180$ data variables

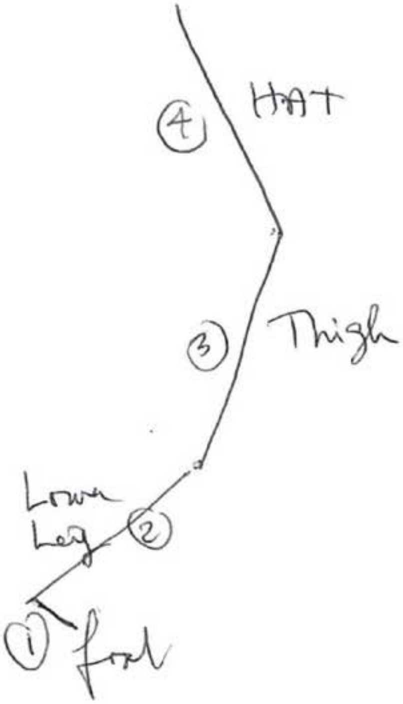
→ We need to simplify to a manageable number.

Gait analysis, assumes
Symmetrical walking

So, we assume sagittal plane
movement

We have then 4 segments:

- (HEAD, ARM, TRUNK) are assumed one segment called HAT
- thigh
- lower leg
- foot



Each segment

$$\left.\begin{array}{ll} \underline{\text{C.g}} & x,\ y \\ & \dot{x},\ \dot{y} \\ & \ddot{x},\ \ddot{y} \\ \underline{\text{Orientation}} & \theta \\ & \dot{\theta}=\omega \\ & \ddot{\theta}=\alpha \end{array}\right\} \text{9 variables}$$

So :-

In a sagittal plane movement,

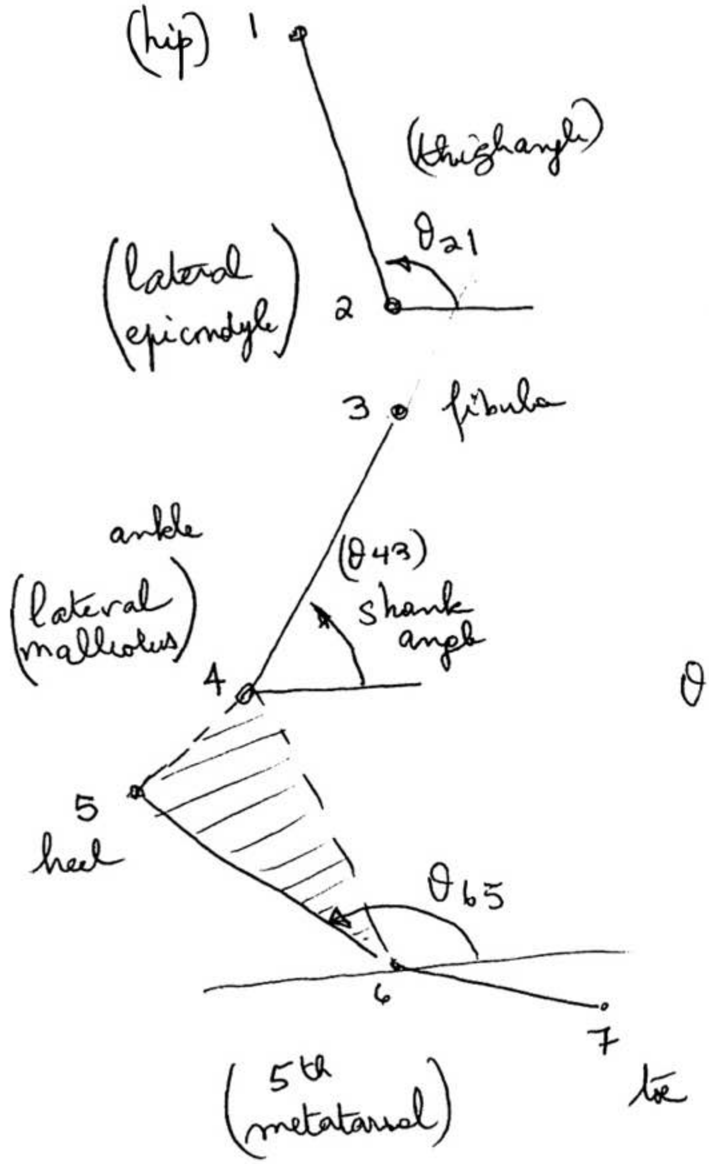
we need to have

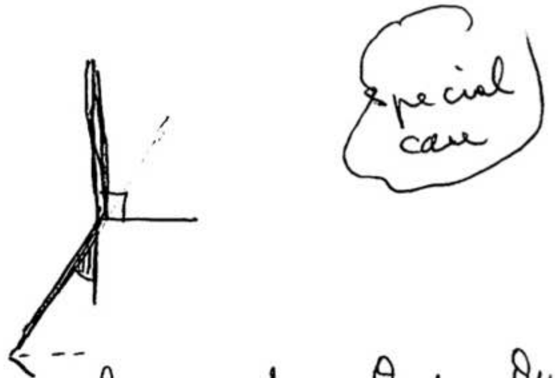
④ × 9 = 36 data variables

segments ←

at any instant in time

Limb Segment Angles (PP 75-77)





knee angle $= \theta_{21} - \theta_{43}$

+ flexion

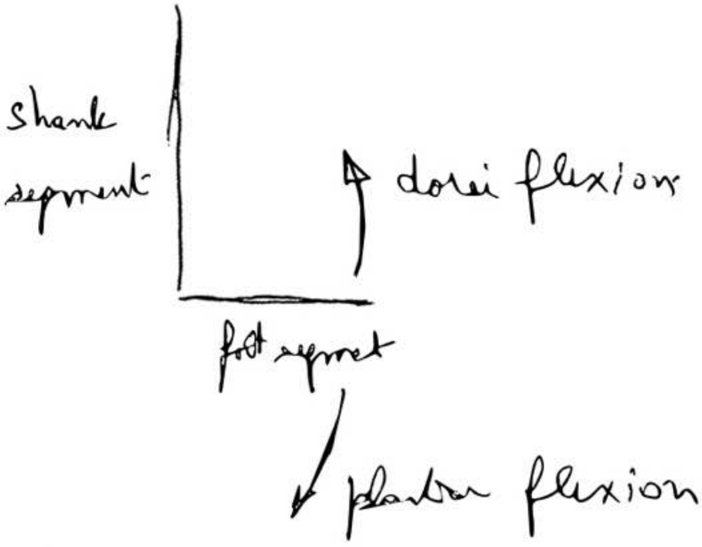
− extension

θ_{65} foot angle

$$** \quad \theta_{ankle} = \theta_{43} - \theta_{65} + 90^\circ$$

+ve plantar flexion

−ve dorsi flexion



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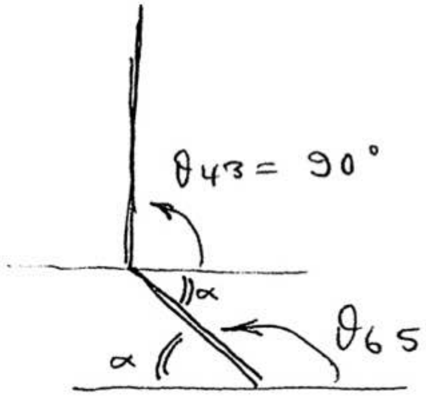


special case

$$\theta_{ankle} = \theta_{43} - \theta_{65} + 90°$$

$$= 90° - (180 - \alpha) + 90$$

$$= \alpha \quad (+ve)$$ plantar flexion

APPENDIX A

KINEMATIC, KINETIC, AND ENERGY DATA

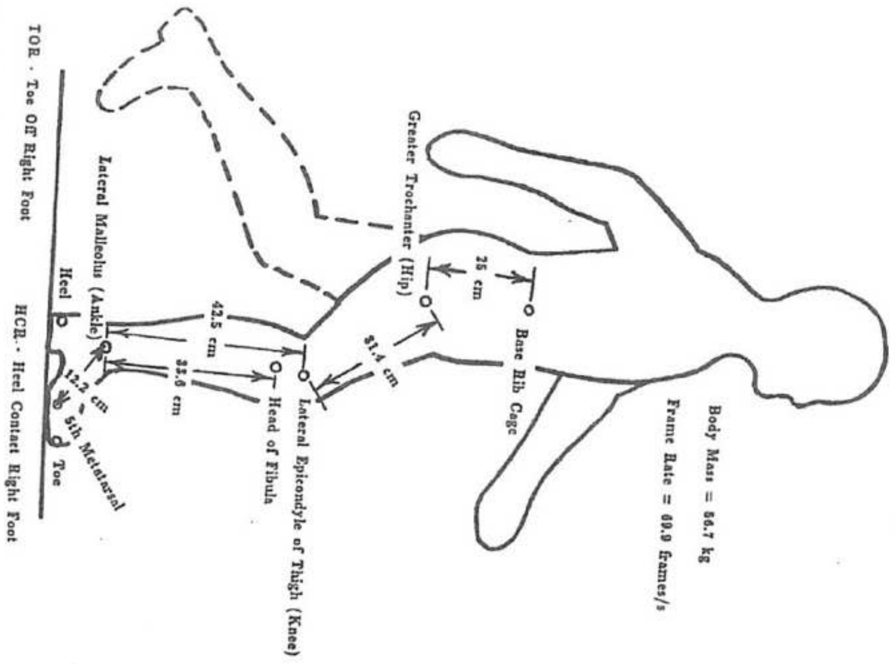


Figure A.1 Walking trial—marker locations and mass and frame rate information.

TABLE A.1 Raw Coordinate Data (cm)

FRAME	TIME S	BASE RIB X	CAGE Y	RIGHT X	HIP Y	RIGHT X	KNEE Y	RIGHT X	FIBULA Y	RIGHT X	ANKLE Y	RIGHT X	HEEL Y	RIGHT X	METAT. Y	RIGHT X	TOE Y
1	0.000	46.98	104.41	44.94	78.58	41.00	47.40	35.91	40.53	9.31	21.44	2.95	24.24	7.53	9.35	11.73	3.63
2	0.014	49.22	104.79	47.31	78.58	45.02	46.89	40.06	40.02	12.70	22.46	7.23	26.02	10.54	10.63	13.72	4.77
3	0.029	51.10	105.17	49.57	78.71	48.68	47.27	44.23	40.15	16.49	23.73	10.64	27.30	14.20	12.03	17.63	6.43
4	0.043	53.13	105.30	51.74	79.21	52.50	47.53	48.43	40.15	20.81	24.37	14.71	27.55	18.78	12.53	22.47	6.94
5	0.057	54.86	105.43	53.33	79.09	56.13	47.91	52.70	40.78	24.96	24.24	18.72	27.42	23.17	12.66	27.12	7.32
6	0.072	56.81	106.06	55.41	79.98	59.87	48.67	56.56	41.29	29.33	24.62	23.09	27.17	28.31	12.41	32.51	6.81
7	0.086	58.25	106.32	56.73	80.49	63.34	49.44	60.29	41.80	33.57	23.73	26.95	26.02	33.44	12.03	38.02	6.81
8	0.100	60.03	106.95	58.89	81.00	66.90	50.84	64.36	42.69	38.78	23.22	31.27	25.01	39.55	11.26	44.38	6.30
9	0.114	61.56	107.08	60.79	81.12	69.96	51.09	67.79	43.20	43.11	22.21	35.73	22.84	45.01	10.24	50.36	6.04
10	0.129	63.54	107.46	62.78	82.01	73.22	51.73	71.31	43.84	48.15	21.06	40.64	20.93	51.46	8.97	57.05	5.28
11	0.143	65.20	107.85	64.69	82.40	76.27	53.00	74.74	44.86	53.23	20.04	45.73	19.02	57.56	8.21	63.54	5.15
12	0.157	66.92	107.85	66.67	82.78	79.01	53.51	77.86	45.11	58.27	18.52	51.27	16.86	63.99	7.44	70.23	5.03
13	0.172	68.77	107.59	68.65	82.65	81.62	54.15	80.73	[illegible]	[illegible]	[illegible]						

Appendix A (pp 296 → 396)

(figure A.1)

- Table A.1 raw data
- Table A.2 filtered data

x & y coordinates of markers located at :

- → base of rib cage
- → greater trochanter (hip)
- → lateral epicondyle of thigh (knee)
- → head of fibula
- → lateral malleolus (ankle)
- → heel
- → 5 the metatarsal
- → toe

frame = 69.9 frame/seconds

$$\Delta t = \frac{1}{69.9} \text{ seconds}$$

$$\boxed{\Delta t = 0.014306 \text{ seconds}}$$

Data is collected each 0.014306 seconds

HCR heel contact (right foot)

TOR toe off (right foot)

Let us study frame (4)

hip : (0.5138, 0.7914)
(from table A.2(a); pp 307)

lateral epicondyle (knee)
(0.5235, 0.4771)
(from table A.2(b); pp 306)

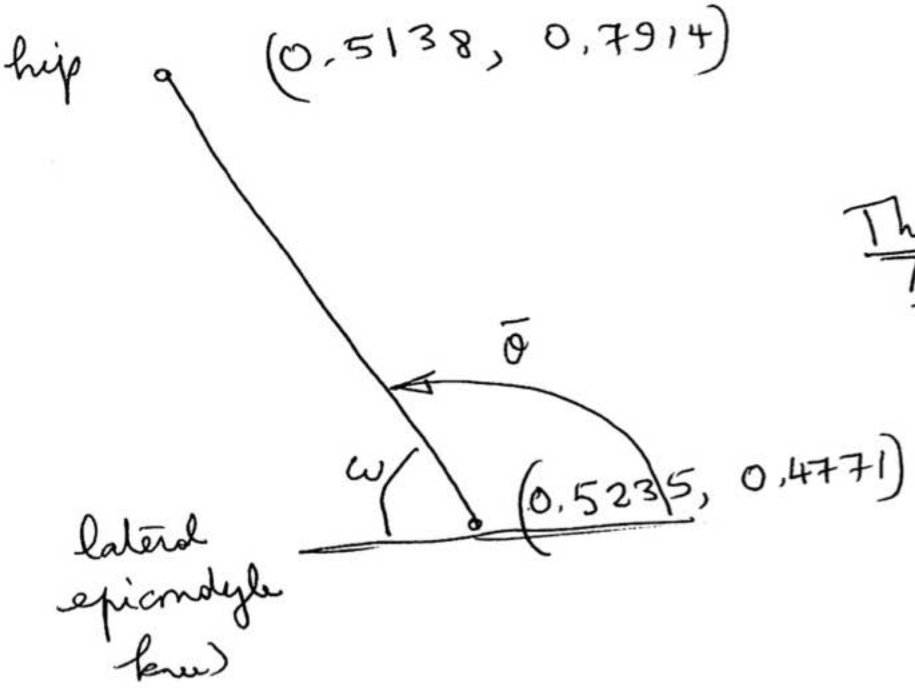
fibula
(0.4829, 0.4049)
(from table A.2.(b); pp 306)

ankle (0.2054, 0.2396)
(from table A.2(c); pp 311)

heel (0.1440, 0.2723)
(from table A.2.(c); pp 311)

5th metatarsal
(0.1854, 0.1201)
(from table A.2(d); pp 316)

Thigh Angle



$$\tan \omega = \frac{0{,}7914 - 0{,}4771}{0{,}5235 - 0{,}5138}$$

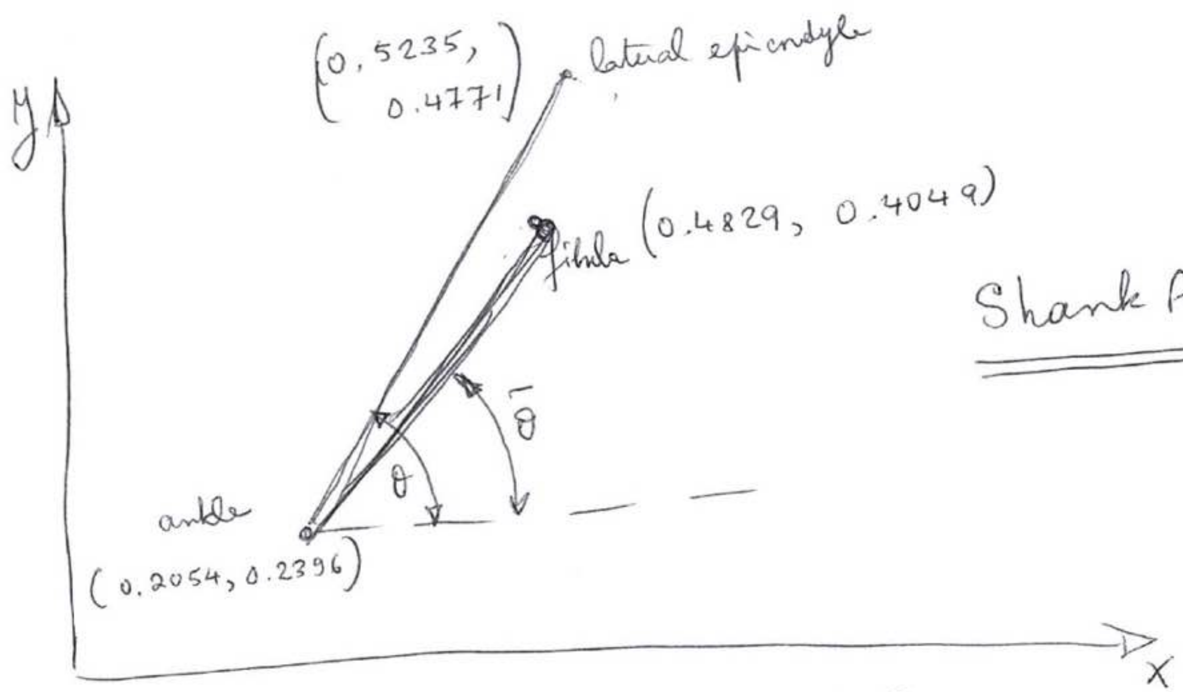
$$\tan \omega = \frac{0{,}3143}{0{,}0097} = 32{,}40206$$

$$\omega = 88.197^\circ$$

$$\boxed{\text{thigh segment angle} = \bar{\theta} = 91.803^\circ}$$

table A.3(c) pp 331

Shank Angle



$$\ast\ \tan\bar{\theta} = \frac{0.4049 - 0.2396}{0.4829 - 0.2054} = \frac{0.1653}{0.2775} = 0.595676$$

$$\bar{\theta} = 30.78^\circ$$

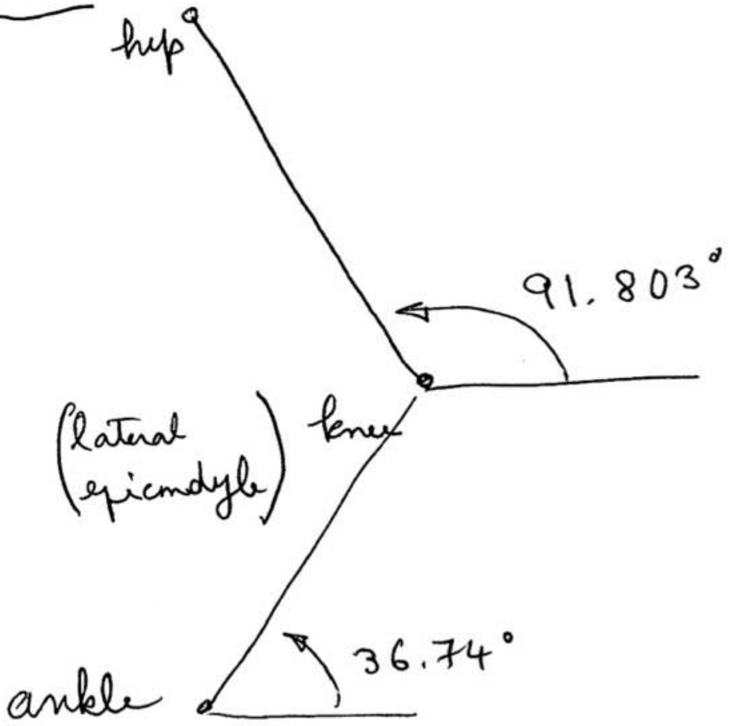
$$\ast\ \tan\theta = \frac{0.4771 - 0.2396}{0.5235 - 0.2054} = \frac{0.2375}{0.3181} = 0.7466$$

$$\theta = 36.74^\circ$$

So, from table A-3(b) pp. 326 leg segment is between ankle - lateral epicondyle and the shank angle is 36.74° at frame 4.

how to calculate knee angle

Method (1)

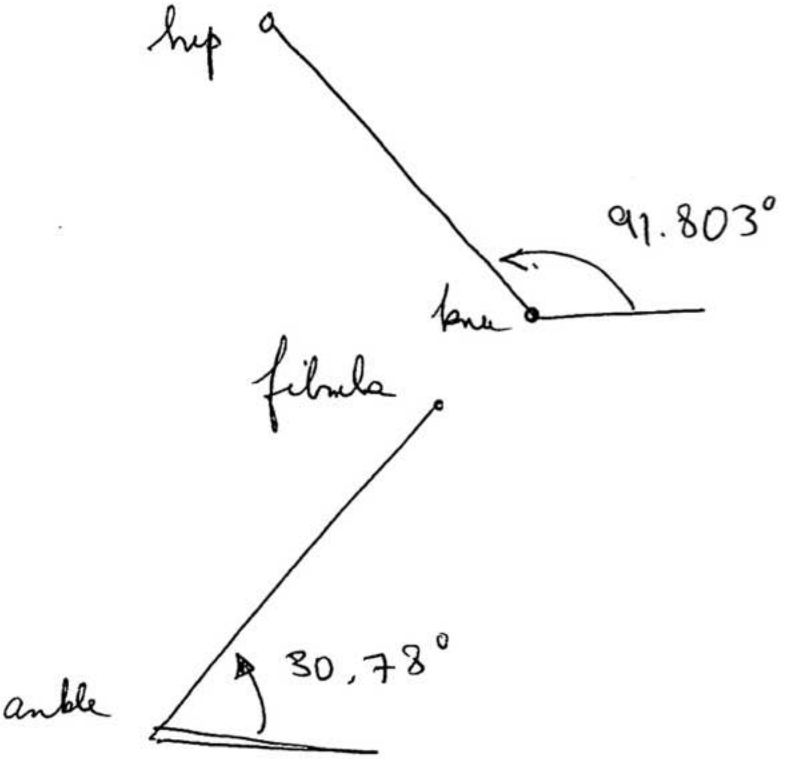


$$\text{knee angle} = 91.803 - 36.74^\circ$$
$$= 55.06^\circ$$

from table A-4; pp 341,
this is not the method used in
this book (knee angle = 61°)

how to calculate knee angle

Method (2)

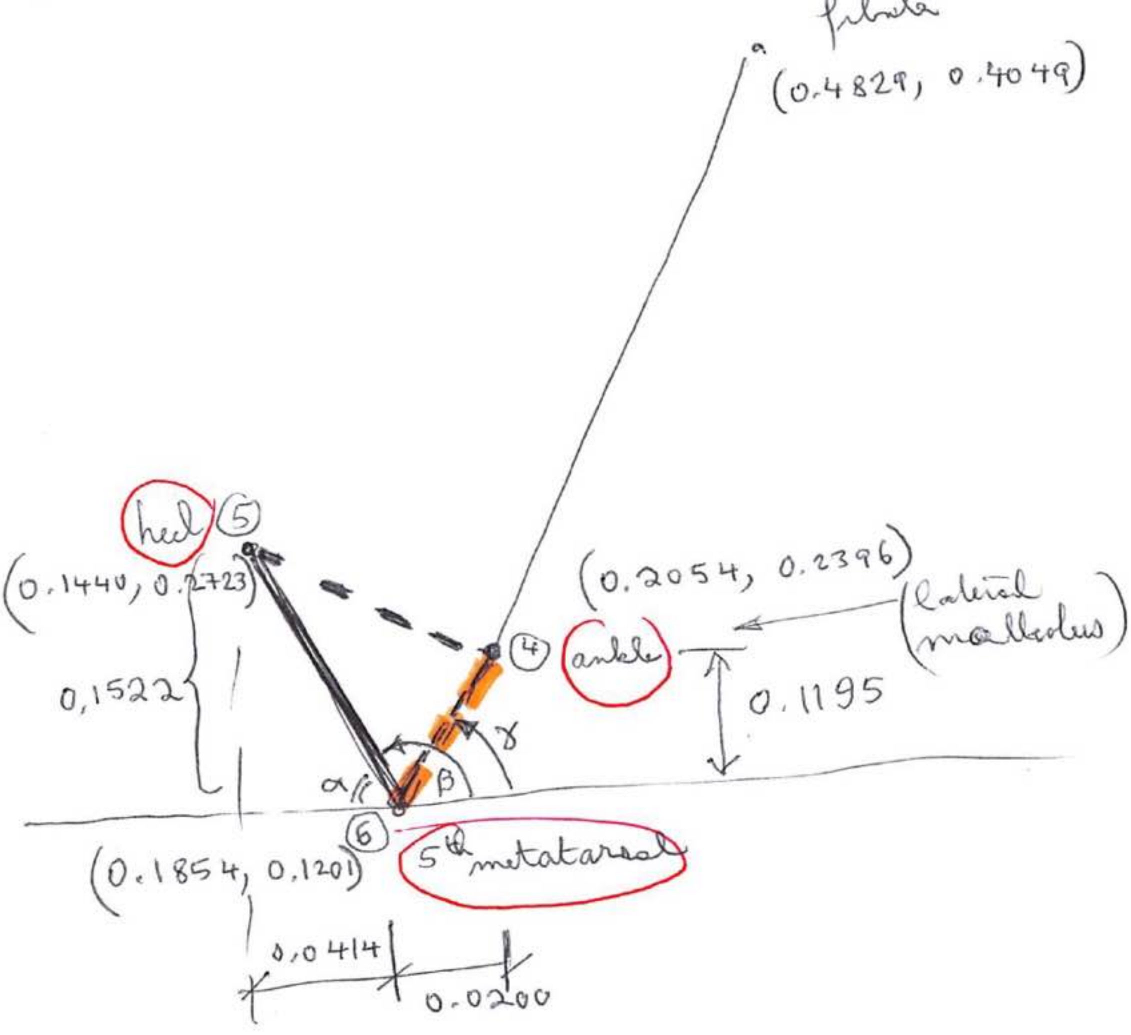


$$\text{knee angle} = 91.803^\circ - 30.78^\circ$$
$$= 61^\circ$$

from table A.4 ; pp341

this is the method that was used in this book (knee angle = 61°)

Foot angle and ankle angle



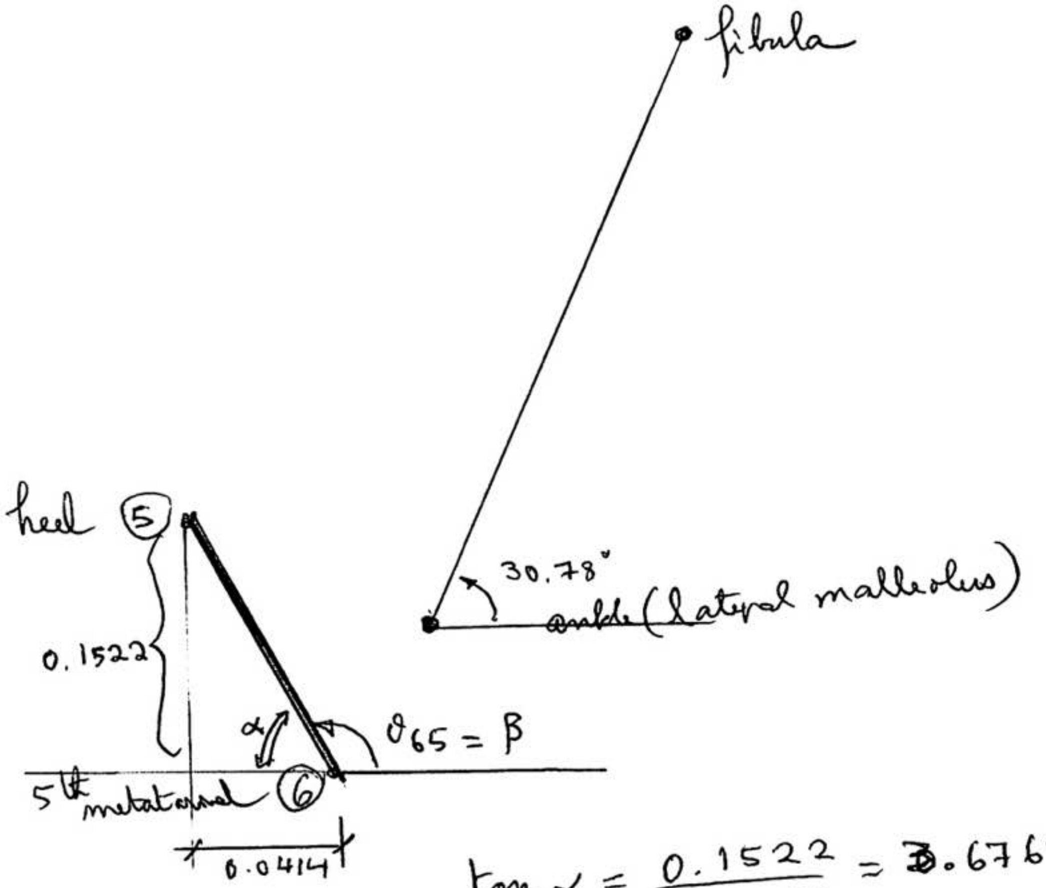
foot segment ankle

$$\tan\gamma = \frac{0.1195}{0.0200} = 5.975$$

$$\boxed{\gamma = 80.46^\circ}$$

So from table A.3(a) pp 321
foot segment is between ankle – 5th metatarsal
and the foot angle is 80.46° at frame 4.

how to calculate ankle angle ?



$$\tan \alpha = \frac{0.1522}{0.0414} = 3.676329$$

$$\boxed{\alpha = 74.75°} \Rightarrow \boxed{\beta = 105.25°}$$

$$\text{ankle angle} = \theta_{43} - \theta_{65} + 90$$

$$= 30.78° - 105.25 + 90$$

$$= \boxed{15.53°}$$

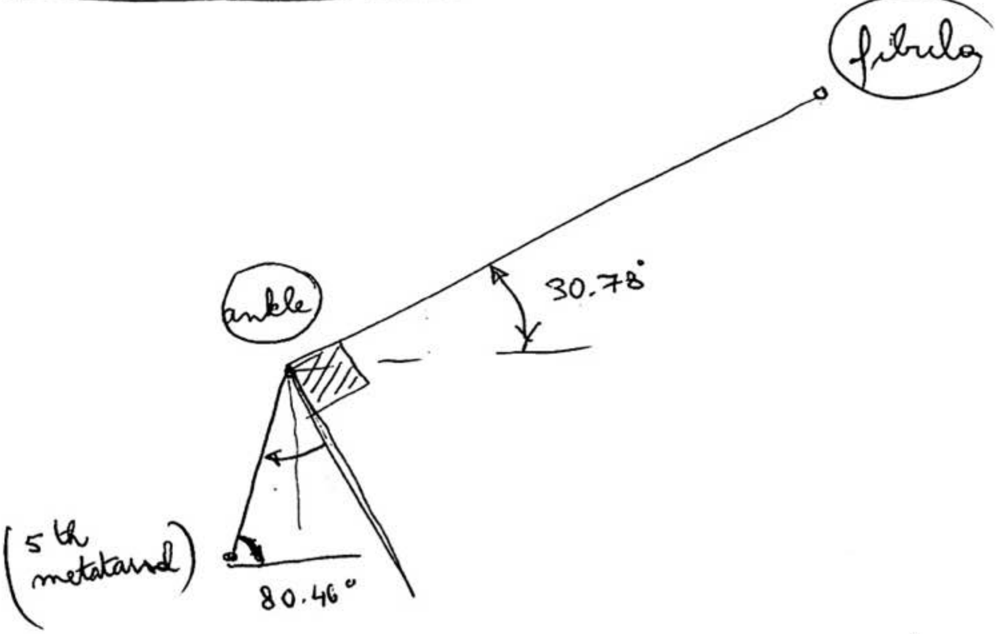
from table A.4. pp 341,

this is the method used in the book

however, there is a mistake in the sign

(ankle angle = 15.53')

So, let us determine the ankle angle at frame ④



$$\text{ankle angle} = \theta_{43} - \theta_{65} + 90^\circ$$

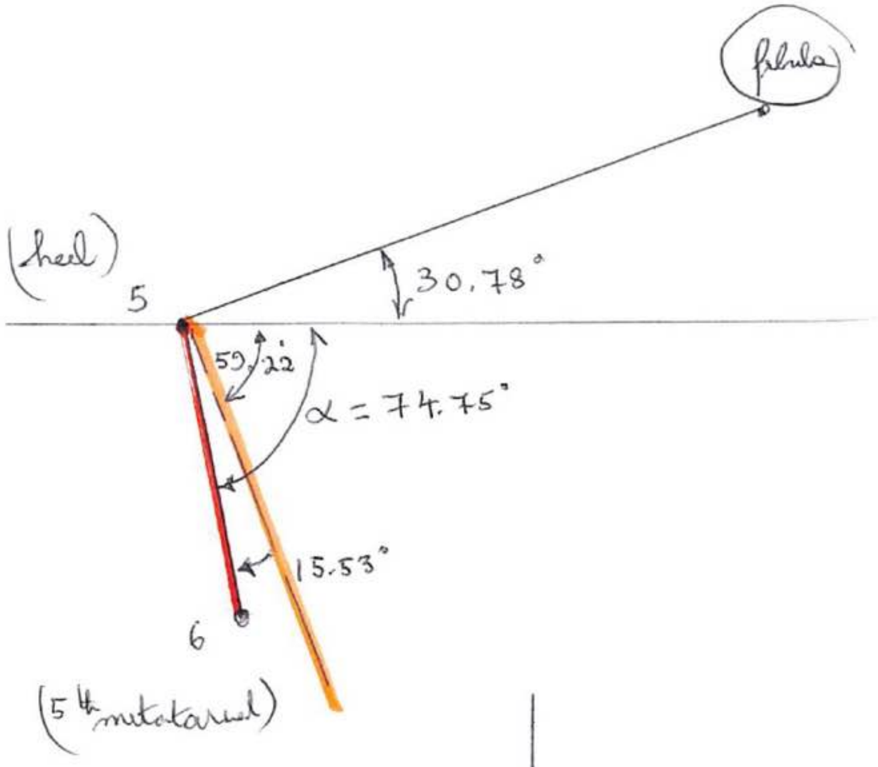
$$= 30.78^\circ - 80.46 + 90^\circ$$

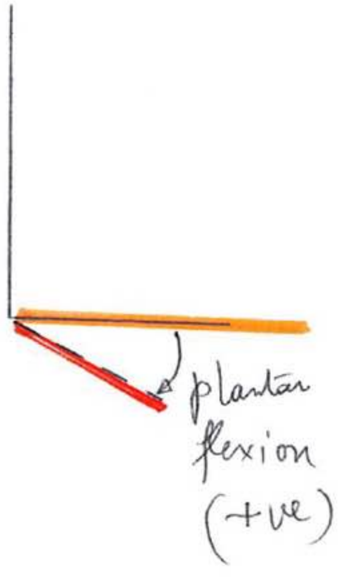
$$= 40^\circ 32$$

from table A.4, page 341,

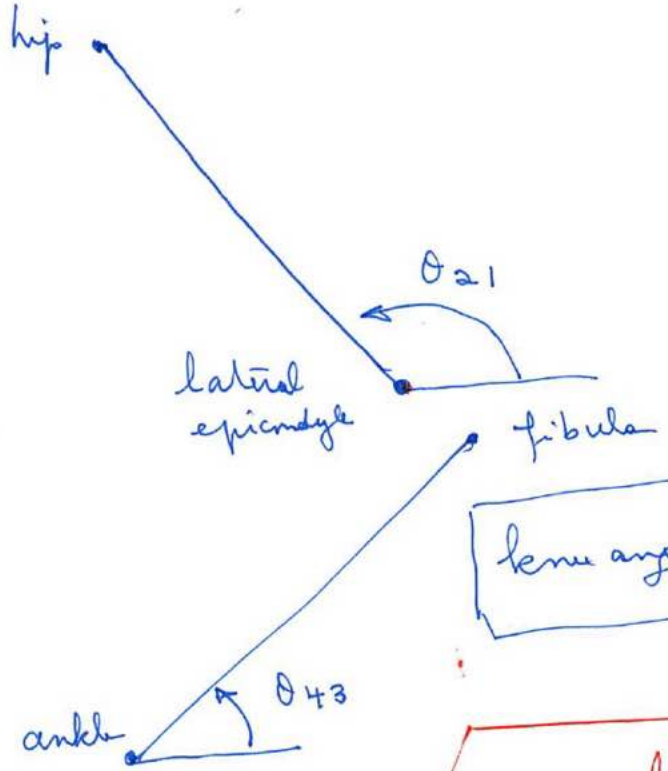
this is not the method used in the book (ankle angle = −16.5°)

ankle angle at frame (4)

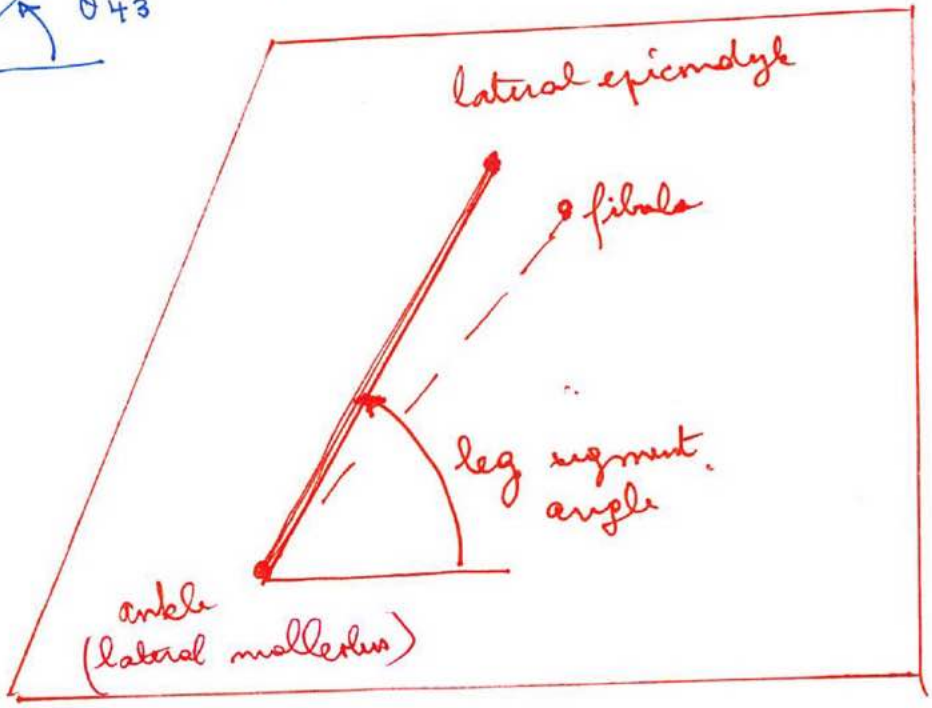




Summary I

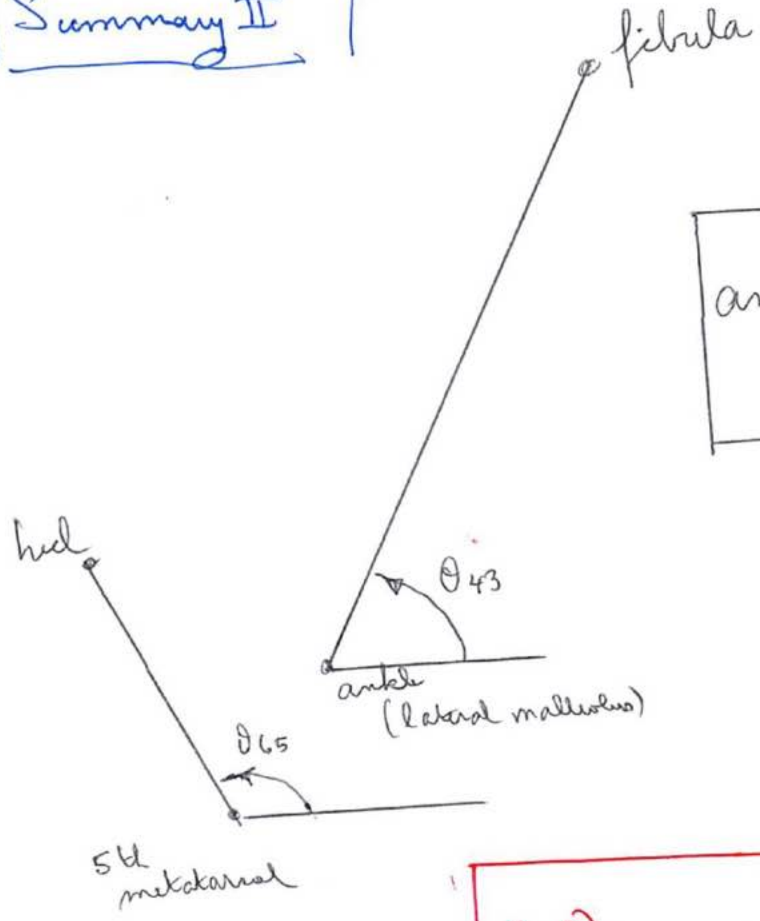


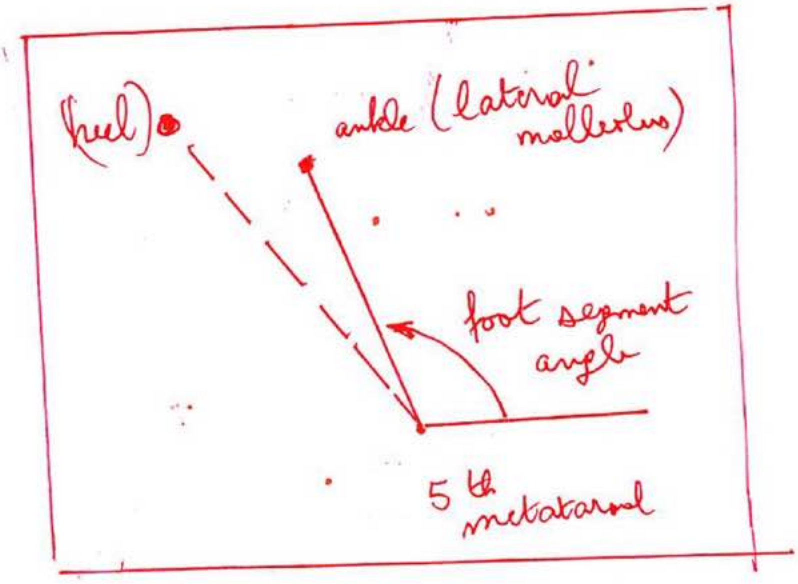
$$\boxed{\text{knee angle} = \theta_{21} - \theta_{43}}$$



Summary II

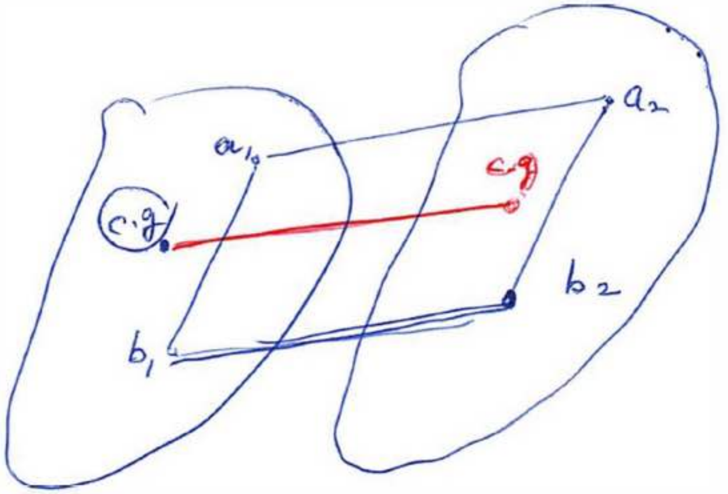
$$\text{ankle angle} = \theta_{43} - \theta_{65} + 90^\circ$$





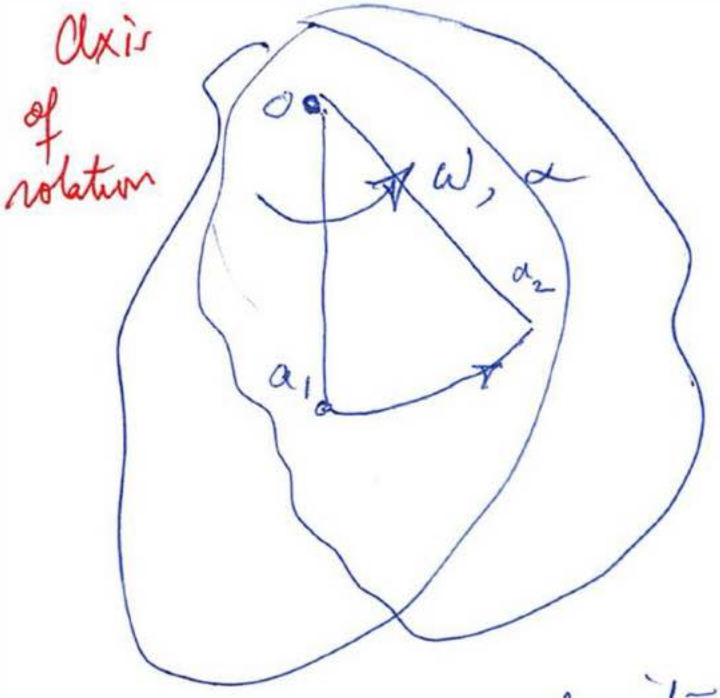
Two Dimensional Motions ①
of a Rigid Body

Ⓘ Translation



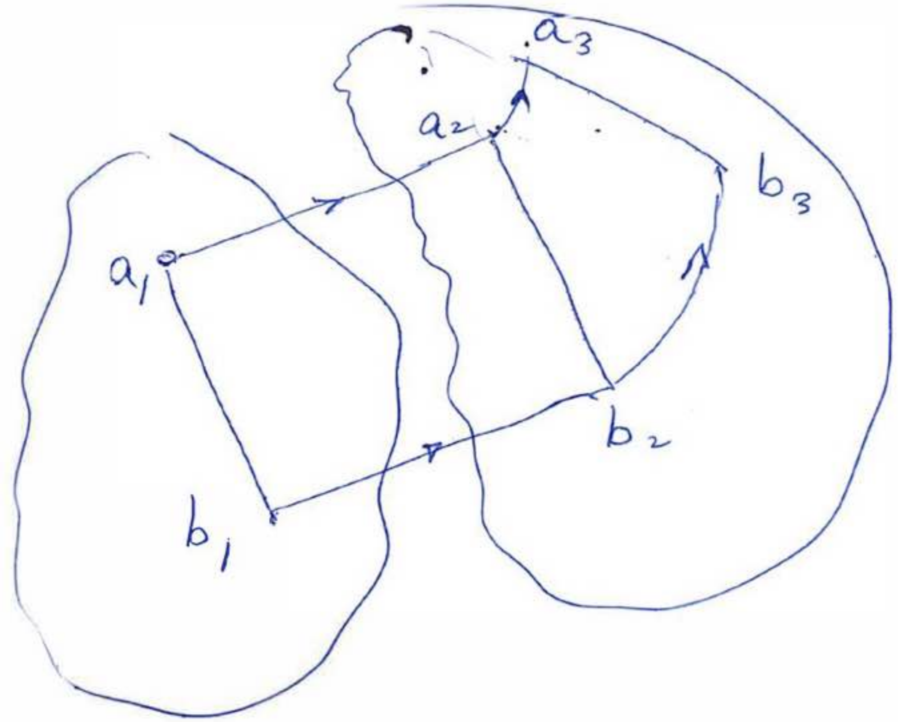
All points have the same linear velocity.

Ⅱ Rotational Motion



ω angular velocity of the body

Ⓘⅈ General Motion

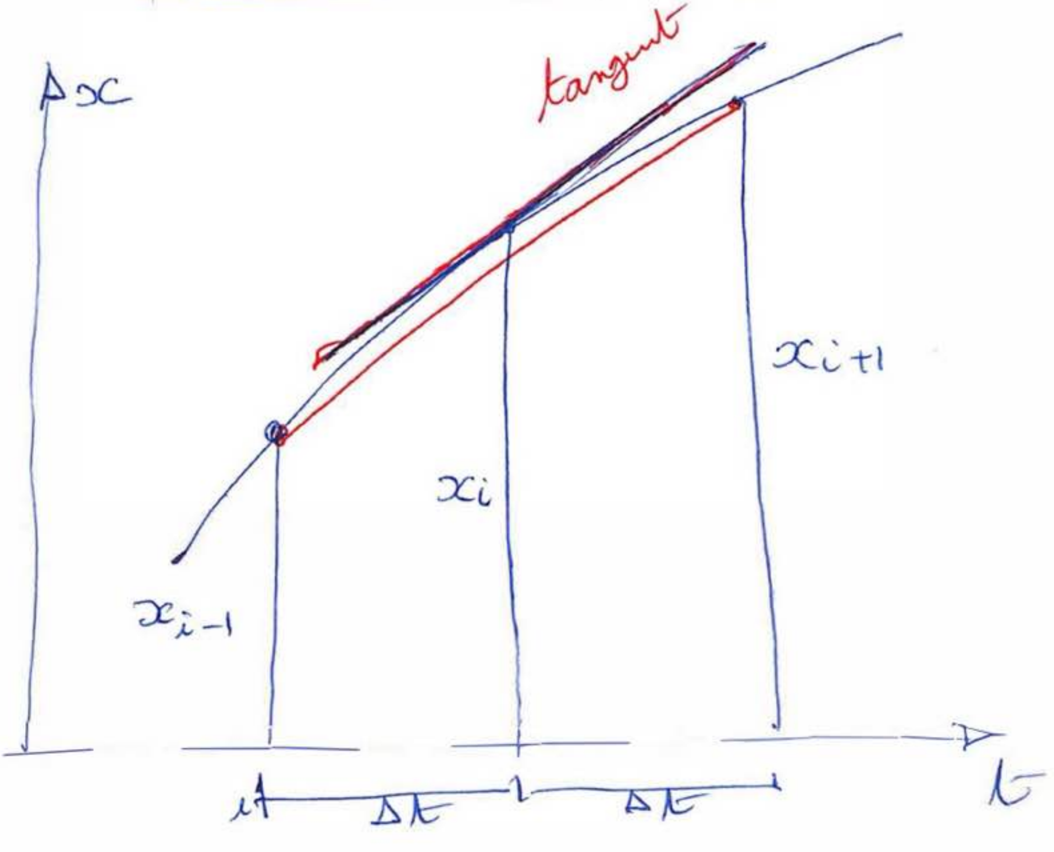


translation

+ Rotation

Numerical Differentiation to calculate linear and angular velocities and accelerations

Ⓘ Linear Velocity of a point

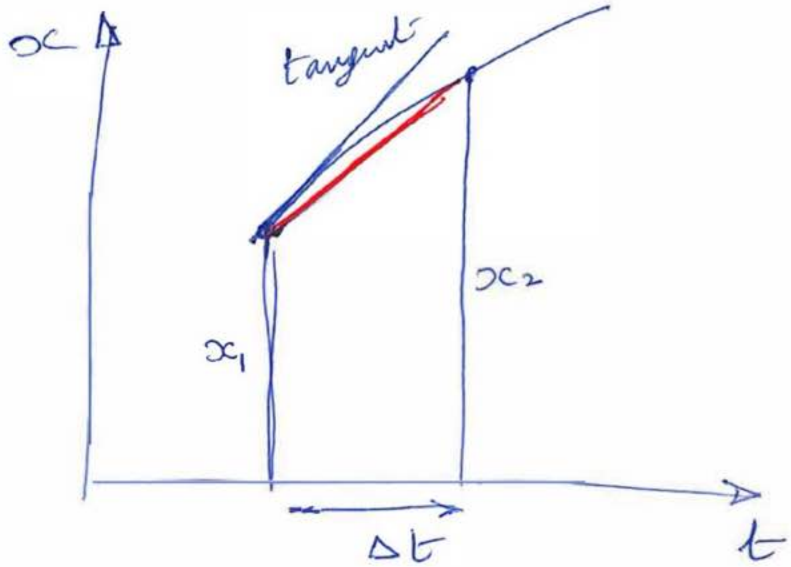


Slope of tangent

$$= V_i = \frac{x_{i+1} - x_{i-1}}{2\Delta t}$$

(central difference)

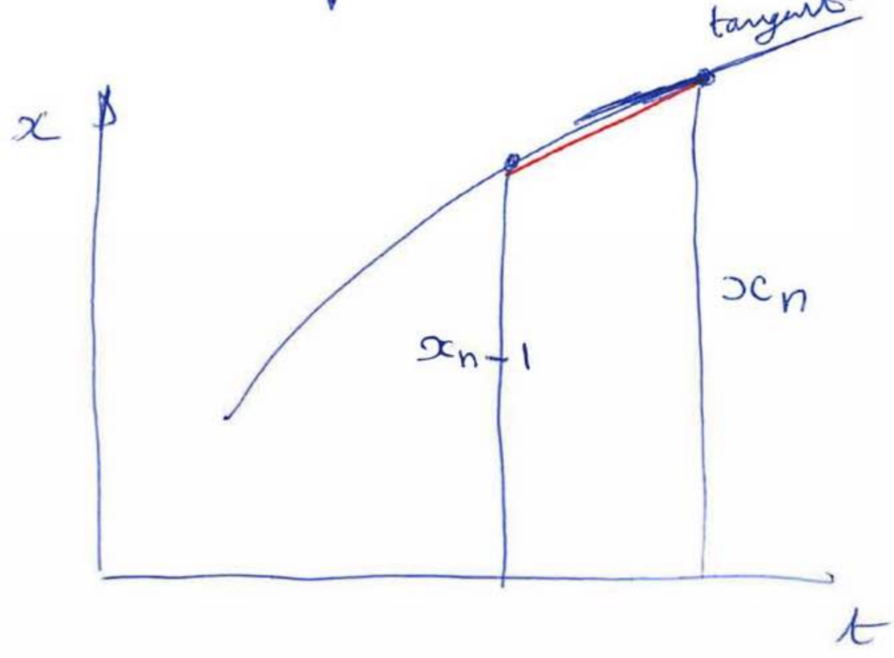
forward difference



$$V_{t=t_1} = \frac{x_2 - x_1}{\Delta t}$$

(Use only at the first point)

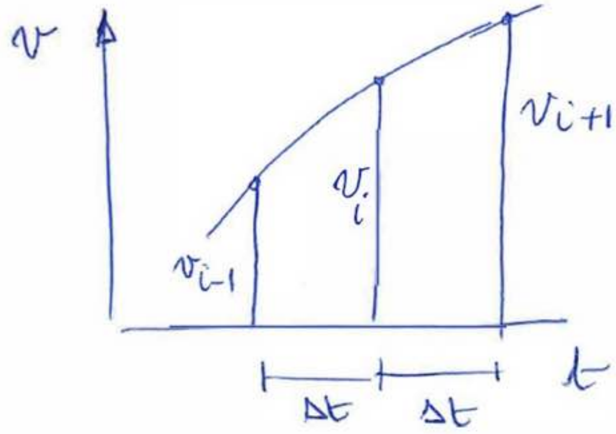
backward difference



$$V_{t=t_n} = \frac{x_n - x_{n-1}}{\Delta t}$$

(Use only at the last point)

(II) Linear acceleration of a point



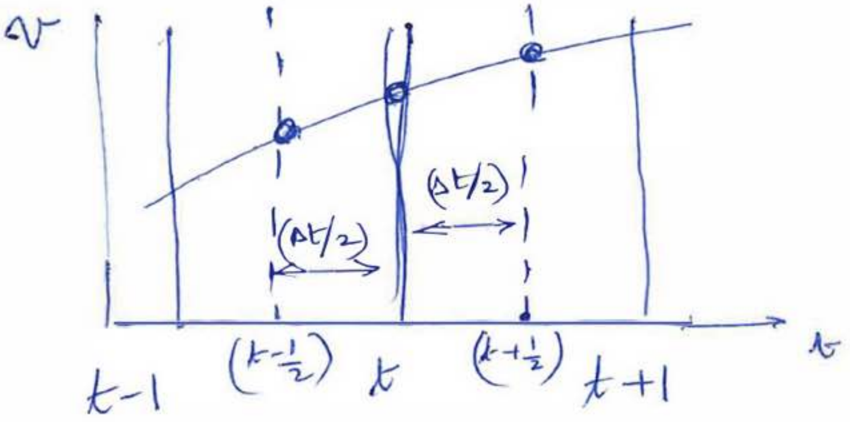
$$a_i = \frac{v_{i+1} - v_{i-1}}{2\Delta t}$$ (3.17) pp 78

$$a_i = \frac{x_{i+2} - x_i - x_i + x_{i-2}}{4\Delta t^2}$$

$$a_i = \frac{x_{i+2} - 2x_i + x_{i-2}}{4\Delta t^2}$$

So, we need 5 successive data points to get accelerations

Another method that uses velocities half way between sample times.



$$a_i = \frac{v_{i+\frac{1}{2}} - v_{i-\frac{1}{2}}}{\Delta t} \qquad \begin{pmatrix}3.18.a\\3.18.b\end{pmatrix}$$

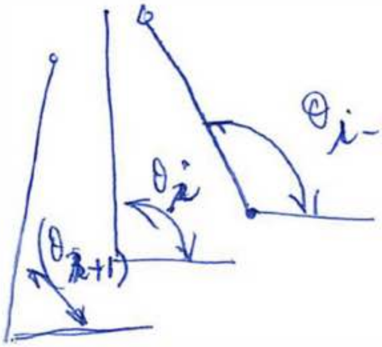
$$a_i = \left(\frac{x_{i+1} - x_i}{\Delta t} - \frac{x_i - x_{i-1}}{\Delta t}\right) \Big/ \Delta t$$

$$\boxed{a_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta t)^2}}$$

(3.18.c)

pp 78

(III) Angular velocity of a rigid body



$$\omega_i = \frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t} \text{ rad/s} \qquad (3.16)$$

$$\alpha_i = \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta t)^2}$$

(angular acceleration of a rigid body)

Example 1

For right knee, table A.2.b pp 306 ⑩

frame	X (m)	V	a
1	0.4075		
2	0.4461		
3	0.4850		
4	0.5235		
5	0.5613		

at frame(4) (Central difference)

$$V_x = \frac{0.5613 - 0.4850}{(2)\left(\frac{1}{69.9}\right)}$$

$$V_x = 2.6667 \text{ m/s}$$

at frame(1) (forward difference)

$$V_x = \frac{0.4461 - 0.4075}{\left(\frac{1}{69.9}\right)}$$

$$V_x = 2.698 \text{ m/s}$$

to find acceleration of knee marker at frame ④

$$a_i = \frac{x_{i+1} - 2x_i + x_{i-1}}{(\Delta t)^2}$$

$$(a_x)_{\text{fram 4}} = \frac{0.5613 - 2(0.5235) + 0.4850}{\left(\frac{1}{69.9}\right)^2}$$

$$\boxed{(a_x)_{\text{fram 4}} = -3.42 \text{ m/s}^2}$$

Summarize table (A.2.[illegible])

$$\underbrace{X \quad VX \quad AX \quad Y \quad VY \quad AY}$$

A.2.b knee & fibula

A.2.a ban rib cage & hip

A.2.c ankle & heel

A.2.d metatarsal & toe

Angular velocity and acceleration of the leg segment

frame	θ	ω	α
1			
2			
3	37.0		
4	36.7		
5	37.3		

$$\omega = \dot{\theta} \text{ at frame } 4 \quad \left(\frac{\theta_{i+1} - \theta_{i-1}}{2\Delta t}\right)$$

$$= \frac{\theta_5 - \theta_3}{2\Delta t}$$

$$= \frac{(37.3 - 37)}{2 \times \frac{1}{69.9}} \times \frac{\pi}{180^\circ}$$

$$\boxed{= 0.183 \text{ rad/s}}$$

Angular acceleration of tibia at frame 4

$$= \frac{\theta_{i+1} - 2\theta_i + \theta_{i-1}}{(\Delta t)^2}$$

$$= \frac{\left[37.3 - 2(36.7) + 37\right]}{\left(\frac{1}{69.9}\right)^2} \times \frac{\pi}{180^\circ}$$

$$= \boxed{76,719 \text{ rad/s}^2}$$

Table	Segment	
3.a	foot	θ
3.b	leg	ω
3.c	thigh	α
3.d	½ HAT	

What about relative joint angular kinematics

(Table A-4) pp341

(knee joint)

frame	θ	ω	α
1	46.7		
2	52.1		
3	56.9		
4	61.0	—	—
5	64.1		

$$\omega_{\text{at frame 4}} = \frac{(64.1 - 56.9)}{(2)\left(\frac{1}{69.9}\right)} \times \frac{\pi}{180}$$

$$= 4.3937 \text{ rad/sec}$$

$$\alpha_{\text{at frame 4}} = \frac{\left[64.1 - 2\times 61.0 + 56.9\right]}{\left(\frac{1}{69.9}\right)^2} \times \frac{\pi}{180}$$

$$= -81.64 \text{ rad/s}^2$$

Anthropometry is the science that deals with the measurements of

a) size
b) mass
c) shape
and d) inertial properties

of the human body.

There are four types of human measurements:

a) length
b) mass weight.
c) location of c.g
d) inertial properties

Because anthropometry is a science, then it is:

- → empirical in nature
- → the methods employed are quantitative to measure some physical dimensions
- → the results are statistical data
- → the value is the biomechanical models that predict human reach, space requirements..

Biomechanical Models

Human Body is considered a system of mechanical links connected at easily identifiable joints

Length segments are obtained by measuring distances between palpable bony landmarks

Drillis and Contini (1966) data

③

Figure 4-1. pp 83

Table 4-1 pp. 86

Length Data

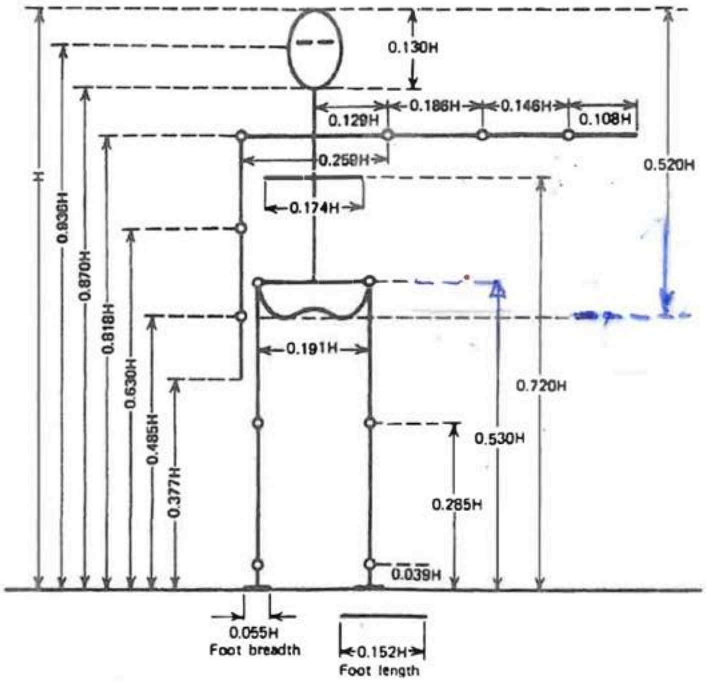


Figure 4.1 Body segment lengths expressed as a fraction of body height *H*

Drillis and Contini data

4.

TABLE 4.1 Anthropometric Data

Segment	Definition
Hand	Wrist axis/knuckle II middle finger
Forearm	Elbow axis/ulnar styloid
Upper arm	Glenohumeral axis/elbow axis
Forearm and hand	Elbow axis/ulnar styloid
Total arm	Glenohumeral joint/ulnar styloid
Foot	Lateral malleolus/head metatarsal II
Leg	Femoral condyles/medial malleolus
Thigh	Greater trochanter/femoral condyles
Foot and leg	Femoral condyles/medial malleolus
Total leg	Greater trochanter/medial malleolus
Head and neck	C7–T1 and 1st rib/ear canal
Shoulder mass	Sternoclavicular joint/glenohumeral axis
Thorax	C7–T1/T12–L1 and diaphragm*
Abdomen	T12–L1/L4–L5*
Pelvis	L4–L5/greater trochanter*
Thorax and abdomen	C7–T1/L4–L5*
Abdomen and pelvis	T12–L1/greater trochanter*
Trunk	Greater trochanter/glenohumeral joint*
Trunk head neck	Greater trochanter/glenohumeral joint*
Head, arms, and trunk (HAT)	Greater trochanter/glenohumeral joint*
HAT	Greater trochanter/mid rib

length of thigh segment

$= 0.530H - 0.285H$

$= 0.245H$

if $H = 180$ cm.

length of thigh segment

$= (0.245)(180) = 44.1$ cm

Whole body density (pp 83–84)

Drillis and Contini developed an expression for body density (d) as a function of ponderal index (c)

- $d = 0,69 + 0,0297\,c$ kg/l (4.1)

$$c = \frac{h}{W^{1/3}}$$

h body height in inches

W body weight in pounds

- $d = 0,69 + 0,9\,c$ kg/l (4.2)

$$c = \frac{h}{W^{1/3}}$$

h body height in meters

W body mass in kgs

Example

person 1 $h = 5$ ft $W = 200$ lbs

person 2 $h = 6$ ft $W = 180$ lb.

for person 1 (short - heavy)

$$C = \frac{h}{W^{1/3}} = \frac{(5 \times 12)}{(200)^{1/3}} = 10.26$$

$$d = 0.69 + 0.0297\,(10.26)$$

$$d = 0.9947 \text{ kg/l}$$

for person 2 (tall - skinny)

$$C = \frac{h}{W^{1/3}} = \frac{6 \times 12}{(180)^{1/3}} = 12.752$$

$$d = 0.69 + 0.0297\,(12.752)$$

$$d = 1.0687 \text{ kg/l}$$

- 1 kg = 2.2 lbs
- 1 lb = 4.45 N
- 1 kg = 9.8 N

Segment Densities

- function of average body density
- Figure 4.2 5 pp 85

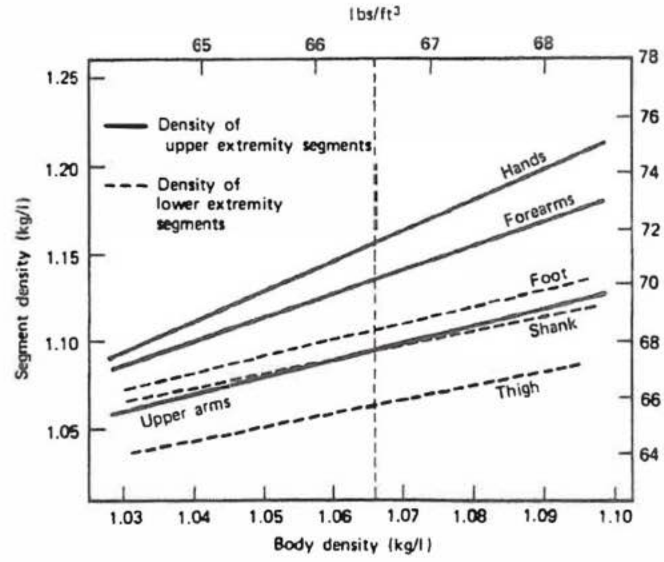
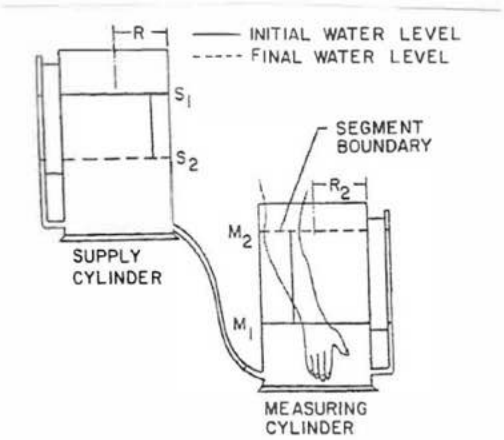


Figure 4.2 Density of limb segments as a function of average body density.

(II) Body Segment Volume and Weight and Segment Mass

In Vivo

* System for Measuring the volume of various body segment



experimental determination

- immersion method
- segment mass = (segment volume) x (Segment density)

* Data for mass → of each segment as a percentage of the total body mass.

TABLE 4.1 Anthropometric Data

Segment	Definition	Segment Weight/Total Body Weight
Hand	Wrist axis/knuckle II middle finger	0.006 M
Forearm	Elbow axis/ulnar styloid	0.016 M
Upper arm	Glenohumeral axis/elbow axis	0.028 M
Forearm and hand	Elbow axis/ulnar styloid	0.022 M
Total arm	Glenohumeral joint/ulnar styloid	0.050 M
Foot	Lateral malleolus/head metatarsal II	0.0145 M
Leg	Femoral condyles/medial malleolus	0.0465 M
Thigh	Greater trochanter/femoral condyles	0.100 M
Foot and leg	Femoral condyles/medial malleolus	0.061 M
Total leg	Greater trochanter/medial malleolus	0.161 M
Head and neck	C7–T1 and 1st rib/ear canal	0.081 M
Shoulder mass	Sternoclavicular joint/glenohumeral axis	—
Thorax	C7–T1/T12–L1 and diaphragm*	0.216 PC
Abdomen	T12–L1/L4–L5*	0.139 LC
Pelvis	L4–L5/greater trochanter*	0.142 LC
Thorax and abdomen	C7–T1/L4–L5*	0.355 LC
Abdomen and pelvis	T12–L1/greater trochanter*	0.281 PC
Trunk	Greater trochanter/glenohumeral joint*	0.497 M
Trunk head neck	Greater trochanter/glenohumeral joint*	0.578 MC
Head, arms, and trunk (HAT)	Greater trochanter/glenohumeral joint*	0.678 MC
HAT	Greater trochanter/mid rib	0.678

total leg 0.161 M

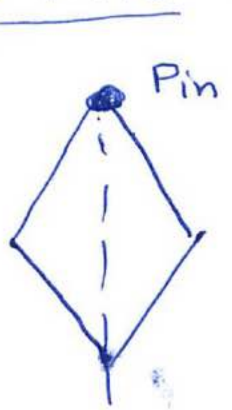
thigh segment (0.1 M)

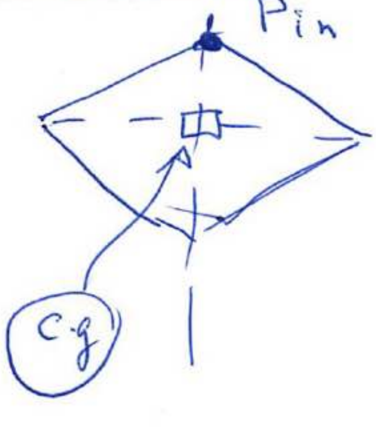
leg segment (0.0465 M)

foot segment (0.0145 M)

Location of Center of Mass

① In- Vitro (Suspension Technique)





② In-Vivo

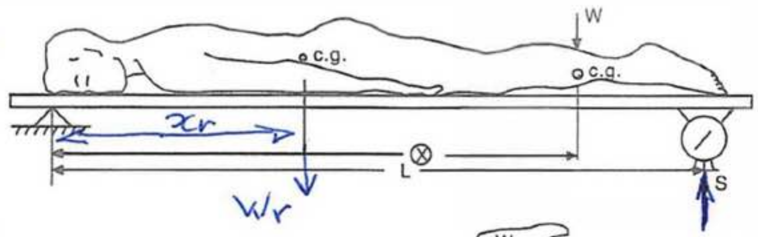
(Moment Substraction method.)

For example to locate experimentally the center of mass of the foot and shank segment.

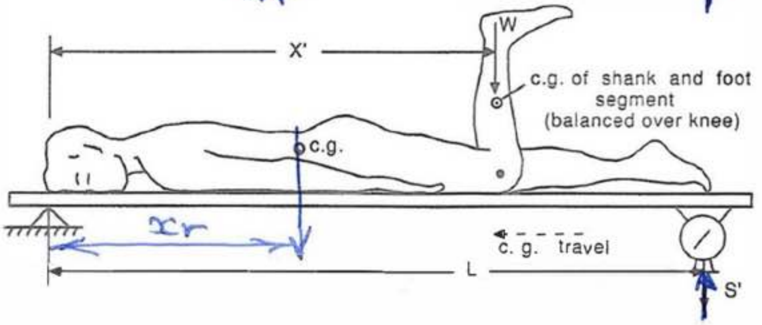
(pp 96-97)

⟶

position 1



position 2



W weight of leg and foot segment

W_r weight of the remainder of the body

$$(W_r)(x_r) + (W)(x) = (S)(L) \quad — ①$$

$$(W_r)(x_r) + (W)(x') = (S')(L) \quad — ②$$

Subtracting (2) from (1):

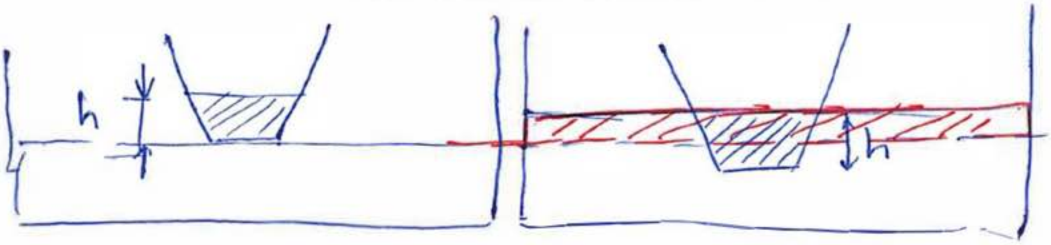
$$W(x - x') = L(S - S')$$

$\Longrightarrow$ if W is known

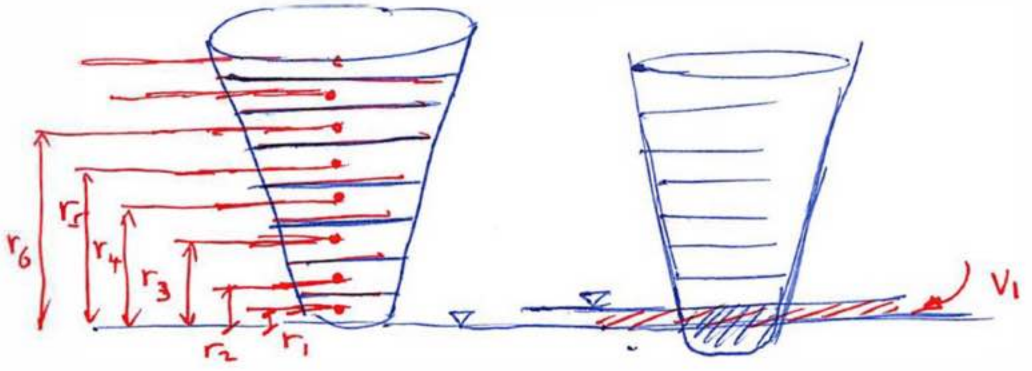
$$\boxed{x = x' + \frac{(S - S')L}{W}} \quad x+$$

③ In-Vivo

Modification of the Immersion Method



So Segment is submerged in known different intervals



$$m_i = \rho V_i$$

$$m_1 r_1 + m_2 r_2 + m_3 r_3 + \dots = \Sigma m_i r_i$$

$$M \bar{r} = \Sigma m_i r_i$$

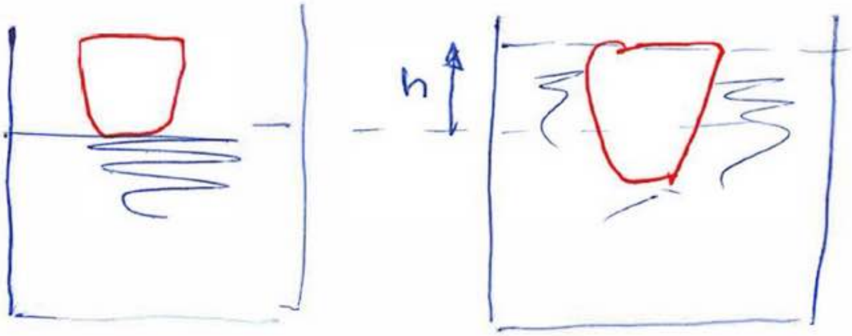
$$\bar{r} = \frac{\Sigma m_i r_i}{M} = \frac{\Sigma V_i r_i}{V}$$

3. Incremental Immersion

- The third method again consists of immersing the body segment in water, but this time we use an *incremental* immersion.

- A good *approximation* can be obtained by:
  - Immersing the segment completely and measuring the volume...
  - Then withdrawing the limb until 1/2 the volume is returned.
  - The water level indicates the location of the center of mass.
  - Only correct if density is uniform.

Approximate Immersion Technique



- $\frac{1}{2}$ of the water displaced is returned by withdrawing the limb

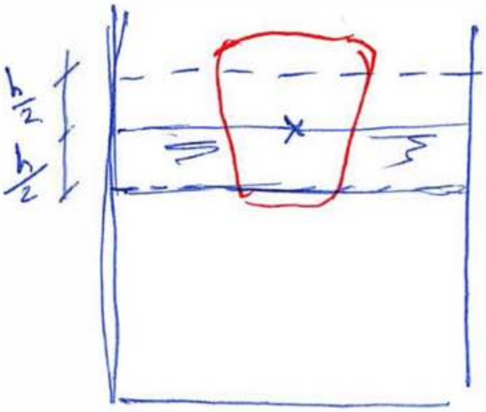
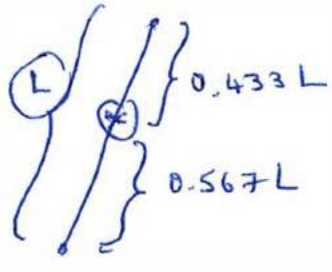


Table 4-1

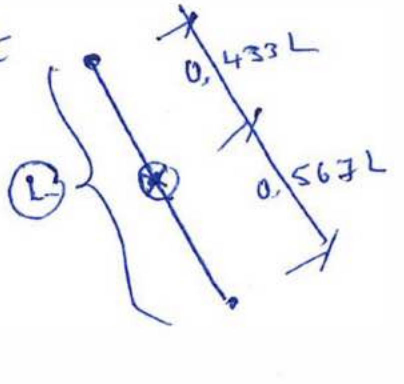
TABLE 4.1 Anthropometric Data

Segment	Definition	Segment Weight/Total Body Weight	Center of Mass/ Segment Length Proximal	Center of Mass/ Segment Length Distal
Hand	Wrist axis/knuckle II middle finger	0.006 M	0.506	0.494 P
Forearm	Elbow axis/ulnar styloid	0.016 M	0.430	0.570 P
Upper arm	Glenohumeral axis/elbow axis	0.028 M	0.436	0.564 P
Forearm and hand	Elbow axis/ulnar styloid	0.022 M	0.682	0.318 P
Total arm	Glenohumeral joint/ulnar styloid	0.050 M	0.530	0.470 P
Foot	Lateral malleolus/head metatarsal II	0.0145 M	0.50	0.50 P
Leg	Femoral condyles/medial malleolus	0.0465 M	0.433	0.567 P
Thigh	Greater trochanter/femoral condyles	0.100 M	0.433	0.567 P
Foot and leg	Femoral condyles/medial malleolus	0.061 M	0.606	0.394 P
Total leg	Greater trochanter/medial malleolus	0.161 M	0.447	0.553 P
Head and neck	C7–T1 and 1st rib/ear canal	0.081 M	1.000	— PC
Shoulder mass	Sternoclavicular joint/glenohumeral axis	—	0.712	0.288
Thorax	C7–T1/T12–L1 and diaphragm*	0.216 PC	0.82	0.18
Abdomen	T12–L1/L4–L5*	0.139 LC	0.44	0.56
Pelvis	L4–L5/greater trochanter*	0.142 LC	0.105	0.895
Thorax and abdomen	C7–T1/L4–L5*	0.355 LC	0.63	0.37
Abdomen and pelvis	T12–L1/greater trochanter*	0.281 PC	0.27	0.73
Trunk	Greater trochanter/glenohumeral joint*	0.497 M	0.50	0.50
Trunk head neck	Greater trochanter/glenohumeral joint*	0.578 MC	0.66	0.34 P
Head, arms, and trunk (HAT)	Greater trochanter/glenohumeral joint*	0.678 MC	0.626	0.374 PC
HAT	Greater trochanter/mid rib	0.678	1.142	—

leg segment

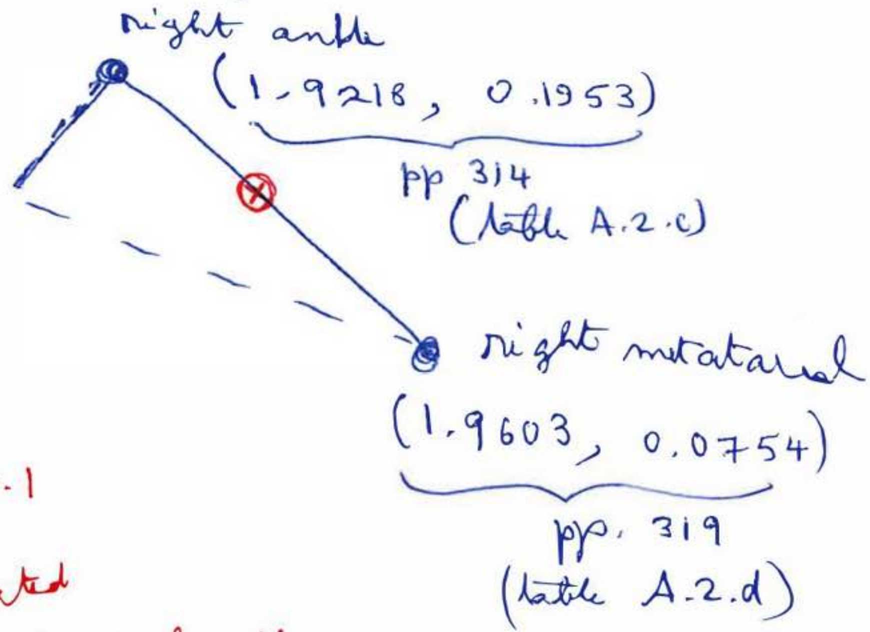


thigh segment



Example ①

Foot Segment at frame 80



from table 4.1

c.g. is located at 50% of the length of the segment

So, coordinates of c.g are:

$$X_{cg} = \frac{1.9218 + 1.9603}{2} = 1.94105$$

$$Y_{c.g} = \frac{0.1953 + 0.0754}{2} = 0.13535$$

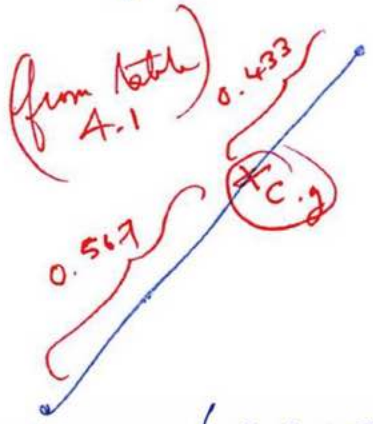
So, go to table (A.3. a) pp 324 for foot segment at frame 80

COM - X = 1.941

COM - Y = 0.135

$\boxed{\text{Example ②}}$

leg segment at frame ④



knee $\underbrace{(0.5235,\ 0.4771)}_{\text{pp 306, table A.2.b}}$

ankle $\underbrace{(0.2054,\ 0.2396)}_{\text{pp 311, table A.2.c.}}$

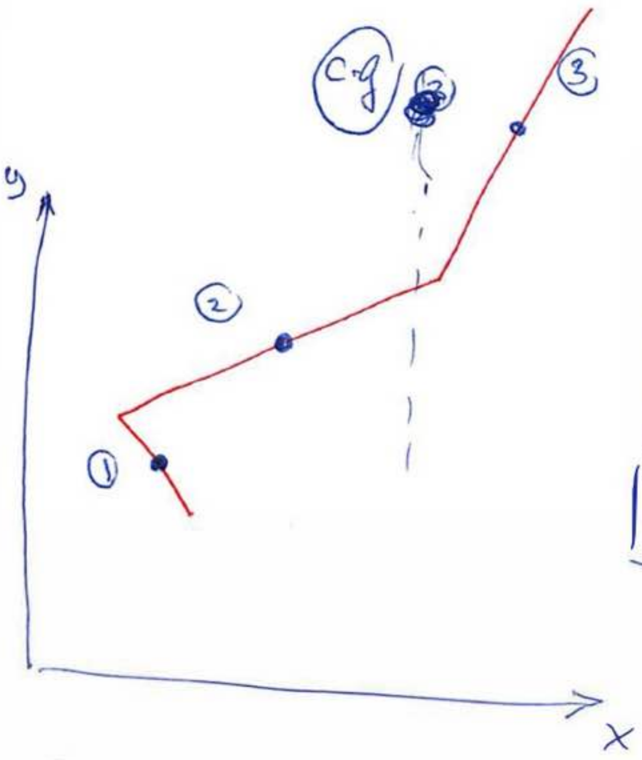
$$x_{cg} = 0.2054 + (0.5235 - 0.2054)(0.567)$$
$$= 0.2054 + (0.3181)(0.567)$$
$$= 0.2054 + 0.180363 = \boxed{0.385763}$$

$$y_{c.g} = 0.2396 + (0.4771 - 0.2396)(0.567)$$
$$= 0.2396 + (0.2375)(0.567)$$
$$= 0.2396 + 0.134663 = \boxed{0.374263}$$

So, go to table (A.3.b) pp 326
for leg segment at frame 4
CoM − x = 0.386
CoM − y = 0.374

Center of Mass of 3 body Segments

at frame ④



	M	$\bar{X}$	$\bar{Y}$	
1	0.0145	0.195	0.180	pp 321
2	0.0465	0.386	0.374	pp 326
3	0.1	0.518	0.655	pp 331

$$\bar{X} = \frac{m_1 x_1 + m_2 x_2 + m_3 x_3}{m_1 + m_2 + m_3}; \quad \bar{y} = \frac{m_1 y_1 + m_2 y_2 + m_3 y_3}{m_1 + m_2 + m_3}$$

$$\bar{X} = \frac{(0.0145)(0.195) + (0.0465)(0.386) + (0.1)(0.518)}{(0.0145 + 0.0465 + 0.1)}$$

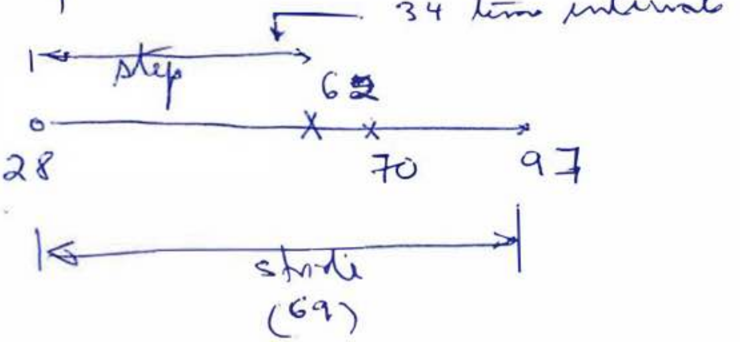
$$\boxed{\bar{X} = 0.450785}$$

$$\bar{Y} = \frac{(0.0145)(0.180) + (0.0465)(0.374) + (0.1)(0.655)}{(0.0145 + 0.0465 + 0.1)}$$

$$\boxed{\bar{Y} = 0.531062}$$

Discussion of problem 2.(b) on page 105

HCR frame 28
TOR frame 70
HCR frame 97
} 69 intervals of time



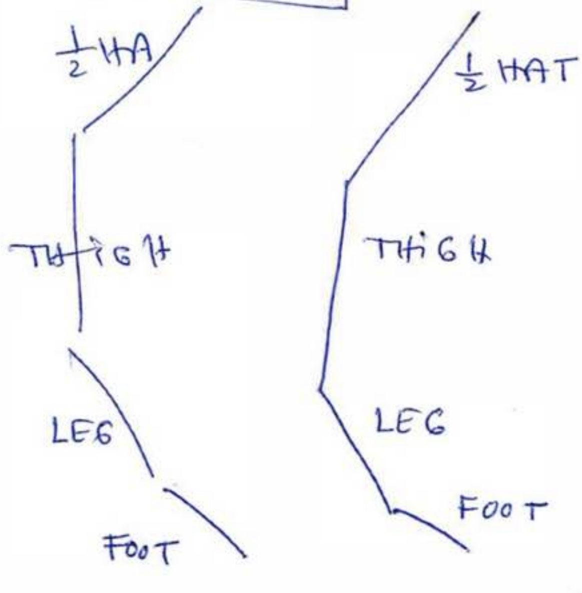
at frame 28: $x_{heel} = 1.2280$ pp. 312

at frame 97: $x_{heel} = 2.6422$ pp 314

stride length = 1.4142 m

step length = 0.7071 m

Right and left sides of body are out of phase with each other by 34 frames (time intervals) at frame (30)



Right (from Left at frame 30)

(Coordinate of right part at frame 64 − step length)

(30 + 34)

For example thigh segment

X c.g Right thigh at frame 30 = (1.110, 0.67) m

page 332

to find c.g. of left thigh at frame 30 :-

$$X_{c.g} = 1.655 - 0.7071 = \cancel{0.9449}$$

0.9479 m

X coord. of c.g. of right thigh at frame 64

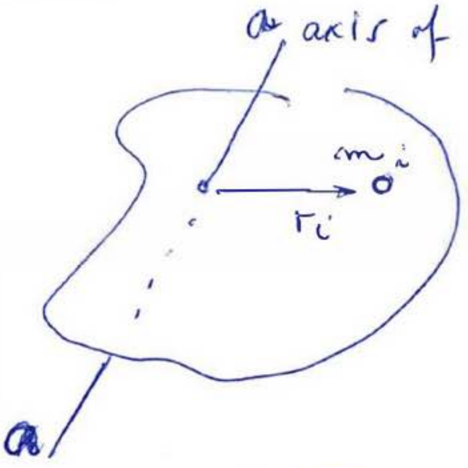
step length

So:

c.g. of left thigh at frame 30 = (0.9479, 0.663)

Y coord of c.g. of right thigh at frame 64

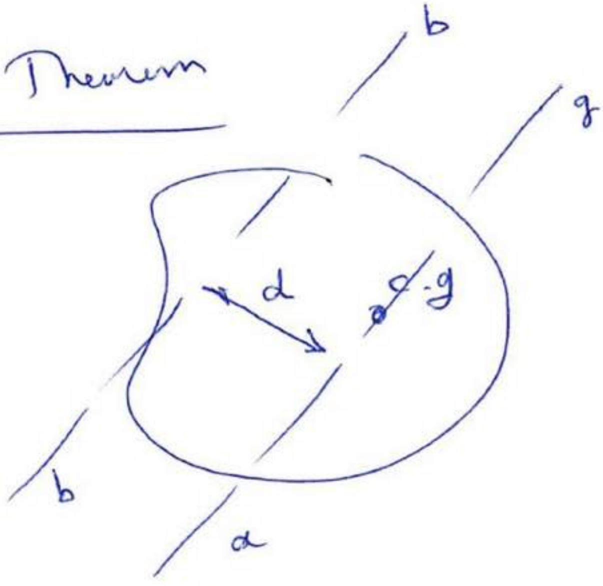
Mass Moment of Inertia and Radius of Gyration



r_i shortest distance from m_i to axis

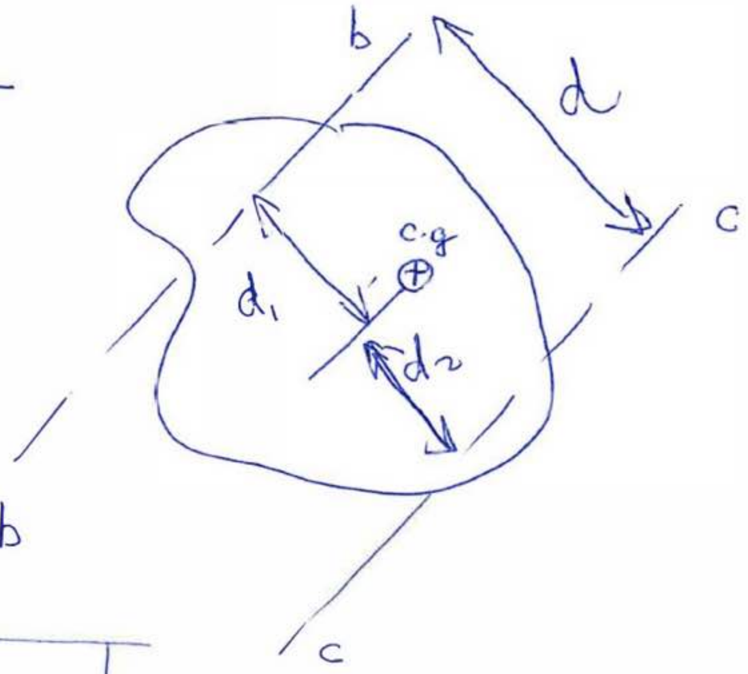
$$I_{aa} = \sum m_i r_i^2$$

Parallel axis Theorem



$$I_{bb} = I_{gg} + Md^2$$

Problem



Given I_{bb}, determine I_{cc}
M
and location of c.g

It is incorrect to calculate

$$\left(\xcancel{I_{cc} = I_{bb} + M d^2} \right)$$

You need first to calculate I_{gg}

$$I_{bb} = I_{gg} + M d_1^2$$

then you calculate I_{cc}

$$I_{cc} = I_{gg} + M d_2^2$$

$$I_{cc} = I_{bb} - M d_1^2 + M d_2^2$$

Radius of Gyration

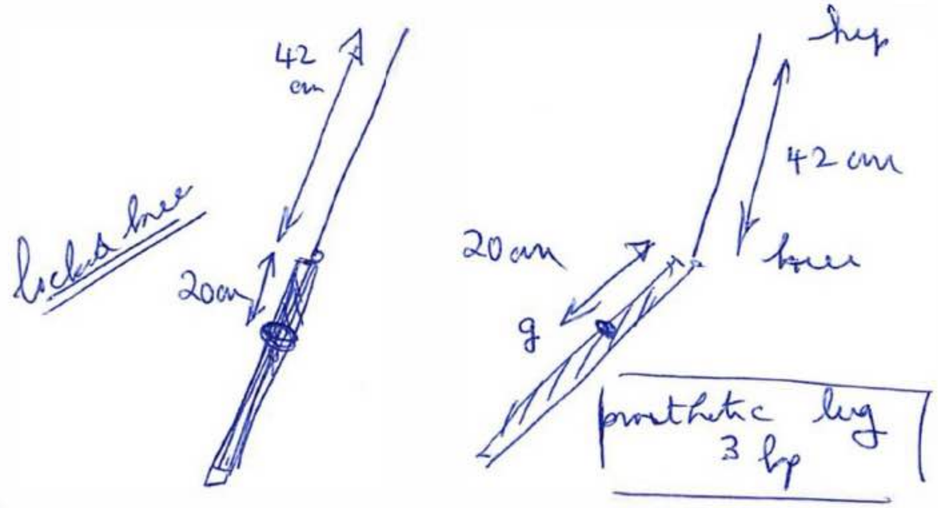
$$I = M k^2$$

$$I_{aa} = M k_{aa}^2$$

and $I_{gg} = M k_{gg}^2$

So: k_{gg} is the radius of gyration about axis g-g. (units of length)

Example 4.3 page 91



k_g of leg about its center of mass = 14.1 cm

What I of the prosthetic leg @ knee joint

$$I_{kk} = I_{gg} + M d^2$$

$$= M(14.1)^2 + M(20)^2$$

$$= 3[0.141]^2 + 3[0.2]^2$$

$$= \boxed{0,06} + 0,12 = \underline{0,18 \text{ kg } m^2}$$

in a locked knee position:

$$I_{hip} = I_{gg} + M d_1^2$$

$$= 0.06 + 3[0.2 + 0.42]^2$$

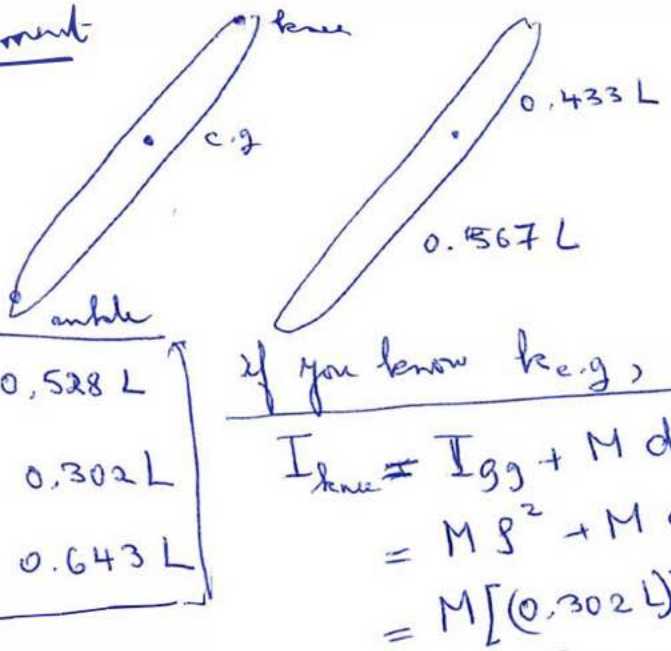
$$I_{hip} = 1,21 \text{ kg} \cdot m^2$$

So $I_{hip} \simeq 20$ time I_{gg}

TABLE 4.1 Anthropometric Data

Segment	Definition	Segment Weight/Total Body Weight	Center of Mass/ Segment Length Proximal	Distal	Radius of Gyration/ Segment Length C of G	Proximal	Distal	Density
Hand	Wrist axis/knuckle II middle finger	0.006 M	0.506	0.494 P	0.297	0.587	0.577 M	1.16
Forearm	Elbow axis/ulnar styloid	0.016 M	0.430	0.570 P	0.303	0.526	0.647 M	1.13
Upper arm	Glenohumeral axis/elbow axis	0.028 M	0.436	0.564 P	0.322	0.542	0.645 M	1.07
Forearm and hand	Elbow axis/ulnar styloid	0.022 M	0.682	0.318 P	0.468	0.827	0.565 P	1.14
Total arm	Glenohumeral joint/ulnar styloid	0.050 M	0.530	0.470 P	0.368	0.645	0.596 P	1.11
Foot	Lateral malleolus/head metatarsal II	0.0145 M	0.50	0.50 P	0.475	0.690	0.690 P	1.10
Leg	Femoral condyles/medial malleolus	0.0465 M	0.433	0.567 P	0.302	0.528	0.643 M	1.09
Thigh	Greater trochanter/femoral condyles	0.100 M	0.433	0.567 P	0.323	0.540	0.653 M	1.05
Foot and leg	Femoral condyles/medial malleolus	0.061 M	0.606	0.394 P	0.416	0.735	0.572 P	1.09
Total leg	Greater trochanter/medial malleolus	0.161 M	0.447	0.553 P	0.326	0.560	0.650 P	1.06
Head and neck	C7–T1 and 1st rib/ear canal	0.081 M	1.000	— PC	0.495	0.116	— PC	1.11
Shoulder mass	Sternoclavicular joint/glenohumeral axis	—	0.712	0.288	—	—	—	1.04
Thorax	C7–T1/T12–L1 and diaphragm*	0.216 PC	0.82	0.18	—	—	—	0.92
Abdomen	T12–L1/L4–L5*	0.139 LC	0.44	0.56	—	—	—	—
Pelvis	L4–L5/greater trochanter*	0.142 LC	0.105	0.895	—	—	—	—
Thorax and abdomen	C7–T1/L4–L5*	0.355 LC	0.63	0.37	—	—	—	—
Abdomen and pelvis	T12–L1/greater trochanter*	0.281 PC	0.27	0.73	—	—	—	1.01
Trunk	Greater trochanter/glenohumeral joint*	0.497 M	0.50	0.50	—	—	—	1.03
Trunk head neck	Greater trochanter/glenohumeral joint*	0.578 MC	0.66	0.34 P	0.503	0.830	0.607 M	—
Head, arms, and trunk (HAT)	Greater trochanter/glenohumeral joint*	0.678 MC	0.626	0.374 PC	0.496	0.798	0.621 PC	—
HAT	Greater trochanter/mid rib	0.678	1.142	—	0.903	1.456	—	—

Leg Segment



$k_{knee} = 0,528\,L$

$k_{c.g} = 0.302\,L$

$k_{ankle} = 0.643\,L$

If you know $k_{c.g}$, can you find k_{knee}?

$$I_{knee} = I_{gg} + M d^2$$

$$= M \rho^2 + M d^2$$

$$= M\left[(0.302\,L)^2 + (0.433\,L)^2\right]$$

$$= M L^2\left[0.091204 + 0.187489\right]$$

$$I_{knee} = M\ 0.278693\ L^2$$

$$k_{knee} = \sqrt{0.278693\,L^2} = \boxed{0.5279\,L}$$

Rotational Equations of Motion (REOM)

Always true : $\sum M_G = I_G \alpha$

<u>For rotational motions only</u>:

$$\sum M_0 = I_0 \alpha$$

<u>What is $\sum M_0$?</u>

Moments of all loads (forces and moments) about the axis of rotation.

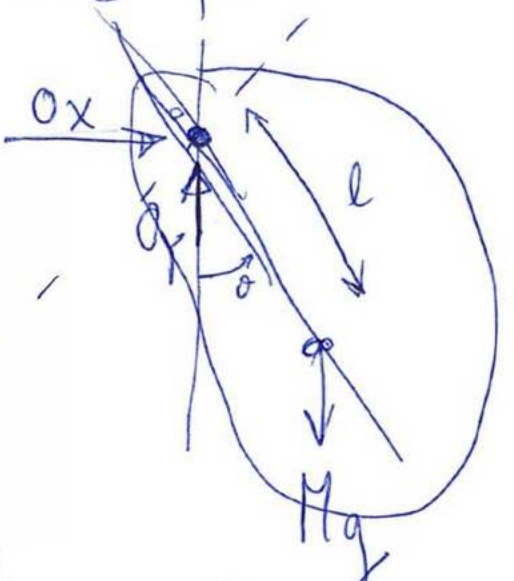
* This is <u>different</u> than the mass moment of inertia about the axis of rotation, I_{00}

How to determine experimentally the mass moments of inertia of segments?

Method ① (In-Vitro)

"Pendulum Axis"

- Cadavers swing as a pendulum around anatomically identified axes, and measure the frequency of oscillation



$$\sum M_o = I_o \alpha$$

$$-Mgl\sin\theta = I_o \ddot{\theta}$$

$$\ddot{\theta} + \frac{Mgl}{I_o}\theta = 0$$

$$\omega^2 = \frac{Mgl}{I_o} = \frac{Wl}{I_o}$$

(Planar Motion)

$$I_o = \frac{Wl}{\omega^2} = \frac{Wl}{4\pi f^2}$$ $\rightarrow$ {equation 3.4, pp 76 from Chaffin book}

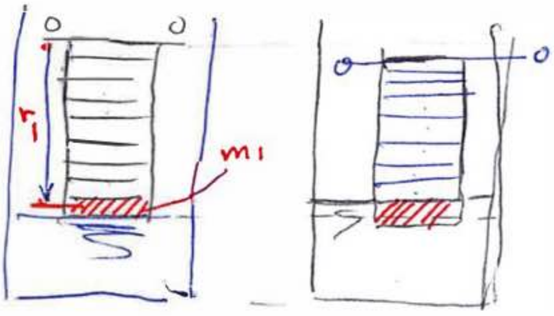
Note that: $\omega = 2\pi f$

and $\tau = \frac{1}{f} = \frac{2\pi}{\omega}$

Method ②

(In Vivo - Extremities only)

(Incremental Immersion Technique)



$$I_{oo} = \sum m_i r_i^2$$

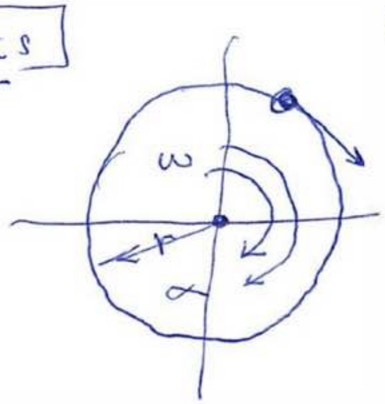
Measure incremental volumes obtained as a body segment is submerged in water in discrete steps.

Then use parallel-axis theorem to find I_{gg},

$$I_{oo} = I_{gg} + M\bar{r}^2$$

(30)

Notes

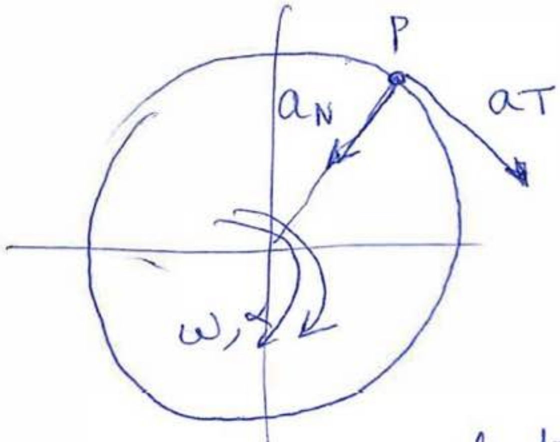


particle (P)

$v_p = \omega r$

(velocity is tangential to the path)

particle moving in a circular path



Acceleration of the particle has 2 components,

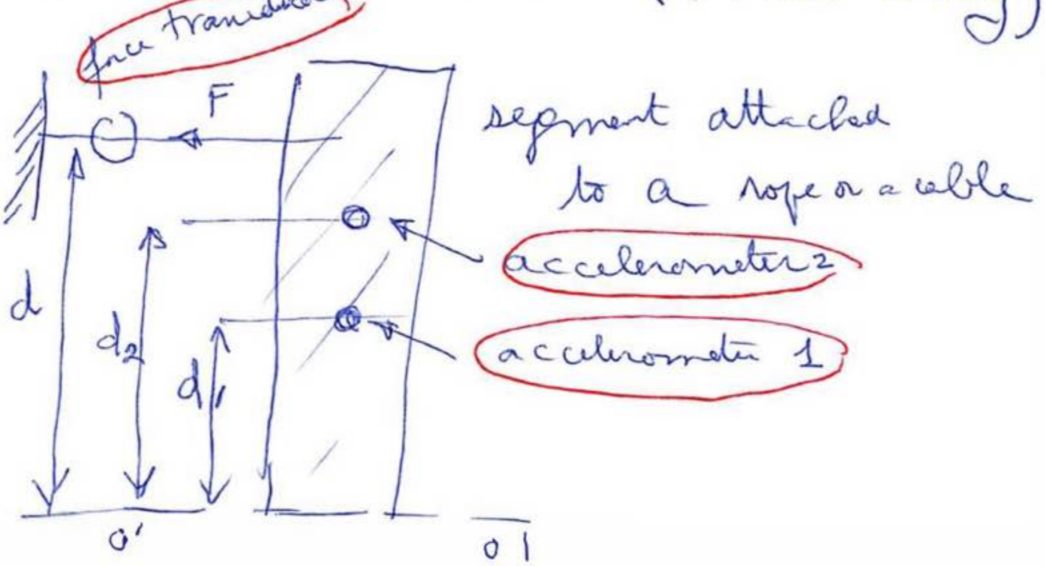
$$a_N = \omega^2 r$$

$$a_T = \alpha r$$

Method ③

(IN-Vivo)

Quick Release Method (Extremities only) (pp 97)



① segment placed on a plane

② tension is exerted onto a cord connected to a force transducer

③ release mechanism is actuated

④ segment is subject to a pure rotation

⑤ two different linear accelerometers are placed on the segment at two different distances from the axis of rotation to measure tangential accelerations

So, you are exerting a torque against a restraining cord. The cord is suddenly released and the angular acceleration of the segment is measured.

Note that at $t=0^+$: $\omega = 0$

$\alpha = \alpha$

$$\text{So,}\quad \left.\begin{aligned}(a_T)_1 &= \alpha\, d_1\\ (a_T)_2 &= \alpha\, d_2\end{aligned}\right\}$$

$$\boxed{\alpha = \frac{(a_T)_2 - (a_T)_1}{d_2 - d_1}}$$

measured experimentally

next, rotational equation of motion

$$\sum M_{o'o'} = I_{o'o'}\,\alpha$$

$$\boxed{I_{o'o'} = \frac{(F)(d)}{\alpha}}$$

NOTES

① Anthropometric data for biomechanical studies in industry is used to define reach and space requirements for a particular population or equipment user.

② Anthropometric data provide new insights into the cause and control of mechanical trauma in the workplace

③ Anthropometric data provided new data on human strength and joint range of motion

④

Table 3.13 Radius of Gyration (K) as a Ratio (C) of Segment Length (L); from Chandler et al., 1975

Segment	Motion Axis	Link	C
Head	x	Head Length	.32
	y		.31
	z		.34
Torso	x	Torso length (Suprasternale to trochanterion)	.43
	y		.35
	z		.21
Upper arm	x	Acromion to radiale	.26
	y		.25
	z		.10
Hand	x	Hand breadth	.50
	y		.46
	z		.27
Thigh	x	Trochanterion height to fibular height	.28
	y		.28
	z		.12
Shank	x	Fibular height	.28
	y		.28
	z		.08
Foot	x	Foot length	.26
	y		.25
	z		.12

Axes to allow the motion are defined as follows:
x = sagittal plane motion,
y = frontal plane motion,
z = horizontal plane motion (around long axis).

approximation

$$I = M\rho^2 \qquad (\rho = \text{radius of gyration})$$

from table above

$$\rho = C L \quad \text{and} \quad C \simeq 0.3$$

So

$$I = M(0.3L)^2$$

$$I = 0.09\, ML^2$$