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Developing Academic Language

Develop Mathematics Concepts and Vocabulary for English Learners

RATIONALE FROM RESEARCH

People use different types of language in the different contexts of their lives. Mathematics, as an academic discipline, is made up of concepts that are most effectively discussed with proficiency in academic language.

Academic language in mathematics instruction includes the specialized words and phrases related to content, procedures, the activity of learning, and expression of complex thinking processes. Echevarria, Vogt, and Short (2004) explain Cummins's term, *cognitive academic language proficiency* (CALP), as "the abstract language abilities required for academic work. A more complex, conceptual, linguistic ability that includes analysis, synthesis and evaluation" (p. 221). Barnett-Clarke and Ramirez (2004) point out, "Not only do students need explicit instruction to read and write mathematical symbols and words, they also need to learn how to express mathematical ideas orally and with written symbols" (p. 57). Rubenstein and Thompson (2001) add that being able to read symbols is linked to understanding. Mathematics content teachers along with English as a second language and support teachers can collaborate in supporting

English learners in acquiring academic English. "On the one hand, content area teachers must recognize and target the key language and literacy practices inherent in their disciplines—such as explaining and arguing with evidence—to enhance the engagement of ELLs with rich content. On the other hand, to help their students to grow, ESL teachers must cultivate a deeper knowledge of the "discipline-specific language and literacy practices that ELLs need in order to perform the activities germane to those disciplines (Council of Chief State School Officers, 2012, p. 4).

Teachers use strategies to develop both mathematical academic language and mathematical ideas. Learning new academic terms is a complex endeavor. Students benefit from multiple exposures to new terms in the course of meaningful activities (Bielenberg & Wong Fillmore, 2005) and from multiple support systems, including activities with visual and tactile scaffolding, as is illustrated in the following teaching example.

In the following teaching example, a set of typical textbook word problems for the second grade prompts the teacher to develop a special lesson.

MATHEMATICS TEACHING EXAMPLE

Addition and Subtraction Word Problems

Number Riddles

Solve each riddle.

1. Kai has 6 dollars. Her brother has 10 dollars. How many more dollars does the brother have?
2. Kai has 40 baseball cards. Her brother has 15 fewer cards. How many baseball cards does her brother have?
3. Kai is 10 years old. Her brother is 3 years younger. How old is the brother?
4. Kai bought a special book for 5 dollars. Did this increase or decrease her savings? *Fill in the blanks:*

This ______ creased her money by ______ dollars. She had ______ dollars left.
(in- / de-) (more / fewer)

Jake Blue teaches a second-grade class that includes several English learners of varying fluency levels. He is modifying a textbook lesson for his class to make the problems meaningful and understandable. The word problems address new, important mathematical relationships that also relate to studying data. He prepares posters with the following headings to write notes with the class: More, Less, Fewer, Increase, Decrease, Compare. As he prepares the lesson, he decides that his main goals for the first day will be to work on "more" and "less/fewer" relationships. They will save the other new terms and the word problems for a future lesson.

As the children sit down on the rug at the front of the room, Mr. Blue starts telling a story to engage the students. "When I was in second grade, I saved up money and one day I had five dollars. Help me count these dollars: one, two, three, four, five. My older brother liked to tease me, and he told me that he had two more dollars than I did. We can write 'I had five dollars. My brother had two more dollars than me.' I will draw a diagram of my five dollar bills all lined up."

"Now I am wondering, how many dollars did my brother have? How many dollars should I draw for my brother? Talk to your neighbor for a minute.... Does he have two dollars? I'll take a quiet hand from someone who can explain." Many students declare that the brother had seven dollars, some showing "five and two more" on their fingers or with linking cubes. Mr. Blue draws his brother's seven dollars next to his five dollars and labels the diagram, with a line showing that they both have at least five dollars.

Figure 2.1 Mr. Blue's diagram comparing the number of dollar bills.

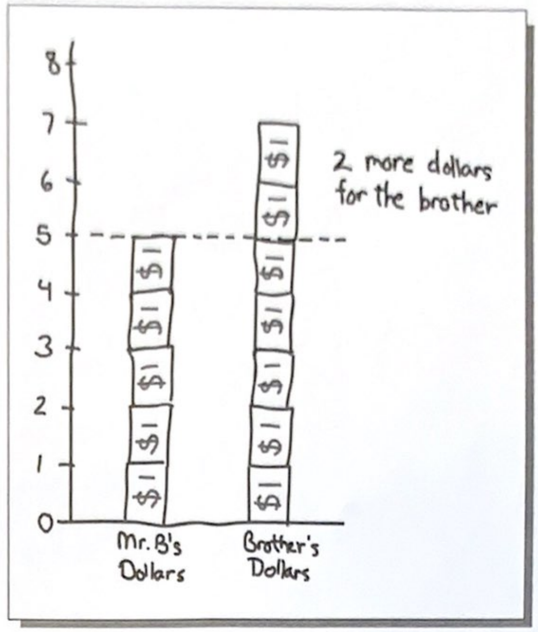

Illustration by Celia Stevenson.

"We can use our linking cubes and also diagrams like this (Figure 2.1) to compare two numbers to see how they are different, to find the difference. What was the difference between my money and my brother's money? Several students share their ideas. Then, Mr. Blue summarizes. "Yes, the difference was the 'two more dollars' that he had. Our amounts of money were different by two dollars." Then, Mr. Blue writes the expression "5 + 2" on the "More" poster and explains, "Today we'll be working on how we use the words more and less in math problems."

"What if I had saved a different number of dollars? Let's do some examples. I'll say a number for my dollars. AJ, you go first and use blue linking cubes to show my dollars. Then, everyone think how many yellow cubes should Pablo show for my brother's dollars? Remember, my brother has two more than my number, like these two red cubes more. I'll start with ten. What are you thinking for two more than ten?" Mr. Blue does several turns and asks the whole class to respond together. Each time, two students show their cube towers next to one another. Gradually, Mr. Blue draws attention to the "more cubes" by asking the students to use two red cubes to show "the brother's extras." He comments, "My blue cube tower needs two more cubes to match my brother's tower. The difference is two cubes."

Next, Mr. Blue asks students to model a problem with the starting amount of six dollars and the difference "four more." He asks, "What should we record if I have six dollars and my brother has four more dollars than me?" (6 + 4). Mr. Blue talks aloud as he constructs a bar diagram, emphasizing that in math problems like this, "We can think about the amount that is the same, or that 'matches,' and then we can tell 'the more part' and add to find the total of the larger amount."

"Now let's all practice saying the whole equation. For example, I could say, 'Four more than six equals ten.' Let's go back to our 'two more than' rule. [He holds up a printed card.] I'll say a number and then I'll call on someone who will say the whole equation. For example, if I say 'Six,' then you would say, 'Two more than six equals eight.'" During oral practice, Mr. Blue records a number sentence for what each student says. Then, he writes "6 + 2 = 8" and tells the students that one way to read this number sentence is to say, "Two more than six equals eight." He asks, "Who noticed that the order of the numbers in the equation and of the sentence on the poster are not the same?" He works through a few examples with the class, and returns to having students show linking cube representations to point out the amount that matches and using the words difference, how they are different, and how much more.

To shift attention to problems where the difference needs to be found, Mr. Blue says, "Now I'm going to use the word more again, but this time we will need to find out something else, so listen carefully. Once I had five dollars and my brother had seven dollars. How many more dollars did my brother have?" Mr. Blue draws two bars on the poster as a visual tool for giving meaning to the problem question.

"What do we need to figure out?" Mr. Blue encourages class discussion and focuses students' attention on comments such as "How they are different," "How much more money," and "How much extra money." The students practice labeling the diagram and pointing out the part of the diagram that refers to the difference. Mr. Blue says, "This part is the answer to the question, 'How many more dollars?'"

"Talk to your neighbor about how you figured out the answer." Students share strategies that include counting up to see how many more dollars Mr. Blue needed to match his brother, thinking about what number goes with five to equal seven, and modeling with cubes. Mr. Blue reminds the students that their ways can be written both with the number sentences $5 + 2 = 7$ and $7 - 5 = 2$, which he records on the poster. But, he realizes that future lessons will need to build meaning for the idea of using subtraction to represent "how many more" and "the difference between" when two amounts are given.

"Now, see what is different about this problem: Once I had five dollars and my brother had seven dollars. How many fewer dollars did I have? How is this problem different?" The class refers to the bar diagram and points to who has fewer dollars, and who has more. Mr. Blue says, "Tell your partner two things that you know about the problem that goes with this diagram. Put up a finger when you hear your partner say 'different,' 'difference,' 'less,' 'fewer,' or 'more.'" Afterward, Mr. Blue told the class, "I heard several of you notice that I would need to get two more dollars to equal my brother's seven dollars and that the difference is two dollars, just like in our other problem. Alan, please use our diagram to talk about that idea.... I heard some students say, 'Mr. Blue has two fewer dollars.' Kamilah, please use our diagram to show us why that idea makes sense."

"We should talk more about our other words for today, less and fewer. Fewer and less both tell us that there isn't as much. It's the opposite of having more. Sometimes problems say 'two less than' or 'two fewer' instead of telling about 'more.' For example, our problem could say that my brother had seven dollars and I had two fewer dollars. Then the question is how much money did I have; or, how much is two fewer dollars than seven dollars? Look, if I write $7 - 2$, we can read this as 'two fewer than seven' or 'two less than seven.' We'll be learning more about that. Who can remind us how our bar diagram tells how much is two less than seven?"

Figure 2.2 A bar diagram for a comparison problem when the difference is given.

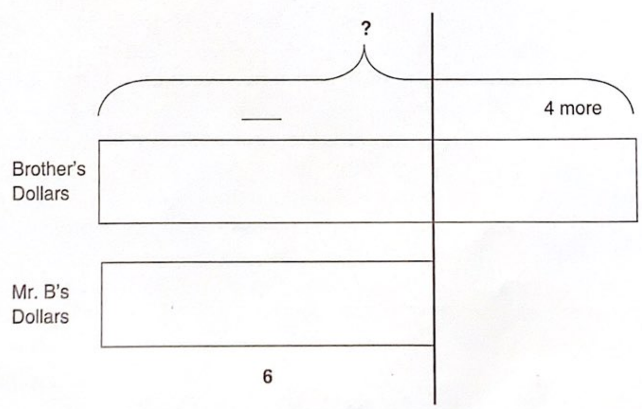

Illustration by Celia Stevenson.

Figure 2.3 A diagram is used to communicate the problem situation and to give meaning to several related phrases.

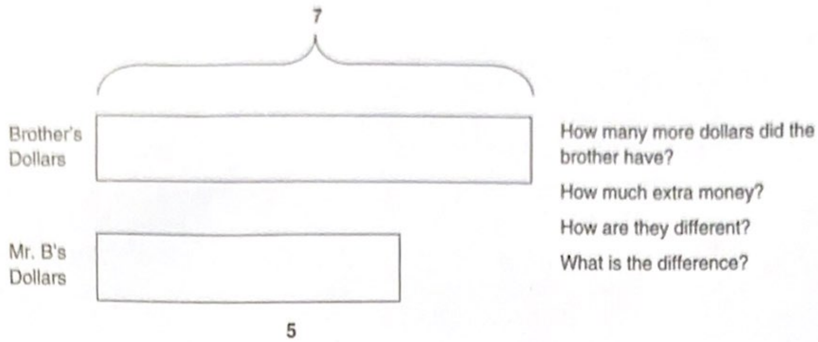

Illustration by Celia Stevenson.

"Now let's practice saying complete sentences, sketching bars, and showing equations. For example, if I say, 'Maria has four dollars and José has six dollars,' you would say, 'José has two more dollars than Maria. Maria has two fewer dollars than José. The difference is two dollars.'" Mr. Blue writes the frame of these sentences and equations on the board:

> _____ has __ dollars and _____ has __ dollars. _____ has ___ *more* dollars than _____. _____ has ___ *fewer* dollars than _____.
>
> The difference is __ dollars.
>
> __ + __ = __ and __ − __ = __.

Mr. Blue then gives students a few more examples orally, has them discuss each one with a partner, asks a few students to share their thinking, and records the equations.

During the next lesson, Mr. Blue uses the same sentence frames and introduces a simple game to practice working with more and less comparison relationships. Students work with a partner and use two number cubes, a copy of the sentence frames, and their own small name cards. The first partner rolls the first number cube and places it on the first number spot on the sentence frame, and the second partner does the same for the second number of dollars. The students work together to read the sentences, figure out the answers, and write the complete sentences and equations on their own papers. They become skilled at the oral part of the activity but show some confusion about the number sentences.

During the game, Mr. Blue goes from group to group. At one group he says, "I just heard you say 'three more.' Did you figure that out by adding or subtracting?" The student responds, "I added on." Mr. Blue asks, "So what's the equation then?" When he gets a response of "two and three more equals five," he says, "Great! Write it down. Keep going." He moves to another group and says, "I just heard you say, 'Maria has more dollars.' How many more?" Mr. Blue continues with this type of interaction as the game progresses.

Afterwards, Mr. Blue draws the student's attention to the four "number riddles." He has a whole-class brainstorming session on ways to show how to solve the first problem as a way of reviewing the language, models, diagrams, and number sentences used the previous day. Then, he hands out the problems to each student and asks them to work in their small groups as they solve the problems and show on their papers different ways of thinking about each problem.

SOURCE: Discussion with Mr. Blue from Coggins, Kravin, Dávila Coates, & Dreux Carroll (2007).

DISCUSSION OF THE MATHEMATICS TEACHING EXAMPLE

Connections to Common Core State Standards

Grade 1 » Operations & Algebraic Thinking » Represent and solve problems involving addition and subtraction. (1.0A.A.1) "Use addition and subtraction . . . to solve word problems . . . using objects, drawings, and equations . . . to represent the problem."

Grade 2 » Measurement & Data » Represent and interpret data. (2.MD.D.10) "Draw a picture graph and a bar graph . . . to represent a data set with up to four categories. Solve simple take-apart and compare problems using information presented in a bar graph."

Standard for Mathematical Practice 6: Attend to precision.
"Mathematically proficient students try to communicate precisely to others . . . . They state the meaning of the symbols they choose, including using the equal sign consistently and appropriately. . . . [S]tudents give carefully formulated explanations to each other."

Mathematics Goals

The goals of this lesson are to help students understand and interpret word problems involving additive comparisons of two distinct quantities and to understand related mathematical academic language terms such as *more*, *less*, and *fewer*. A secondary goal is to use multiple representations to demonstrate how additive or subtractive comparison relationships can be represented using language (e.g., word problems), diagrams, and symbols (e.g., equations). Also, during the lesson, connections between addition and subtraction are reinforced in several ways and a foundation is built for future work with word problems involving fractions and decimals. The students begin to see mathematical similarities between various word problems.

Mr. Blue introduces the class to a new meaning for the symbolic expression, "6 + 2," when he writes it to represent two more dollars than the six that he started with. He also teaches students how to read such symbolic expressions as saying "two more than six," noting that the digit 2 is after the 6, but it comes second in the symbolic expression. He apparently wants to prepare students for future algebraic expressions where $x + 2$ is interpreted as "two more than x." Of course, it may also be interpreted as "x and two more."

By first introducing the lesson with a situation involving "two more dollars than five," Mr. Blue is able to use it as a foundation for his representations for comparing two distinct amounts. The "two more dollars" represent the difference between his brother's amount of money and his amount of money.

Language Goals

A concurrent goal of this lesson is to develop students' ability to read and understand academic language in mathematics, particularly in relation to the specific terms *more, less, fewer, compare,* and *difference*. The mathematical meaning of the word difference is very carefully built and linked to the word *different*. Another goal is for students to develop their ability to use academic language in communicating with the teacher and other students and in solving and discussing simple mathematical problems.

Notice that the teacher models and writes frames of the sentences and equations as additional scaffolding for the desired use of the academic language. Also, several times, forms of response were simple enough for students at Proficiency Level 1 and Level 2 to participate, such as when showing a number of linking cubes or saying a particular sum.

Support Through Primary Language and Cognates

Whenever new concepts and language are introduced in the mathematics classroom, English learners need multiple support systems. In this lesson, new vocabulary is appropriately introduced through an activity with visual and tactile scaffolding. Students have a number of opportunities to communicate and verify their understanding. It is not clear whether all students, and particularly the beginning English speakers, understand the lesson since there is no mention of informal assessment nor observed use of primary language support. It is hoped that paraprofessionals, parent volunteers, older student-tutors, or classmates provide primary language support to the English learners during the initial phase of the lesson and during partner discussions. Use of primary language can increase opportunities to create links to prior knowledge in the student's primary language and increase the opportunity for every student to understand the foundational ideas of the lesson. Beginning English learners will also be able to verify their understanding of the new concepts if they know they can use their primary language to ask more complicated questions.

The teacher could point out that some words are very similar across languages, which are referred to as "cognates," or words that share a common root, for example, *diferencia* and *difference;* or *para comparar* and *to compare*.

The focus during mathematics instruction involving word problems is to read for meaning and to become aware of similarities between problems. Discussing a word problem in multiple languages can enhance the depth of understanding and make it possible for English learners to think about mathematical relationships in the primary language.

Developing Specific Math Vocabulary

In the teaching example, the students first experience the idea of "more" through discussion and hands-on modeling. The teacher posts a few targeted vocabulary words as headings on large class posters and has the class help develop explanations and illustrations. Even though the comparative words *more, less,* and *fewer* may be familiar in everyday contexts such as "I have more cookies," in this lesson they are indicating specific mathematical quantities such as "three less than. . . ." It may be helpful to point out to students the similarities and differences between "Who has fewer dollars?" and "How many fewer dollars does Maria have (than José)?"

The teacher made sure that new academic language was layered on top of the activity and discussion and that specific definitions and illustrations were jointly developed, discussed, posted, and referred to during the lesson. The teacher avoided the temptation to treat *more* and *fewer* as "key words" that magically say to add or subtract. Students might intuitively think that "more" indicates addition, but Mr. Blue carefully prepares them to see that in cases such as in Problem 1, "more" is part of a comparative phrase, "how many more." The phrase indicates that the difference between two amounts is to be found. This difference could be represented in a subtraction equation or as a missing addend problem. The consistent message to students is that they must read for meaning in a mathematics classroom, while developing facility with specific academic vocabulary.

Opportunities to Use Academic Language

The questions, verbal and physical models, diagrams, "talk to your neighbor" directions, and partner games all lead to an increased likelihood that students would have frequent meaningful encounters with the mathematics vocabulary. The positive environment and friendly, relaxed atmosphere also lead to the students' earnest efforts to learn the new concepts and words. When necessary, Mr. Blue scaffolds the students' sharing by repeating parts or all of the sentence frame used earlier in the lesson ("_____ has ___ more dollars than ____"). He asks the students to practice expressing their ideas using this frame.

When the students work in their small groups on the number riddles, Mr. Blue may systematically provide quick interactions and questions for students who need additional support as they attempt to use new mathematical academic language. He may plan when he will focus on specific students and when to model specific phrases when he interacts with these students.

Teaching Decisions

It is worth noting that the teacher chose to develop a whole-class lesson around four challenging word problems for his class that includes several English learners. He did not skip the important yet difficult topic of comparison problems. He uses word problems in order to develop this new mathematics concept and related academic language, which is likely to also have benefits when students work with data in bar graphs. By incorporating a variety of strategies and keeping the context interesting, yet simple, and the numbers small, the teacher is able to focus the students' attention on the vocabulary and the concepts.

Mr. Blue effectively balances between using "teachable moments" and waiting for a future opportunity to further discuss more abstract ideas. He keeps a tight time schedule, but manages to include several minutes of modeling, oral practice, and rehearsal at the beginning of the lesson. He checks all students' understanding on a regular basis. He has chosen to do a whole-class lesson, with modeling of several different representations during the main lesson, but without expecting students to immediately use the representations independently. He has decided to save the increase/decrease concept and vocabulary for another day. By constantly assessing the readiness of his students to take in more information, and through his informal observations and interactions with students during the lesson, Mr. Blue appropriately maintains the focus of the lesson and accomplishes reasonable goals and objectives with his students.

Related Teaching Example for Middle School Students

Middle school English learners, and all learners, benefit from carefully planned introductions to new content. Older students, just like the second-grade students in this chapter's Mathematics Teaching Example, benefit when a teacher introduces new material through an oral story related to a compelling mathematics problem, accompanied by visual aids, diagrams, and a routine that purposefully introduces, emphasizes, and posts key vocabulary. They can work at times as a class, in small groups, and individually, just as the Grade 2 students did during their teacher's modified textbook lesson. When an entire class understands a contextual mathematics example, students can use it as an anchor for talking to one another about future related problems. A shared focus problem can provide a base for learning new academic vocabulary, related concepts, and representations.

For example, many teachers begin their unit on linear functions with investigations of situations involving a constant rate of change. A class might work in small groups to gather data on the height of a stack of cups or the cost of ordering items with an included delivery charge. The lesson can be structured to emphasize development of new concepts and related academic language. Such lessons include frequent opportunities to speak with peers, including in native languages. And, the teacher has the opportunity to monitor the engagement and understanding of students with limited English.

Connections of Academic Language to Standard for Mathematical Practice 6: Attend to Precision

> "Students . . . try to communicate precisely to others . . . state the meaning of . . . symbols . . . students give carefully formulated explanations to each other."

In the Grade 2 lesson, the teacher models clear, age-appropriate, precise communication and concept development before working at a symbolic level. He introduces a new type of mathematical relationship through an informal oral story that is accompanied by visual aids, diagrams, and a routine that purposefully develops mathematical meaning for key vocabulary (*more, less, fewer, increase, decrease,* and *compare*). The students gradually expand their understanding of subtractive differences and additive comparison problems along with their ability to use related vocabulary to discuss sample situations. Students participate in whole-class discussions and partner activities, and use sentence frames while discussing the relationships. They write number sentences (equations), which are a specialized type of academic language. In the middle school example, students would likely begin with less formal language such as "goes up" to describe the constant increase and would move on to understand and use phrases such as "rate of change" and "constant."

TEACHING TIPS

- Monitor adult use of language to ensure that it is positive and optimistic to promote a safe environment where students feel comfortable taking risks to use mathematical language they are just learning, and where it is accepted that mistakes will occur while appropriate language use is being developed.
- Model proper mathematical use of language (e.g., say "trapezoid" rather than "the red block") and incorporate rephrasing whenever speaking with students.
- Begin teaching a new concept through experiences that draw out and expand students' understanding, make use of informal language, and make explicit connections between student language and academic language; for example, help students' word *times-ing* become *multiplying*.
- Create learning environments, and give enticing partner or small-group problem-solving assignments that compel students of various language abilities and varying levels of mathematical understanding to engage in meaningful use of mathematical language.
- Engage students in activities where they are asked to explain an idea or procedure to a younger student or give directions on how to draw or locate an object to promote use of mathematical language.
- Present students with problems that are incorrectly solved, tell them that the solution is incorrect, and ask them to discuss and explain the mistake and misunderstanding and describe a correct solution method.

Figure 2.4 A word wall entry may be created as a part of a whole-class discussion once students are grasping the underlying concept.

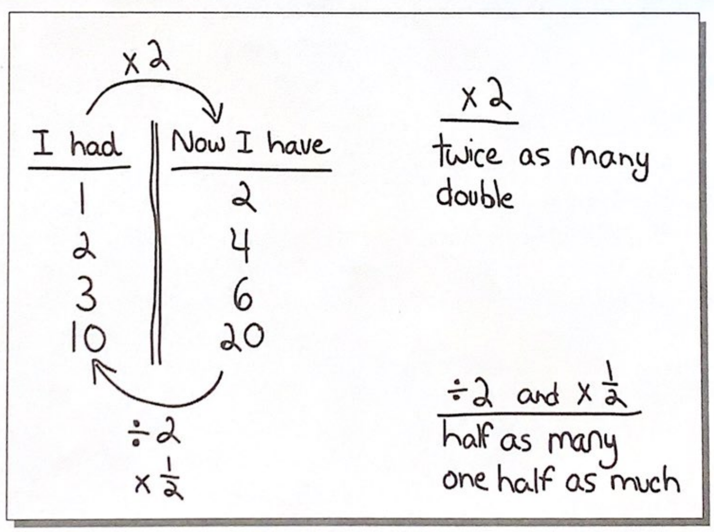

Illustration by Celia Stevenson.

Figure 2.5 Students may be asked to fill out a word card as either a learning-support task or as a formative assessment task. The teacher varies the labels for the categories to maximize thinking about the particular word.

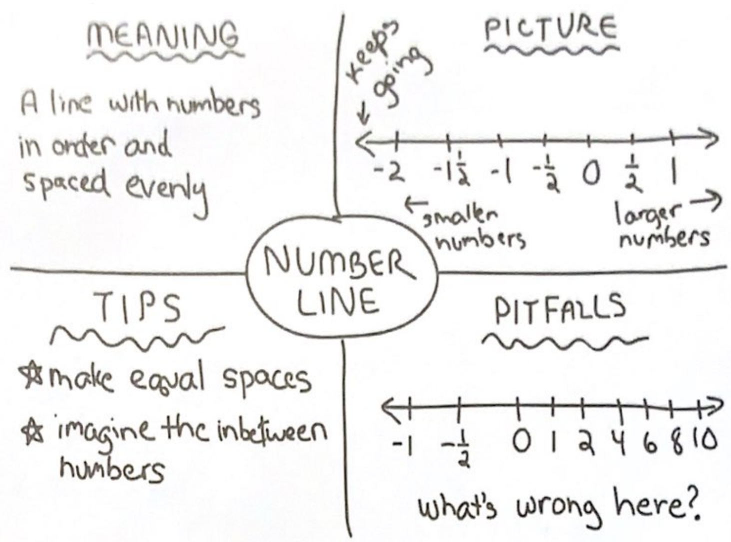

Illustration by Celia Stevenson.

- Use choral response for single number or word answers. For example, say, "The shape on the wall that has only two parallel sides is a _____, everyone tell me."
- After a new concept has been developed, explicitly connect it to academic vocabulary and create a class routine for students to record and refer to new key vocabulary.
- Explicitly encourage students to use new academic language by announcing, "Today, I'm listening for these words . . . ," then keep track of usage and record and share sample student sentences.
- Engage students in movement and use gestures that demonstrate actions associated with important verbs such as *combine* when teaching concepts that underlie new vocabulary.
- Use visual aids such as a number line or a hundreds chart while teaching and practicing vocabulary and associated concepts such as less than, equivalent, and times as much.
- Introduce each word wall entry through a specific, interactive instructional process and refer frequently to individual word wall cards and posters. For example, work together with students to develop an illustrated poster that represents a new concept.
- Ask questions that are in a familiar repetitive frame, and model frames for appropriate responses while maintaining a focus on meaning and not repetition for its own sake.
- Ask groups of students to make statements that incorporate a desired phrase such as "twice as much as" as they analyze everyday items, such as the nutritional information written on two different cereal boxes.

DISCUSSION OF ACADEMIC LANGUAGE DEVELOPMENT

Mathematics vocabulary words and phrases generally are *abstract* terms that relate to critically important concepts, often concepts that are not yet understood. Truly understanding and using such vocabulary is one indication of success in learning a major new idea. In contrast, being able to repeat a definition of a mathematics term does not necessarily indicate understanding of the concept. Moschkovich (2002) points out that true proficiency with academic language involves much more than learning vocabulary and phrases. Students should also learn communication skills such as explaining and justifying mathematical ideas. Academic language development can be fostered in a classroom where important mathematical problems, assessment tasks, and concepts are addressed by both small and large groups of students. Support in developing practices such as explaining, questioning, conjecturing, and proving is an essential aspect of preparing students for success with today's mathematics standards. Also, a wide variety of mathematics assessment practices should be used to establish what students actually know.

Some mathematics vocabulary, such as *odd, area,* and *operations,* have specialized meanings that must be pointed out as different from everyday usage and that must be carefully taught like any other academic vocabulary associated with a new abstract idea. Other mathematics vocabulary may represent a word, with its concept, that is already understood in students' primary language. In such cases, links need to be made through discussion and illustration to associate the new English word with the known concept. For example, the Spanish word *cuadrilátero* supports the development of meaning for the English word *quadrilateral* because *cuatro lados* literally means "four sides."

Why is it usually not productive to go over new key vocabulary at the beginning of a math lesson?

In language arts, we want students to be able to read new material without being distracted by unfamiliar vocabulary. Therefore, presenting and discussing vocabulary words before the students read the new material helps them understand the material by "front-loading vocabulary." This strategy is often not as productive in mathematics. Mathematics vocabulary words often represent the main idea of an entire chapter or unit of study, while a new reading word is usually a supporting contextual element. For example, presenting a vocabulary word such as circumference to students as "the distance around a circle" will not be meaningful unless students have prerequisite understanding of distance as a one-dimensional measurement of length and clear understanding of perimeter. An entire lesson may be centered on teaching the concept, and its word, *circumference.*

A new math *concept* that underlies a vocabulary word must be systematically built up through experiences, through activation of prior knowledge, and use of informal language. In addition, the formal vocabulary must be purposefully associated with the new concept and frequently practiced.

Consider the word *parallel.* What reaction and long-range success would you expect if you taught a fourth-grade lesson by focusing on a formal dictionary definition of parallel lines? How else might you start a lesson on parallel lines? For example, you might ask students to draw two lines that "stay the same distance apart" to represent train tracks or as a space for writing a heading on a poster. Students could physically check the measurements between the lines. You might say, "Talk with your partner about what you know about parallel lines, lines that stay the same distance apart. If you could draw very, very long lines, would they cross? Where?" As the concept of parallel becomes part of each student's experience, the formal vocabulary *parallel lines* can be introduced and associated with its physical meaning. As a product of such a class discussion, a wall poster might be made about parallel lines, using both academic and informal language as well as illustrations.

Why is it suggested that we not emphasize key words in math problems?

When students are encouraged to focus on key phrases such as "in all," "how many," and "less," they often fail to focus carefully on the meaning of a story problem, on the mathematical structure implied, and what operation actually makes sense. Relying on key words leads to mistakes in choice of operation when such common phrases are used for a different purpose, such as, "In all of the boxes there were some . . ." or ". . .Then, Jim had five fewer stickers, so he only had eleven left. How many did he start with?" A key word focus also leads to a general lack of individual skill at interpreting story problems, identifying the underlying mathematical relationships, and making plans for a solution.

In reality, there are not very many types of situations that are commonly associated with each operation, so it makes sense to help students focus on understanding such key relationships as part-part-total addition and subtraction situations, and sharing, or partitive, division situations, rather than focusing on key words. When the focus is on understanding the action involved and the mathematical relationships behind important phrases such as "three times as many," students are more likely to be able to generalize to other problems and to find a solution no matter which quantity is the missing quantity in a problem.

Why do we need to differentiate discussion goals for levels of English language development?

Students of varying proficiency levels can engage in the same mathematics investigations and lessons. However, ability to *speak* about the mathematics and to use the new target vocabulary will vary according to language level. A student at Proficiency Level 2 is likely to understand spoken language and to model "three more than six" with cubes or possibly to answer, "It's nine." Students at Level 3 or Level 4 are more able to initiate use of a newly posted target phrase such as "half of" with a statement such as, "Look, five is half of ten!" They are also more able to refer to posted discussion phrases such as, "I know this is correct, because . . ." to explain their reasoning in general terms.

A lesson plan should include carefully planned opportunities to learn about and to use new vocabulary in a meaningful context. Questions and instructions should vary between literal and complex forms. Students at beginning levels of language proficiency can be encouraged to respond orally to simply stated questions, while students at more advanced levels will benefit from clear, interesting questions that entice them to speak and to use new vocabulary. All students should be thinking about the same important, high-level mathematics.

ACADEMIC LANGUAGE RESEARCH

It can take five to seven years for students to acquire academic English at a level that enables them to fully benefit from academic instruction in English (Cummins, 2000). According to the Council of Chief State School Officers Framework for English Learner Proficiency Development Standards, "Several modalities of language use by students and teachers are components of learning a discipline such as mathematics" (2012).

Given the complex nature of mathematical academic language, and the ever-increasing content demands across the grades, it is not surprising that it takes considerable time, and purposeful teaching, for academic English to be fully acquired.

Academic language for mathematics involves much more than learning vocabulary. Solomon and Rhodes (1995) discuss various ways that researchers describe academic language. They point out that Cummins and many others emphasize that academic language, referred to by Cummins as *cognitive academic language proficiency* (CALP), is less context-embedded in nature than conversational language (basic interpersonal communication skills, or BICS) and is associated with high cognitive demand. Others stress that academic language is a compilation of unique language functions and structures. Examples of language functions are comparing, classifying, predicting, justifying, persuading, and solving problems. Dale and Cuevas (1987) point out examples of special mathematical syntactic structures such as comparative structures (e.g., greater than four) and logical connectors (e.g., if . . . then . . .). Besides language functions, Spanos, Rhodes, Dale, and Crandall (1988) differentiate between four aspects of academic language: technical vocabulary terms, which are complex strings of words, ordinary vocabulary that has different meanings in math, and synonymous words and phrases (e.g., *add, plus,* and *combine*), and various mathematical symbols and notations. The teachers in Solomon and Rhodes's study tended to focus on academic language "in terms of discrete aspects of language, such as vocabulary, lexis, and syntax," particularly in terms of the language students need to understand the lesson or unit being studied, as opposed to more theoretical components.

Barnett-Clarke and Ramirez (2004) bring up the importance of language when thinking about mathematics, using what Bickmore-Brand (1990) calls "internal chatter." Teachers can model this kind of reflective talk as they work through a problem that presents a challenge to their students. Internal chatter may occur in English, in the primary language, or in a combination of languages. The important point is that knowing academic terms and phrases facilitates thinking, learning, and problem solving.

Mathematics classrooms where lessons purposefully and supportively have students collaborate and talk about their mathematical ideas are instructional environments where Standard 1, "English language learners communicate for social, intercultural, and instructional purposes within the school setting" (Teachers of English to Speakers of Other Languages [TESOL], 2006), can be addressed. Students benefit when they have multiple forms of interaction and support as they learn new concepts, skills, and related language.

DISCUSSION AND PROJECT QUESTIONS ■

1. Think about the vocabulary term *greater than* and its symbol. How would you teach it to second graders?
2. Name a specific academic language term or terms and a targeted grade level. Describe what you would do to maximize students' acquisition and use of the term(s) during a lesson.
3. Why is it advisable to teach vocabulary to English learners through movement and action? What else is advised in this chapter?
4. What can you do to ensure that English learners include academic language as they participate in oral discourse?
5. Discuss teaching decisions that support the learning of the English learners in Mr. Blue's class. Include comments on the choice of problems and representations, and the students' types of involvement during the lesson. What would you add or modify, and why?
6. Consider situations involving teaching students' academic language and mathematics while addressing Standard for Mathematical Practice 6, "Attend to precision . . . try to communicate precisely to others . . . state the meaning of . . . symbols . . . students give carefully formulated explanations to each other."
   a. Identify and comment on specific examples from this chapter's Teaching Example that relate to Standard for Mathematical Practice 6.
   b. Choose a particular grade level and mathematics learning target. Outline your ideas for teaching English learners at several levels of proficiency the related academic language and mathematics with attention to Standard for Mathematical Practice 6.