Chapter 4

Fixed Interest Rate Mortgage Loans

This chapter deals with various approaches to pricing and structuring fixed interest rate mortgage loans. By *pricing* a loan, we refer to the rate of interest, fees, and other terms that lenders offer and that borrowers are willing to accept when mortgage loans are made. As a part of the pricing process, we also stress the supply and demand for loanable funds, the role of inflation, and how both affect the rate of interest. As to loan structuring, we review the many innovations in mortgage payment patterns that have evolved from changes in the economic environment.

Another major objective of this chapter is to illustrate techniques for determining the yield to the lender and actual cost to the borrower when various provisions exist in loan agreements. Lenders on real estate commonly include various charges and fees in addition to the interest rate as a condition of making a loan. These charges may include loan discounts, origination fees, prepayment penalties, or prepaid interest. In addition, various amortization or loan repayment schedules can be agreed upon by the borrower and lender to facilitate financing a particular real estate transaction. Because these provisions often affect the cost of borrowing, the methodology used to compute the yield to the lender (cost to the borrower) is heavily stressed.

Determinants of Mortgage Interest Rates: A Brief Overview

Changing economic conditions have forced the real estate finance industry to go through an important evolution. These changing conditions now require lenders and borrowers to have a better understanding of the sources of funds used for lending and the nature of how risk, economic growth, and inflation affect the availability and cost of mortgage funds.

When considering the determinants of interest rates on mortgage loans, we must also consider the demand and supply of mortgage funds. Most mortgage lenders are intermediaries, or institutions that serve as conduits linking flows of funds from savers to borrowers. Borrowers use the savings in the form of mortgage credit. The market rate of **interest** on mortgage loans is established by what borrowers are willing to pay for the use of funds over a specified period of time and what lenders are willing to accept in the way of compensation for the use of such funds. On the demand side of the market, it can be safely said that the demand for mortgage loans is a **derived demand,** or is determined by the demand for real estate.

When supplying funds to the mortgage market, lenders also consider returns and the associated risk of loss on alternative investments in relation to returns available on mortgages. Hence, the mortgage market should also be thought of as part of a larger capital market, where lenders and investors evaluate returns available on mortgages and all competing forms of investment, such as bonds, stocks, and other alternatives, and the relative risks associated with each. Should lenders believe that a greater return can be earned by making more mortgage loans (after taking into account the costs and the risk of loss) than would be the case if they invested in corporate bonds or business loans, more funds would be allocated to mortgage loans, and vice versa. Hence, lender decisions to allocate funds to mortgages are also made relative to returns and risk on alternative loans and investment opportunities.

The Real Rate of Interest: Underlying Considerations

When discussing market interest rates on mortgages, we should keep in mind that these interest rates are based on a number of considerations. We pointed out earlier that the supply of funds allocated to mortgage lending in the economy is, in part, determined by the returns and risks on all possible forms of debt and investment opportunities.

One fundamental relationship that is common to investments requiring use of funds in the economy is that they earn at least the **real rate of interest.**[1] This is the minimum rate of interest that must be earned by savers to induce them to divert the use of resources (funds) from present consumption to future consumption. To convince individuals to make this diversion, income in future periods must be expected to increase sufficiently from interest earnings to divert current income from consumption to savings. If expected returns earned on those savings are high enough to provide enough future consumption, adequate amounts of current savings will occur.

Interest Rates and Inflation Expectations

In addition to the real rate of interest, a concern that all investors have when making investment decisions is how *inflation* will affect investment returns. The rate of inflation is of particular importance to investors and lenders making or purchasing loans made at fixed rates of interest over long periods of time. Hence, when deciding whether to make such commitments, lenders and investors must be convinced that interest rate commitments are sufficiently high to compensate for any expected loss in purchasing power during the period that the investment or loan is outstanding; otherwise, an inadequate real return will be earned. Therefore, a consensus of what lenders and investors expect inflation to be during the time that their loans and investments are outstanding is also incorporated into interest rates at the time investments and loans are made.

To illustrate the relationship between the **nominal interest rate,** or the contract interest rate agreed on by borrowers and lenders, and real rates of interest, suppose that a $10,000 loan is made at a nominal or contract rate of 10 percent with all principal and interest due at the end of one year. At the end of the year, the lender would receive $11,000, or $10,000 plus $10,000 times (.10). If the rate of inflation during that year was 6 percent, then the $11,000 received at the end of the year would be worth about $10,377 ($11,000 ÷ 1.06). Thus, although the nominal rate of interest is 10 percent, the *real* rate on the mortgage is

[1] If the reader can visualize an investment portfolio containing investments in all productive activities in the economy based on the weight that any particular activity has to the total value of all productive activity in the economy, the rate of current earnings on such a portfolio would be equivalent to the real rate of interest. Such a rate would also be the rate required by economic units to save rather than consume from the current income.

just under 4 percent ($377 ÷ $10,000 = 3.77%). Therefore, we conclude that if the lender wanted a 4 percent real rate of interest, the lender would have to charge a nominal rate of approximately 10 percent to compensate for the expected change in price levels due to inflation.[2]

We can summarize by saying that the nominal interest rate on any investment is partially determined by the real interest rate *plus a premium* for the expected rate of inflation. In our example, the real rate of 4 percent plus an inflation premium of 6 percent equals 10 percent. Note that this premium is based on the rate of inflation *expected* at the time that the loan is made. The possibility that inflation will be more or less than expected is one of many risks that lenders and investors must also consider.

We should also point out that the nominal interest rate is usually expressed as an *annual* rate of interest. However, depending on the type of loan, the nominal rate could be an annual rate compounded daily, monthly, quarterly, annually, or continuously. We will explore the effects of compounding, accrued interest, and payment patterns in more detail throughout this chapter.

Interest Rates and Risk

In addition to expected inflation, lenders and investors are also concerned about various *risks* undertaken when making loans and investments. Lenders and investors are concerned about whether interest rates and returns available on various loans and investments compensate adequately for risk. Alternatively, will a particular loan or investment provide an adequate risk-adjusted return?

Many types of risk could be discussed for various investments, but they are beyond the scope of this book. Consequently, we will focus on risks affecting mortgage loans. Many of these risks are, however, present to greater and lesser degrees in other loans and investments.

Default Risk

One major concern of lenders when making mortgage loans is the risk that borrowers will default on obligations to repay interest and principal. This is referred to as **default risk,** and it varies with the nature of the loan and the creditworthiness of individual borrowers. The possibility that default may occur means that lenders must charge a premium, or higher rate of interest, to offset possible loan losses. Default risk relates to the likelihood that a borrower's income may fall after a loan is made, thereby jeopardizing the receipt of future mortgage payments. Similarly, a property's value could fall below the loan balance at some future time, which could result in a borrower defaulting on payments and a loss to the lender.

Interest Rate Risk

An additional complication in lending and investing arises from the uncertainty in today's world about the future supply of savings, demand for housing, and future levels

[2] Actually, the nominal rate of interest should be (1.06 × 1.04) − 1, or 10.24 percent, if a real rate of 4 percent is desired. For convenience throughout this text, we will *add* the real rate and premium for expected inflation as an approximation to the nominal interest rate. We should point out that the relationship of expected inflation and interest rates has long been a subject of much research. While we show a very simple, additive relationship in our discussion, there may be interaction between real interest rates and inflation. The specific relationship between the two is not known exactly. Hence, the student should treat this discussion at a conceptual or general level of interpretation.

of inflation. Thus, interest rates at a given point in time can only reflect the market consensus of what these factors are expected to be. Investors and lenders also incur the risk that the interest rate charged on a particular loan may be insufficient, should economic conditions change drastically *after* a loan is made. The magnitude of these changes may have warranted a higher interest rate when the loan was made. The uncertainty about what interest rate to charge when a loan is made can be referred to as **interest rate risk.**

For example, **anticipated inflation** may have been 6 percent at the time our $10,000 loan was made. But if *actual* inflation turns out to be 8 percent, this means the interest rate that should have been charged is 12 percent. In this case, we say that the anticipated rate of inflation at the time the loan was made was 6 percent. However, because **unanticipated inflation** of 2 percent occurred, the lender will lose $200 in purchasing power (2% of $10,000) because the rate of interest was too low. This does not mean that lenders did not charge the "correct" interest rate *at the time the loan was made.* At that time, the inflation was expected to be 6 percent. Therefore, to be competitive, a 10 percent interest rate had to be charged. However, the additional 2 percent was unanticipated by all lenders in the market. It is unanticipated inflation that constitutes a major component of interest rate risk to all lenders.

The possibility that too low an interest rate was charged at the time the loan was made is a major source of risk to the lender. Hence, a premium for this risk must also be charged or reflected in the market rate of interest. Interest rate risk affects all loans, particularly those that are made with fixed interest rates, that is, where the interest rate is set for a lengthy period of time when the loan is made. Being averse to risk, lenders must charge a premium to incur this risk.

Prepayment Risk

Some mortgage loans allow borrowers to prepay loans before the maturity date without a penalty. This, in effect, gives borrowers the *option* to prepay the loan, refinance, or pay off the loan balance if a property is sold. If loans are prepaid when interest rates fall, lenders must forgo the opportunity to earn interest income that would have been earned at the original contract rate. As funds from the prepaid loans are reinvested by lenders, a lower rate of interest will be earned. When interest rates increase, however, the loan is not as likely to be prepaid. The risk that the loan will be prepaid when interest rates fall below the loan contract rate is referred to as **prepayment risk.**

Other Risks

There are additional risks that lenders and investors consider that may vary by type of loan or investment. For example, the *liquidity* or *marketability* of loans and investments will also affect the size of the premium that must be earned. Securities that can be easily sold and resold in well-established markets will require lower premiums than those that are more difficult to sell. This is called **liquidity risk.**

Legislative risk is another risk associated with mortgage lending that also may result in a premium. It can refer to changes in the regulatory environment in which markets operate; for example, regulations affecting the tax status of mortgages, rent controls, state and federal laws affecting interest rates, and so on, are all possibilities that lenders face after making loans for specified periods of time. Lenders must assess the likelihood that such events may occur and be certain that they are compensated for undertaking these risks when loans are made.

A Summary of Factors Important in Mortgage Loan Pricing

We can now see that the interest rate charged on a particular mortgage loan will depend on the real interest rate, anticipated inflation, interest rate risk, default risk, prepayment risk, and other risks. These relationships can be summarized in general as follows:

$$i = r + p + f$$

In other words, when pricing or setting the rate of interest (i) on a mortgage loan, the lender must charge a premium (p) sufficiently high to compensate for default and other risks and a premium (f) that reflects anticipated inflation to earn a real rate of interest (r) that is competitive with real returns available on other investment opportunities in the economy. If lenders systematically *underestimate* any of the components in the above equation, they will suffer real economic losses.

Pricing decisions by lenders are rendered complex because mortgage loans are made at fixed interest rates for long periods of time. For example, if we assume that a mortgage loan is to be made with a one-year maturity, the interest rate charged at origination should be based on what the lender expects each of the components discussed above to be during the coming year. More specifically,

$$i_1 = r_1 + p_1 + f_1$$

or the mortgage interest rate (i) at origination (time t) would be based on the lender's expectations of what the real rate of interest, the rate of inflation, and risk premiums (for risks taken in conjunction with making the mortgage loan over and above the level of risk reflected in the real rate of interest) should be for the term of the loan.

Understanding Fixed Interest Rate Mortgage (FRM) Loan Terms

As previously discussed in Chapter 2, there are many terms and options in mortgage loan agreements that are very important. We begin our analysis of fixed interest rate loans with a discussion of some of the most elementary terms:

- Loan amount
- Loan maturity date
- Interest rate
- Periodic payments

The *loan amount* identifies the amount borrowed and what the borrower is legally required to repay. The *loan maturity date* is the date by which the loan must be fully repaid. While these terms are relatively easy to understand, when analyzing and comparing various loan alternatives, the interest rate and its affect on periodic payments can be more complex and will be discussed further.

When dealing with fixed interest rate loans, in addition to the loan amount and maturity, lenders generally quote what is referred to as a *nominal annual rate of interest.* To elaborate, say a 30-year loan is made for $60,000 and the interest rate is 12 percent. That rate of interest is referred to as the *nominal rate because no reference is made as to how interest is to be calculated or how frequent payments will be.* If, in this case, *interest* is to be calculated *monthly* and *payments* are to be made *monthly,* one interpretation of the 12 percent *nominal interest rate* would be an annual rate of 12 percent interest *compounded monthly.* Recall from Chapter 3 that another way to interpret an annual rate of interest, compounded monthly, is to *calculate its equivalent annual rate of interest.* That is, a rate of interest *compounded annually* that would be equivalent to a loan with an annual rate of interest *compounded monthly*. This can be done in our example as follows:

STEP 1:
Solution: Find the FV of an amount that earns interest at an annual rate *compounded monthly*:

Function: $FV(PV, i, n)$

$PV = -\$60{,}000$

$i = 12\%/12 = 1\%$ or .01

$n = 12$

Solve for $FV = \$67{,}609.50$

STEP 2: Find the equivalent annual interest rate *compounded annually*:

$FV = \$67{,}609.50$

$n = 1$

$PV = \$60{,}000.00$

Solve for $i = 12.6825\%$

In Step 1, using our approach outlined in Chapter 3, we consider a $60,000 deposit made today and compounded monthly for 12 months at 12 percent, leaving an *FV* of $67,609.50. We then solve for the *annual* compound rate equivalent in Step 2 by changing the compounding period to 1. This produces $i = 12.6825$ percent. In other words, a loan quoted with a 12 percent annual rate of interest compounded *monthly* is equivalent to a loan with an annual rate of 12.6825 percent compounded *annually*. The difference in these rates is due to the fact that when interest on a loan is compounded monthly and paid monthly, the incremental value is greater than receiving cash flows annually.[3] This is true even though the interest rate may be quoted as 12 percent in both cases. So, a loan with an annual rate of 12 percent compounded *monthly* is worth the equivalent of a loan with an annual rate of 12.6825 percent compounded *annually*. The latter can be thought of as an *effective* or *equivalent* annual rate of interest. Alternately, a loan with an annual rate of 12.6825 percent compounded *annually* is equivalent to a 12 percent annual rate compounded *monthly*.

So why do lenders quote *nominal* rates of interest (12% in our case)? If interest is to be compounded monthly (or for other periods), why not quote equivalent annual rates (12.6825% in our case)? If the latter approach was used, uniformity would be achieved because interest rates on all loans could be quoted based on an equivalent annual rate *regardless* of how frequently interest is calculated and payments are made (daily, monthly, quarterly, annually). The answer partially lies in the evolution of banking, financial instruments, simplicity when making interest calculations, and a general lack of knowledge (understanding) among people working in finance-related fields. *In the discussion that follows, and throughout this book, we will follow the practice used by lenders and use the nominal rate of interest for all mortgage loan examples. In most cases, interest will be calculated monthly and payments will be made monthly. However, the reader should be aware that interest could be calculated and payments made over very different time periods.*

[3] The reader may recall that we discussed this concept with annual percentage yield (APY) in Chapter 3.

Calculating Payments and Loan Balances—Fixed Interest Rate Loans

The Importance of Accrued Interest and Loan Payments

A very important concept that must be understood when calculating payments or outstanding loan balances for real estate loans is: (1) the relationship between *accrued interest* and *loan payments* for a given period and (2) how any differences between them will affect loan balances. For example, as discussed above, many loans require interest to be *accrued monthly* ($i/12$); that is, if a fixed interest rate mortgage (FRM) loan is made in an amount of \$60,000 (PV) at a 12 percent interest rate and interest (i) is to be accrued monthly, the dollar amount of interest accrued as of the end of the first month would be calculated as

$$\$60{,}000 \times (.12/12) = \$600$$

We should also point out that ($i/12$) is referred to as the **accrual rate**. The amount of interest *accrued* and *owed* to the lender at the end of the month will be \$600.

The borrower and lender may also negotiate payments (PMT). The ratio of these payments to the loan amount is referred to as the **pay rate**. If the borrower and lender agree that payments (PMT) to be made at the end of each month are to be equal to accrued interest, then the monthly pay rate and the monthly accrual rate are the same.[4] This means that dollar payments (PMT) will be \$600, or *exactly equal* to \$600 accrued interest. When the monthly accrual rate and the monthly pay rate are equal, the outstanding loan balance remains unchanged. So, in our example, the loan balance would remain \$60,000 at the end of the month. *We should again stress at this point that, in any given period, the pay rate and accrual rate do not have to be equal.*

Loan Amortization Patterns

In the previous section, we emphasized the relationship between accrued interest and payments on mortgage loans. We used the example of an interest-only loan which indicated that the pay rate and accrual rate were equal. When considering other loan types, we will see that the monthly accrual and pay rates are frequently *not* equal. There are many situations when lenders and borrowers consider *different* loan structures and vary the pay rate and accrual rate. In these cases, loan balances will be affected and will change depending on the difference between the two.

Differences between Accrued Interest and Payments

We now consider situations where the pay rate, and therefore, monthly *payments* are (1) greater than, (2) equal to, or (3) less than monthly accrued interest. We then consider the effect that each case has on loan balances. At this point in our discussion of **constant payment mortgage (CPM)** loans, we will use examples for fixed interest rate loans that are classified in four very general ways:

Type of CPM Loan	Pay Rate	Loan Balance at Maturity
1. Fully amortizing	Greater than accrual rate	Fully repaid
2. Partially amortizing	Greater than accrual rate	Not fully repaid
3. Interest only	Equal to accrual rate	Equal to amount borrowed
4. Negative amortizing	Less than accrual rate	Greater than amount borrowed

[4] We will use *monthly time periods* for accrued interest and payments to limit the number of possible examples. The reader should be aware that accrual and payment periods do not have to be equal.

Notice that the first loan type, which we refer to as **fully amortizing**, means that the pay rate will *exceed* the accrual rate. This means that monthly payments will *exceed accrued interest* by an amount sufficient to pay the accrued interest due each month and *fully repay the loan by the maturity date.*

The second loan type, or the **partially amortizing loan**, refers to the case when the borrower and lender agree that, like the fully amortizing loan, the pay rate will result in a payment that will *exceed accrued interest,* but not by as much as the payment for the fully amortizing loan. Therefore, the loan will *not be fully repaid at maturity.* It will be only *partially repaid.*

The third loan type, or the **interest-only loan,** is sometimes called a **zero amortizing loan.** As we have discussed, in this case the pay rate will equal the accrual rate. Consequently, the loan balance at the end of each month will remain the same as the original loan amount. The *full, original, loan amount will have to be paid at maturity.*

Finally, the fourth loan type, or the **negative amortizing loan,** represents the case where borrowers and lenders agree that the pay rate will be *less than* the accrual rate. As a result, payments will *not* equal the amount of interest due and the loan balance will actually *increase* each month. At maturity, the *loan balance will be greater than the original loan amount.*

We will now illustrate payments for each category of loan. We also should note at this point that each category of loan will have constant, or level, monthly payments. We will discuss other monthly payment patterns later in this chapter.

Fully Amortizing, Constant Payment Mortgage (CPM) Loans

The most common loan payment pattern used in real estate finance during the post-depression era, and one which is still very prevalent today, is the fully amortizing, constant payment mortgage (CPM). (**Amortization** means the process of loan repayment over time.) The CPM loan payment pattern is used most extensively in financing single family residences and, to a lesser extent, income-producing properties such as multifamily apartment complexes and shopping centers. This payment pattern means simply that a level, or constant, monthly payment is calculated on an original loan amount at a fixed rate of interest for a given term. Each monthly payment includes interest and *some* repayment of principal. At the end of the term of the CPM loan, the original loan amount, or **principal**, is completely repaid, or has been fully amortized. The lender has earned and the borrower has paid a fixed rate of interest on the monthly loan balance.

To illustrate how the monthly loan payment calculation is made, we turn to our previous example of a $60,000 loan made at a 12 percent (nominal) rate of interest for 30 years. What are the constant monthly mortgage payments on this loan, assuming it is to be fully amortized at the end of 30 years? Based on our knowledge of discounting annuities from Chapter 3, the problem involves no more than finding the present value of an annuity and can be formulated as follows:

$$PV = \sum_{t=1}^{n} \left[\frac{PMT_t}{1 + \frac{i}{12}} \right]^t$$

where

PV = present value

PMT = payment

i = fixed nominal, interest rate on mortgage

n = number of months loan will remain outstanding because *PMT* is a constant, this is also equivalent to

$$PV = PMT \times \sum_{t=1}^{n} \frac{1}{(1 + \frac{i}{12})^t}$$

and

$$PMT = \frac{PV}{\sum_{t=1}^{n} \frac{1}{(1 + \frac{i}{12})^t}}$$

Solution:	Function:
$n = 30 \times 12 = 360$	*PMT* (*n*, *i*, *PV*, *FV*)
$i = 12\%/12 = 1\%$ or .01	
$PV = -\$60{,}000$	
$FV = 0$	
Solve for *PMT* = $617.17	

In this case, we are interested in solving for *PMT*, or the constant monthly payment (annuity) that will fully repay the loan amount (*PV*) and earn the lender 12 percent interest compounded monthly. The required payment will be $617.17.

Consider the fully amortizing loan pattern illustrated in Exhibit 4–1. The initial, relatively low principal reduction, shown in column 6, results in a high portion of interest charges in the early monthly payments. Note that the ending loan balance after the first six months (column 6) is $59,894.36; thus, only $105.64 has been amortized from the original balance of $60,000 after six months. Interest paid during the same six-month period totals $3,597.38. The explanation for the high interest component in each monthly payment is

EXHIBIT 4–1
Fully Amortizing Loan Pattern

(1) Month	(2) Beginning Loan Balance	(3) Monthly Payment	(4) Interest (.12 ÷ 12)	(5) Amortization*	(6) Ending Loan Balance
1	$60,000.00	$617.17	$600.00	$17.17	$59,982.83
2	59,982.83	617.17	599.83	17.34	59,965.49
3	59,965.49	617.17	599.65	17.52	59,947.97
4	59,947.97	617.17	599.48	17.69	59,930.28
5	59,930.28	617.17	599.30	17.87	59,912.41
6	59,912.41	617.17	599.12	18.05	59,894.36
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
358	1,815.08	617.17	18.15	599.02	1,216.06
359	1,216.06	617.17	12.16	605.01	611.06
360	611.06	617.17	6.11	611.06	-0-

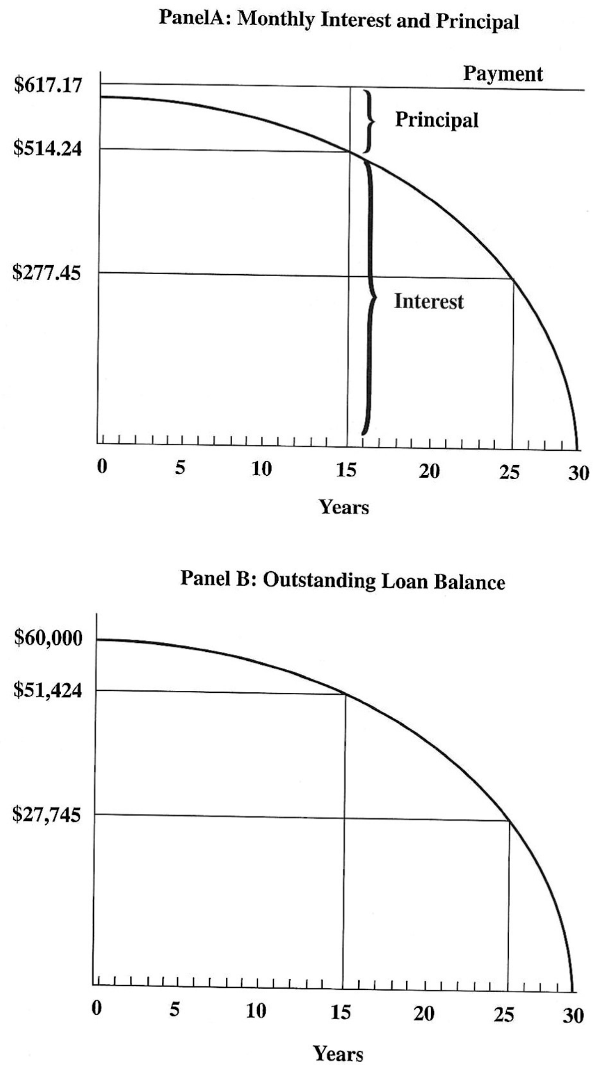
*Amortization increases each month by the factor $1 + i/12$; that is, 17.17(1.01) = 17.34, and so on.

vthat the lender earns an annual 12 percent return (1 percent monthly) on the outstanding monthly loan balance. Because the loan is being repaid over a 30-year period, obviously the loan balance is reduced only very slightly at first and monthly interest charges are correspondingly high. Exhibit 4–1 also shows that the pattern of accrued interest in the early years of the loan reverses as the loan begins to mature. Note that during the last months of the loan, accrued interest declines sharply and amortization increases.

Interest, Principal, and Loan Balance Illustrated

Exhibit 4–2 (panel A) illustrates the loan payment pattern over time, by indicating the relative proportions of interest and principal in each monthly payment over the 30-year term of the loan. Exhibit 4–2 (panel B) shows the rate of decline in the loan balance over the same 30-year period. It is clear that the relative share of interest as a percentage of the total monthly mortgage payment declines very slowly at first. Note in panel A that halfway into the term of the mortgage, or after 15 years, interest still makes up $514.24 of the $617.17 monthly payment and principal the difference ($617.17 = $514.24 = $102.93). Further, the loan balance after 15 years (panel B) is approximately $51,424. Total mortgage payments

EXHIBIT 4–2
Monthly Payment, Principal, Interest, and Loan Balances for a Fully Amortizing, Constant Payment Mortgage



Mortgage Loan Constants Fully Amortizing Loans

Concept Box 4.1

Prior to widespread use of financial calculators and spreadsheets, performing loan calculations had to be done manually. Therefore, a series of tables containing what were referred to as **loan constants** *were developed. These factors continue to be used* in the vocabulary of the lending industry. Equivalent to the pay rate discussed earlier in the chapter, these constants are simply the present value of a monthly annuity for various interest rates and time periods. These can be applied to any FRM loan to determine the constant monthly payment.

Related to our example, if we desired we could calculate the loan constant as

Solution:

$PV = -\$1$

$n = 360$

$FV = 0$

$i = 12\%/12 = 1\%$ or .01

Solve for ***PMT*** = .01286 (or loan constant)

Function:

PMT (*PV, n, FV, i*)

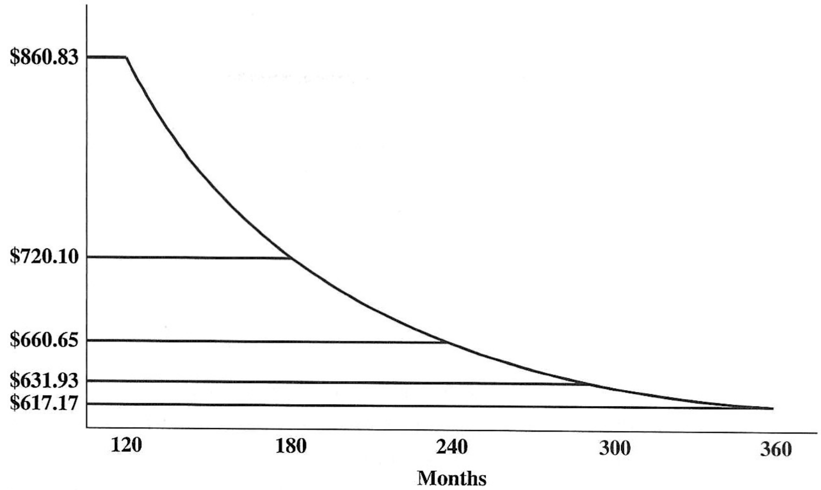
This constant, or payment, can be applied to any fully amortizing loan made at 12 percent for 30 years (360 months) to determine the monthly payment. The term *loan constant* continues to be used in lending negotiations, loan agreements, and other discussions between lenders and borrowers. The loan constant can be multiplied by any beginning loan amount to obtain the monthly mortgage payments necessary to amortize the loan fully by the maturity date.

		Interest Rate			
Years	Months	9%	10%	11%	12%
5	60	.020758	.021247	.021742	.022244
10	120	.012668	.013215	.013775	.014347
15	180	.010143	.010746	.011366	.012002
20	240	.008997	.009650	.010322	.011011
25	300	.008392	.009087	.009801	.010532
30	360	.008046	.008776	.009523	.010286

In the table above, we provide a sample of monthly mortgage loan constants (or pay rates) for various interest rates and loan maturities. Returning to our problem of finding the monthly mortgage payment for a $60,000 loan made at 12 percent for 30 years, we locate the 12 percent column and look down until we find the row corresponding to 30 years, where the loan constant is .010286 (rounded).* Loan constants are often stated on an annualized basis by multiplying the monthly loan constant by 12. In the above example, the annualized loan constant is $.010286 \times 12 = .123432$, or 12.34% (rounded). Note that this is greater than the interest rate of 12 percent. The difference is how the loan gets amortized. If the annualized constant was 12 percent, it would be an interest-only loan. Note that the annualized loan constant still assumes that payments are made monthly.

* Because of rounding (to six decimal places), the loan constant is .010286. When we multiply $60,000 by the rounded constant, we get a monthly payment of $616.16. The more exact solution is $617.17. Hence, readers should be aware that small discrepancies between their solutions and ours may occur when financial calculators are used, because calculator solutions may be rounded off to eight or more decimal places. We have attempted to carry out solutions to at least six decimal places before rounding.

EXHIBIT 4–3
Relationship between Monthly Mortgage Payments and Maturity Periods: Fully Amortizing Loans



of $111,090.60 ($617.17 × 180 months) have been made through the 15th year, with approximately $8,576 (or $60,000 – $51,424) of the loan repaid at that point. This pattern reverses with time. Note in panel A that after 25 years, interest makes up only $277.45 of the monthly payment, and the loan balance (panel B) has declined sharply to $27,745.

The Effect of Maturity on Monthly Payments—Fully Amortizing Loans

Exhibit 4–3 shows the effect of loan maturity on monthly payments for the base case mortgage loan. The exhibit shows the extent to which levels of constant monthly payments decline as maturities negotiated by the lender and borrower increase. In our base case example of a $60,000, fully amortizing, 30-year loan made at 12 percent interest, monthly payments would decline from $860.83 for a maturity period of *10 years* to $617.17 for a maturity of *30 years*. This indicates the importance of loan maturity and its effects when negotiating a loan structure.

Partially Amortizing, Constant Payment Mortgage (CPM) Loans

In many cases, loans may be structured to accomplish one or more goals. For example, the borrower may desire (1) a payment that is lower than what would be available with a fully amortizing loan and/or (2) a nonzero outstanding loan balance on the maturity date. This can be shown in the following example: A borrower and lender agree that a $60,000 loan made at 12 percent interest for 30 years will have a $40,000 balance (sometimes referred to as a **balloon payment**) on the maturity date. It will have constant monthly payments.

To solve for those payments, we modify the equation that was used to calculate the CPM with full amortization as follows:

$$PV = \sum_{t=1}^{n} \frac{PMT_t}{(1 + \frac{i}{12})^t} + \frac{FV_n}{(1 + \frac{i}{12})^n}$$

The above equation differs in that the term FV is introduced and represents the desired outstanding balance (also frequently referred to as a balloon payment), at period n, or maturity.

The calculator solution for our problem can be modified as follows:

Solution:

$n = 12 \times 30 = 360$

$i = 12\%/12 = 1\%$ or .01

$PV = -\$60{,}000$

$FV = \$40{,}000$

Solve for $PMT = \$605.72$

Function:

$PMT\ (n, i, PV, FV)$

Note that the payment for the partially amortizing loan is $605.72, which is *less than* the $617.17 payment that we calculated for the fully amortizing loan. We should also point out that the loan constant, or pay rate, is $605.72 ÷ $60,000 = .010095. This is different from the *loan constant* that would apply if the loan was fully amortizing, which was $617.17 ÷ $60,000 = .010286.

It should be noted that the $605.72 payment also is slightly closer to the $600 accrued interest calculated earlier. This means that loan amortization will be only $5.72 at the end of month one. A more detailed schedule demonstrating the monthly relationships between payments, accrued interest, and the loan balance is shown in Exhibit 4–4.

EXHIBIT 4–4
Partially Amortizing Loan Pattern

Month	Beginning Loan Balance	Monthly Payment	Interest (.12 ÷ 12)	Amortization	Ending Loan Balance
1	$60,000.00	$605.72	$600.00	$ 5.72	$59,994.28
2	59,994.28	605.72	599.94	5.78	59,988.50
3	59,988.50	605.72	599.88	5.84	59,982.66
4	59,982.66	605.72	599.83	5.90	59,976.76
5	59,976.76	605.72	599.77	5.95	59,970.81
6	59,970.81	605.72	599.71	6.01	59,964.79
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
358	40,605.03	605.72	406.05	199.67	40,405.35
359	40,405.35	605.72	404.05	201.67	40,203.69
360	40,203.69	605.72	402.04	203.69	40,000.00

Zero Amortizing, or Interest-Only—Constant Payment Mortgage (CPM) Loans

Another pattern for fixed interest rate and constant payment loans that is frequently used in real estate finance is referred to as the interest-only loan. As the name implies, constant monthly payments will be "interest only."

Solution:

$n = 360$

$i = 12\%/12 = 1\%$ or .01

$PV = -\$60{,}000$

$FV = \$60{,}000$

Solve for $PMT = \$600.00$

Function:

$PMT\ (n, i, FV, PV)$

Because the loan is interest only, the payment is simply equal to accrued interest, or 12 ÷ 12 × $60,000 = $600. This means, of course, that the loan balance at the end of each month will remain at $60,000. The monthly pattern of accrued interest, payments, and loan balances for the interest-only loan are detailed in Exhibit 4–5.

EXHIBIT 4–5
Interest-Only Loan Pattern

Month	Beginning Loan Balance	Monthly Payment	Interest (.12 ÷ 12)	Amortization	Ending Loan Balance
1	$60,000.00	$600.00	$600.00	$0.00	$60,000.00
2	60,000.00	600.00	600.00	0.00	60,000.00
3	60,000.00	600.00	600.00	0.00	60,000.00
4	60,000.00	600.00	600.00	0.00	60,000.00
5	60,000.00	600.00	600.00	0.00	60,000.00
6	60,000.00	600.00	600.00	0.00	60,000.00
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
358	60,000.00	600.00	600.00	0.00	60,000.00
359	60,000.00	600.00	600.00	0.00	60,000.00
360	60,000.00	600.00	600.00	0.00	60,000.00

Negative Amortizing, Constant Payment Mortgage (CPM) Loans

A final category of fixed rate, constant payment loans to be discussed in this chapter is referred to as a negative amortizing loan. This pattern may occur when: (1) the borrower and lender agree that the loan balance at maturity will be *greater* than the initial loan amount; that is, $FV > PV$ or (2) payments are negotiated to be lower than the periodic interest due on the loan.

To illustrate the first case, if $60,000 is borrowed but the amount due at maturity will be $80,000, then monthly payments will be $594.28. This result can be obtained as shown in the calculator solution below.

Solution:

$n = 360$

$i = 12\%/1\%$ or .01

$PV = -\$60,000$

$FV = \$80,000$

Solve for *PMT* = $594.28

Function:
PMT (*n*, *i*, *FV*, *PV*)

We should also point out that the *pay rate,* calculated as $594.28 ÷ $60,000, is now equal to .009905. This is lower than the accrual rate, which is .12 ÷ 12, or .010000. When the pay rate used to determine monthly payments is *less* than the interest rate specified in the note, also referred to as the "accrual rate," negative amortization occurs. This is because payments are not large enough to meet monthly interest requirements. The difference between payments actually made and those that would be made on an interest-only loan is deferred and becomes additional amounts owed by the borrower to the lender. These amounts must also earn interest. The rate at which they earn interest is usually the same as the interest rate on the note. In this example, it would be .12 ÷ 12, or .010000 per month. The monthly payment pattern, accrued interest, and loan balance for the negative amortizing loan are shown in Exhibit 4–6.

The second example for negative amortization loans is to set the desired payment pattern, and then to solve for the loan balance that will include negative amortization. For

EXHIBIT 4–6
Negative Amortizing Loan Pattern

Month	Beginning Loan Balance	Monthly Payment	Interest (.12 ÷ 12)	Amortization	Ending Loan Balance
1	$60,000.00	$594.28	$600.00	$(5.72)	$60,005.72
2	60,005.72	594.28	600.06	(5.78)	60,011.50
3	60,011.50	594.28	600.12	(5.84)	60,017.34
4	60,017.34	594.28	600.17	(5.90)	60,023.24
5	60,023.24	594.28	600.23	(5.95)	60,029.19
6	60,029.19	594.28	600.29	(6.01)	60,035.21
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
358	79,394.97	594.28	793.95	(199.67)	79,594.65
359	79,594.65	594.28	795.95	(201.67)	79,796.31
360	79,796.31	594.28	797.96	(203.69)	80,000.00

example, in the above problem, if we assume that the loan is negotiated with monthly payments *preset* at $400, the loan balance at the end of year five could be calculated as

Solution:	Function:
$PV = -\$60,000$	$FV(PV, n, i, PMT)$
$n = 60$	
$i = 12\%/12$	
$PMT = \$400$	
Solve for FV = $76,333.93	

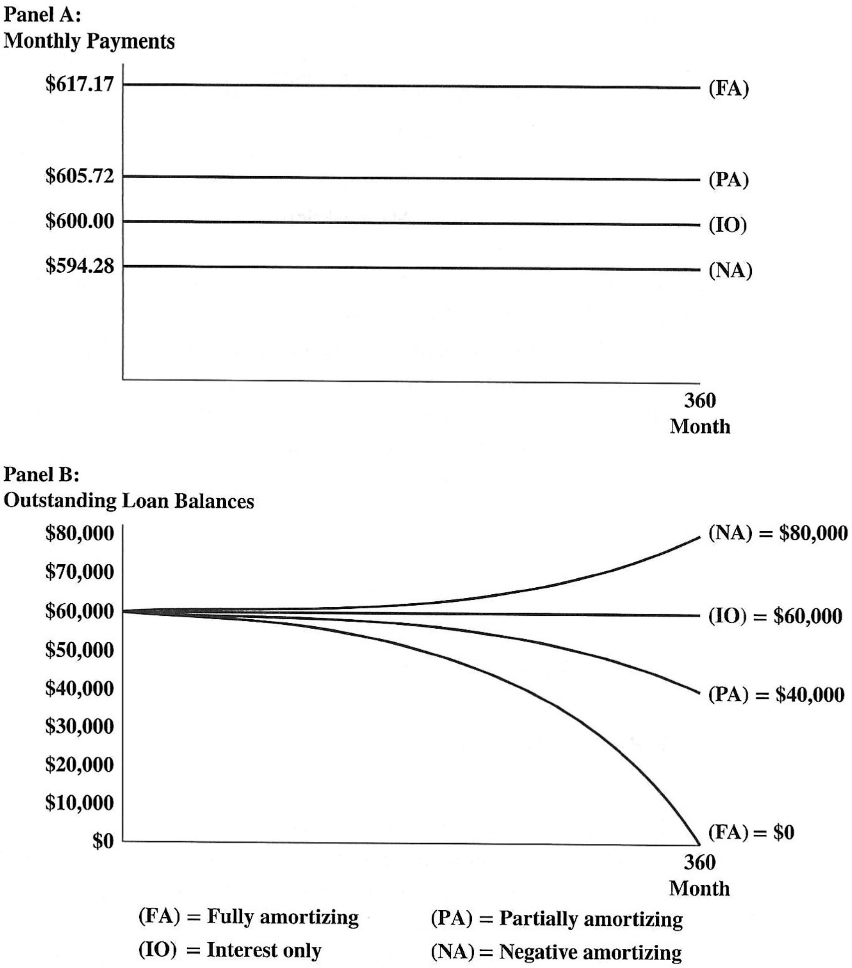
Note that because the monthly payments have been set at $400, which is below the $600 in monthly interest being accrued, $200 per month in interest is *not being collected.* Over the 60-month period, this amount must be added to the loan balance and also must earn 12 percent interest compounded monthly. This results in a total of $16,333.93 being added to the loan balance. Note that the balance outstanding at the end of year 5 is now $76,333.93. When compared to the initial loan amount of $60,000, we can see that negative amortization has increased the loan balance by $16,333.93.

Summary and Comparisons: Fixed Interest Rate, Constant Payment Mortgage (CPM) Loans with Various Amortization Patterns

At this point, we have discussed four types of fixed interest rate, constant payment mortgage (CPM) loans, categorized by amortization pattern. Exhibit 4–7 (panel A), provides a diagram showing the four distinct monthly mortgage payments for our base case example (a $60,000 loan made at 12% for 30 years). Looking at the monthly loan balance pattern for each of the four cases, the reader should begin to understand the trade-offs involved when considering these different loan types. Furthermore, the strategy and objectives of borrowers should be apparent. Holding all else constant, *monthly payments* range from the highest amount when loans are *fully amortizing* to the lowest amount when they are *negatively amortizing.*[5] Moving to panel B, we can see that loan balances are highest when payments

[5] Obviously, there are many combinations in between.

EXHIBIT 4–7
Base Case: Summary and Comparison of Monthly Payments and Loan Balances for Selected Fixed Interest Rate, Constant Payment Mortgage (CPM) Loan Patterns



are lowest and lowest when payments are highest. Risk is also considerably different under each of the four scenarios. It should be clear that if the *same property* is being financed by the *same borrower* in each case, risk will be greatest in the case involving the negative amortizing loan.

We should emphasize that we have deliberately kept our base case example the same throughout the chapter in order to emphasize the most important aspects of *mortgage loan mechanics* and *loan structuring*. Structuring mortgage loans in real estate financing is, in many cases, critical to the success or failure of a transaction; therefore, knowledge of the relationships discussed in this chapter should prove valuable to the reader. For instance, many of these concepts will prove to be beneficial in later chapters when applications involving structuring partnerships, mortgage-backed securities, and other financing vehicles and arrangements are considered. Another important observation should be made regarding the interest rate in our base case example, which has been kept constant. In reality, we would expect the interest rates on loans to increase as risk increases. Finally, there are additional factors that affect risk when structuring loans. These factors include varying loan maturities and varying the amount borrowed. Changes in these factors tend to increase or decrease risk, depending upon the type of payment and loan amortization patterns chosen.

Determining Loan Balances

Because most mortgage loans are repaid before they mature, mortgage lenders and real estate investors must be able to determine balances on mortgage loans at any time. As previously indicated, even though most loans are made for terms of 25 or 30 years, mortgages are usually repaid from 8 to 12 years after they are made. Therefore, it is very important to know what the loan balance will be at any time when financing real estate. There are many ways to find loan balances. We will demonstrate several approaches in this section.

To illustrate one approach, let us return to our base case example of the $60,000 fully amortizing loan made at 12 percent interest for a term of 30 years. After 10 years, the borrower decides to sell the property. To do so, the existing loan must be paid off. How much will have to be repaid to the lender after 10 years? We could construct a loan balance schedule as in Exhibit 4–1; however, this is time consuming and unnecessary. Finding loan balances can be accomplished in other ways. First, one can simply find the present value of the $617.17 payments at the 12 percent contract rate of interest *for the 20 years remaining until maturity.* For example

Solution:

$n = 20 \times 12 = 240$

$i = 12\%/12 = 1\%$ or .01

$PMT = -\$617.17$

$FV = 0$

Solve for $PV = \$56{,}051.02$

Function:
$PV\ (n, i, FV, PMT)$

The unpaid balance after 10 years is $56,051.02. This exercise also points out another interesting fact. Because the payments include interest and a reduction of principal each month, by "removing" all interest from all remaining payments, it follows that only the principal can remain. Discounting the $617.17 monthly payments at an annual rate of 12 percent compounded monthly is a process that really amounts to "removing interest" from the payments. Hence, after "removing interest" by discounting, we ascertain that the unamortized or unpaid balance must be $56,051.02.

Another way of finding a loan balance using a financial calculator is to calculate future value (*FV*). This may be done by entering payments (*PMT*), the initial loan amount as a present value (*PV*), the number of periods (*n*) (e.g., months) that the loan has been amortized, and the interest rate (*i*). Future value (*FV*) then can be calculated as the outstanding loan balance.

Solution:

$PV = -\$60{,}000$

$n = 10 \times 12 = 120$

$i = 12\%12 = 1\%$ or .01

$PMT = \$617.17$

Solve for $FV = \$56{,}050.24$ (difference due to rounding of payment)

Function:
$FV\ (PV, n, i, PMT)$

An alternative method for finding mortgage balances at any time in the life of the mortgage is to divide the interest factor for the *FV* of $1 per period, for the year in which the balance is desired, by the *FV* of $1 per period factor for the original term of the mortgage, and then subtract that result from 1. The result of this computation is the loan balance expressed as a percentage of the original amount.

Solution:	Function:
% of loan balance:	*FV* (*PV*, *n*, *i*, *PMT*)
$1 - \frac{FV \text{ at } n = 10 \text{ year}}{FV \text{ at } n = 30 \text{ year}}$	

STEP 1: *FV* at 10 years:
$n = 120, i = 12\%, PMT = -1, PV = 0$

Solve for *FV* = $230.03869

STEP 2: *FV* at 30 years:
$n = 360, i = 12\%, PMT = -1, PV = 0$

Solve for *FV* = $3,494.9641

STEP 3: % of loan outstanding
1 – ($230.03869 ÷ $3,494.9641) = 93.42%

This is a more general formula that can be used regardless of the original dollar amount of the loan or payments because it gives a solution in percentage form. In this case, we determine that approximately 93.42 percent, or $56,052, of the original loan balance ($60,000) is still outstanding. This solution is nearly the same result that we obtained by discounting. The difference is due to rounding.

It should also be noted that many financial calculators may be purchased with a loan amortization function preprogrammed in them. Finding a loan balance becomes simply a matter of inputting the loan information and pressing an amortization key.

Finding Loan Balances—Other Amortization Patterns

We now consider how to solve for loan balances when loans are prepaid at the *end of 10 years* for the other three CPM loan categories discussed in this section.

Solution:

Partial Amortizing	Interest Only	Negative Amortizing
PV = –$60,000	*PV* = –$60,000	*PV* = –$60,000
i = 12%/12	*i* = 12%/12	*i* = 12%/12
n = 120	*n* = 120	*n* = 120
PMT = $605.72	*PMT* = $600	*PMT* = $594.28
Solve for *FV* = $58,684	**Solve for *FV*** = $60,000	**Solve for *FV*** = $61,316

The reader should note that simply substituting the loan payment that we calculated previously for each loan amortization pattern and solving for *FV* yields the appropriate loan balance at the end of 10 years.

A Note on Interest Rates, Loan Yields, and Early Payoff

At this point, it is not unusual for some readers to wonder: Is the cost to borrowers or the yield[6] to lenders different if the loan is repaid early or prior to maturity? It should be stressed that *regardless of when loans are repaid,* either at maturity or any time prior to maturity, each loan pattern discussed here will yield 12 percent compounded monthly to the lender. Similarly, the cost for the borrower also will be an annual rate of 12 percent compounded monthly, *regardless of when the loan is repaid.* This is the case because each loan pattern is structured around an accrual rate of 12 percent. Thus, interest is accrued and will eventually be paid on all loan balances outstanding. This is the case for fully, partially, zero, or negative amortizing loans.

However, we also should stress that if *additional financing fees* are charged by the lender, the effective cost of borrowing to the borrower will usually be greater than the interest rate. Yields to the lender also will vary depending on when the loan is repaid. We now turn to the topic of loan fees and the cost of borrowing.

Loan Closing Costs and Effective Borrowing Costs

Loan closing costs are incurred in many types of real estate financing, including residential property, income property, construction, and land development loans. Closing costs that affect the cost of borrowing are additional finance charges levied by the lender. These charges constitute additional costs to the borrower and must be included as a part of the cost of borrowing. Generally, lenders refer to these additional charges as *loan fees.*

Loan fees can be classified into two categories: loan origination fees and loan discount fees. **Loan origination fees** are intended to cover expenses incurred by the lender for processing and underwriting loan applications, preparation of loan documentation and amortization schedules, obtaining credit reports, and any other expenses that the lender believes should be recovered from the borrower. Lenders usually charge these costs to borrowers when the loan is made, or "closed," rather than charging higher interest rates. They do this because if the loan is repaid soon after closing, the additional interest earned by the lender as of the repayment date may not be enough to offset the fixed costs of loan origination. For example, assume that the prevailing interest rate on a $60,000 mortgage is 12 percent and the lender believes that expenses equal to $1,000 will be incurred to close the loan. If the lender chose to increase the interest rate to 12.25 percent to recover these origination costs, an additional $150 (approximately) would be collected during the first year ($60,00 × .0025). If the loan was repaid after the first year, the lender would not recover the full $1,000 in origination costs. This is why lenders attempt to "price" these origination costs separately

The second category of loan fees is **loan discount fees,** or **points.**[7] This charge also represents an additional finance charge, but its primary purpose is to *adjust the yield* on a mortgage loan. In the context of real estate lending, loan discounting amounts to a borrower and lender negotiating the terms of a loan based on a certain loan amount. The lender then discounts the loan by charging a fee, which will be deducted from the loan funds advanced to the borrower. Payments made by the borrower, however, are based on the contract amount of the loan. For example, assume a borrower and lender agree on a $60,000 loan at 12 percent interest for 30 years. The lender actually disburses $58,200 to the borrower by including a loan discount charge of 3 percent (points), or $1,800. The borrower is required to repay $60,000 at 12 percent interest for 30 years. However, because the borrower actually receives $58,200 but must repay $60,000 plus interest, it is clear that the actual borrowing cost to the buyer is greater than 12 percent.

Why do pricing practices such as discounting exist? Many reasons have been advanced. One reason given by lenders is that mortgage rates tend to be somewhat "sticky" in upward and downward moves. For example, suppose that the prevailing rate is 12 percent and

[6] When the term *yield* is used in lending it is usually referred to from the lender's perspective, who evaluates loans on the basis of profitability. Also, in addition to interest, lenders frequently charge fees on loans and use the term *yield* to include interest and fees on loans that they make.

[7] Lenders in some areas of the country refer to loan discounts as "discount points" or simply "points." In conventional mortgage lending, the borrower usually pays this charge, which adds to financing costs. In this chapter, we are concerned with conventional lending situations where the borrower pays the loan discount as a part of origination fees.

market pressures begin to push rates upward. However, instead of all lenders moving the rate to perhaps 12.25 percent, one or more lenders may continue to quote 12 percent as the loan rate. However, in lieu of raising the interest rate, these lenders may charge borrowers loan discount points. This practice is also referred to as the borrower **"buying down the interest rate."** That is, instead of paying 12.25 percent interest, the borrower will buy down the interest rate to 12 percent by paying discount points to the lender.

Another reason points may be charged is because many mortgage loans are originated by lenders who have entered into contracts with investors to assemble, or package, then sell them a specific number, or dollar amount, of such loans. Such contracts with investors may require that loans are to be sold to yield investors a *rate of interest* very close to the rate that lenders expect to charge borrowers. Therefore, in order for originating lenders to earn a profit, points are charged to borrowers to provide lenders with revenue for performing origination services prior to the delivery of loans to investors.

Another situation may provide lenders with an opportunity to replace revenue in a declining interest rate environment. In these cases, interest rates may begin to decline before the date that the loan is made to the borrower but after the date on which the lender and investor agree on the yield on the packaged mortgages to be sold. In this case, loans will be originated at a lower interest rate, and the lender will charge discount points in order to offset the decline in interest rates being charged to borrowers as the loans are being assembled or packaged.

A final reason for loan discount fees is that lenders believe that, in this way, they can better price the loan relative to the *risk* they take. For example, in the beginning of this chapter we referred to the risk premium component (p) of the interest rate. However, the risk for some individual borrowers is slightly higher than it is for others. Further, these loans may require more time and expense to process and control. In this case, instead of raising the interest rate, discount points may be charged by the lender (in addition to origination fees) to compensate for the slightly higher risk.

The practice of using loan origination fees and discount points has historically prevailed throughout the lending industry. It is important to understand (1) how these charges affect borrowing costs and (2) how to include them in computing effective borrowing costs on loan alternatives when financing any real estate transaction.

Loan Fees and Borrowing Costs

To illustrate loan fees and their effects on borrowing costs in more detail, consider the following problem: A borrower would like to finance a property for 30 years at 12 percent interest. The lender indicates that loan fees(origination and discount points) equal to 3 percent of the loan amount will be charged to obtain the loan. What is the actual interest cost of the loan?

We structure the problem by determining the amount of the loan fee $[.03 \times (\$60{,}000) = 1{,}800]$. Second, we know that the monthly mortgage payments based on $60,000 for 30 years at 12 percent will be $617.17. Now we can determine the effect of the origination fee on the interest rate being charged as follows:

Contractual loan amount	$60,000
Less: Loan fees	1,800
Net cash disbursed by lender	$58,200
Amount to be repaid:	
Based on $60,000 contractual loan amount, $617.17 for 30 years.	

In other words, the amount actually disbursed by the lender will be $58,200, but the repayment will be made on the basis of $60,000 plus interest at 12 percent compounded monthly, in the amount of $617.17 each month. Consequently, the lender will earn a yield on the $58,200 actually disbursed, which must be greater than 12 percent.

Using a financial calculator, we can calculate the **effective interest rate** for the loan, assuming it is outstanding until maturity, as 12.41 percent. This yield is obviously higher than the 12 percent contract, or nominal, rate of interest specified in the note or mortgage.

Solution:

$n = 30 \times 12 = 360$

$PMT = -\$617.17$

$PV = \$58,200$

$FV = 0$

Solve for

i (monthly) = 1.034324% monthly rate

i (annualized) = 1.034324% × 12 = 12.41% effective interest rate

Function:
$i\,(n, PMT, PV, FV)$

This computation forms the basis for a widely used rule of thumb in real estate finance; that is, for every 2 percentage points in the origination fee charged to the borrower, the effective cost to the borrower, or investment yield earned by the lender, increases by approximately one-fourth of a percent above the contract rate. Note that in our solution, we obtained an effective rate of 12.41 percent, versus 12.375 percent using the approximation. While this estimate is close to the yield calculated in one example, we have assumed that the loan remains outstanding until maturity. However, most loans on the average are "pre-paid" or paid off long before maturity. Hence, this rule of thumb, while helpful, generally provides a very rough estimate of the effective cost (yield) for most mortgage loans.[8]

We should point out that, in the above example, the 12.41 percent effective interest rate is an *annual rate of interest, compounded monthly*. Many analysts prefer to calculate the **effective annual interest rate** (recall that this is equivalent to an annual rate of interest, compounded annually). We can easily calculate this as follows:

STEP 1:

Solution:

$i = 12.41\%$

$PV = -\$60,000$

$n = 12$

Solve for $FV = \$67,884.47$

Function:
$FV\,(PV, i, n)$

STEP 2:

Solution:

$FV = \$67,884.47$

$n = 1$

$PV = \$60,000,00$

Solve for $i = 13.14\%$

[8] This rule of thumb will become very inaccurate if the payoff period is very short relative to the maturity and when the level of interest rates increases.

Truth-in-Lending Requirements and the Annual Percentage Rate (*APR*) Concept Box 4.2

Because of problems involving loan fees and the potential abuse by some lenders of charging high fees to unwary borrowers, Congress passed the federal Truth-in-Lending Act to provide disclosure regarding the effects of loan fees to *consumers purchasing residences.** As a result of this legislation, the lender must disclose to consumers the **annual percentage rate (*APR*)** being charged on the loan. (We will deal with the *APR* more extensively in Chapter 8.) Calculation of the *APR* is generally done in the manner shown for calculating the effective interest rate, assuming that the loan is repaid at maturity. The *APR* in our chapter example would be disclosed to the borrower at closing as 12.41 percent. The *APR*, then, does reflect origination fees and discount points and treats them as additional income or yield to the lender regardless of what costs, if any, the fees are intended to cover. Other fees charged by the lender as a condition for obtaining the loan also may have to be included in the *APR*.†

* See Regulation Z of the Federal Reserve Board, 12 C.F.R., sec. 226, as amended.

† Generally, the *APR* disclosed to the borrower is the effective interest rate computed under the assumption that the loan will be outstanding until maturity. The lender *may* round the *APR* to the nearest quarter percent; however, it must fall in a range that is within one-eighth of 1 percent above or below the APR that is calculated based on federal guidelines as disclosed in Regulation Z. If the reader desires greater accuracy in these computations, consult *Computational Procedures Manual for Supplement 1 to Regulation Z of the Federal Reserve Board: Calculator Instructions* (Office of the Comptroller of the Currency, February 1978).

Note that because of the effects of (1) monthly compounding and (2) discount points, the *effective annual interest rate* is 13.14 percent, whereas the nominal interest rate is 12 percent and the effective interest cost is 12.41 percent (annual rate of interest compounded monthly) for the loan in our example.

Loan Fees and Early Repayment: Fully Amortizing Loans

An important effect of loan fees and early loan repayment must now be examined in terms of the effect on interest rate. We will show in this section that when loan fees are charged and the loan is paid off before maturity, the effective interest rate of the loan increases even further than when the loan is repaid at maturity.

To demonstrate this point, we again assume our borrower obtained the $60,000 loan at 12 percent for 30 years and was charged an $1,800 (3 percent) loan origination fee. At the end of *five years,* the borrower decides to sell the property. The mortgage contains a due-on-sale clause; hence, the loan balance must be repaid at the time the property is sold. What will be the effective interest rate on the loan as a result of both the origination fee and early loan repayment?

To determine the effective interest rate on the loan, we first find the outstanding loan balance after five years to be $58,598.16. To solve for the yield to the lender (cost to the borrower), we proceed by finding the rate at which to discount the monthly payments of $617.17 *and* the lump-sum payment of $58,598.16 after five years so that the present value of both equals $58,200, or the amount actually disbursed by the lender.

This presents a new type of discounting problem. We are dealing with an annuity in the form of monthly payments for five years *and* a loan balance, or single lump-sum receipt of cash, at the end of five years. To find the yield on this loan, we proceed as follows:

STEP 1: Solve for remaining balance:

Solution:

$n = 25 \times 12 = 300$

$i = 12\%/12 = 1\%$ or .01

$PMT = -\$617.17$

$FV = 0$

Solve for $PV = \$58,598.16$ (remaining balance)

Function:

$i\ (n, PMT, FV, PV)$

STEP 2: Next, solve for the interest payment, holding a 30-year loan, for five years, and discounted by the loan origination fee:

Solution:

$n = 5 \times 12 = 60$

$PMT = -\$617.17$

$PV = -\$58,200$

$FV = -\$58,598.16$

Solve for i (monthly) = 1.069%

Solve for i (monthly) = 1.069% × 12 = 12.82%

Function:

$i\ (n, PMT, FV, PV)$

An Excel template is included on the Website for this book that may be used to calculate the same yield shown in this example (*www.mhhe.com/bf16e*).

This formulation simply says that we want to find the interest rate (i) that will make the present value of both the \$617.17 monthly annuity and the \$58,598.16 received at the end of five years equal to the amount disbursed, or \$58,200. From the above analysis, we can conclude that the actual yield (or actual interest rate) that we have computed to be approximately 12.82 percent is higher than both the contract interest rate of 12 percent and the 12.41 percent yield computed assuming that the loan was outstanding until maturity. This is true because the \$1,800 origination fee is earned over only five years instead of 30 years, which is equivalent to earning a higher rate of compound interest on the \$58,200 disbursed. Hence, when this additional amount earned is coupled with the 12 percent interest being earned on the monthly loan balance, this increases yield to 12.82 percent.[9]

Relationship between Yield and Time

Based on the preceding discussion, we can make some general observations about the relationship of mortgage yields and the time during which mortgages are outstanding. The first observation is that the effective interest rate on a mortgage will always be equal to the contract rate of interest when no finance charges are made at the time of loan origination or repayment. This follows because, as we saw in Exhibit 4–2, the level payment pattern assures the lender of earning only a given annual rate of interest, compounded monthly, on the monthly outstanding loan balance. Hence, the outstanding mortgage balance can be repaid at any time, and the lender's yield (borrower's cost) will not be affected. It will be equal to the contract rate of interest.

The second observation is that if origination or financing fees are charged to the borrower, the following occurs: (1) the effective yield will be higher than the contract rate of interest and (2) the yield will increase as repayment occurs sooner in the life of the mortgage. These relationships can be explained by referring to Exhibit 4–8, where the two curves, *A* and *B*, represent the mortgage yield pattern under two assumptions. Line *A* represents the effective yield, or cost, when no financing fees are charged to the borrower. In our previous example, the yield would remain at 12 percent, equal to the contract rate of

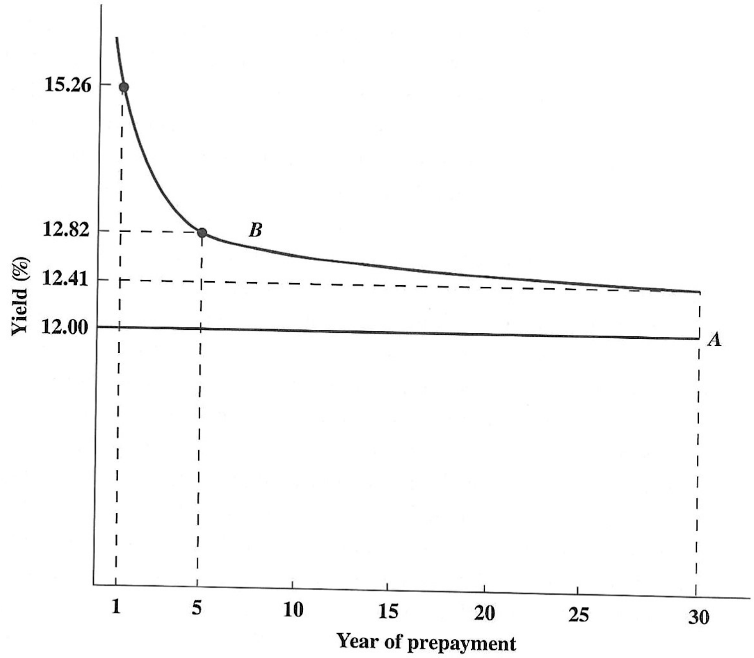
[9] If the loan is repaid in less than one year, the yield becomes larger and approaches infinity, should the loan be repaid immediately after closing.

Web App

There are numerous companies offering mortgage rate information on the Internet, such as www.bankrate.com. Find a quote for a 15-year fixed rate mortgage on a $150,000 primary residence valued at $200,000. What are the current interest rate, discount points, and other lender fees for the loan you found?

Calculate the effective cost of the mortgage over the lifetime of the loan. Calculate the effective cost of paying off the mortgage after seven years. How do these rates compare to the stated *APR?* What accounts for the discrepancy between the effective rate and the *APR?*

EXHIBIT 4–8
Relationship between Mortgage Yield and Financing Fees at Various Repayment Dates



interest regardless of when the loan is repaid; hence, the horizontal line is over the range of 0 to 30 years. Curve *B* represents a series of loan yields computed under the assumptions that a 3 percent origination fee is charged to the borrower and that the loan is prepaid in various years prior to maturity. In our example, we note that the yield earned by the borrower is 15.26 percent if the loan is repaid one year after closing and that it diminishes and eventually equals 12.41 percent after 30 years. Hence, we can again conclude that if financing fees are charged to the buyer, the effective yield to the lender (cost to the borrower) can range from one that is extremely high if prepaid, say, after one year (the yield in that case would be approximately 15.26 percent) to a yield that would be considerably lower if repaid at maturity, or 12.41 percent. If a borrower knows when he or she expects to repay a loan, this method of computing the effective borrowing cost should be used. This is particularly important if the borrower is comparing alternative loans with different terms.

Prepayment Penalties

Many borrowers mistakenly take for granted that a loan can be prepaid in part or in full anytime before the maturity date. This is not the case; if the mortgage note is silent on this matter, the borrower may have to negotiate the privilege of early repayment with the lender. In our example using the 30-year mortgage loan, the note will usually specify that 360 payments are to be made at the end of each month until the loan is repaid. Any payments made in advance of this monthly schedule, or in amounts greater than the scheduled amount are prepayments and may be subject to penalties. However, many mortgages provide explicitly that the borrower can pay a **prepayment penalty** should the borrower desire to prepay the loan.

One rationale for a prepayment penalty is that the lender may be trying to recover a portion of loan origination costs not charged to the borrower at closing. This may have been done by the lender to compete for the loan by making initial closing costs lower for the borrower. Another reason for prepayment penalties is that the lender has agreed to extend funds for a specified time, 30 years in our present example. Early payment from the lender's view may represent an unanticipated inflow of funds that may or may not be readily reinvested during periods when mortgage rates are unstable or are expected to decline. However, if interest rates undergo a sustained increase over long periods of time, lenders usually welcome early repayments since they may be able to loan out funds again at higher rates of interest.

Another possible reason for prepayment penalties is that they are not included in the computation of the *APR;* hence, they are not included in the *APR* disclosure to the borrower. Borrowers may not be able to determine the effect of these penalties on borrowing costs and, therefore, the penalties represent a technique lenders use to increase yields. Some states have begun prohibiting the enforcement of prepayment penalties to individuals financing residences if the loan has been outstanding more than some specified minimum number of years. Also, in some areas where penalties are allowed, lenders may waive them if the buyer of a property agrees to originate a new loan with the same lender. Some states have passed consumer protection laws that allow for prepayment if the borrower has a job transfer to another state, joins the military, and for other unanticipated reasons. (In the case of loans on commercial real estate, prepayment penalties are common practice. Prepayment penalties may have to be paid even when the borrower wants to sell the mortgaged property. The existence and amount of penalties must be negotiated when such loans are originated.)

Because of the use of prepayment penalties, we want to know the effective mortgage loan yield (interest rate) when both a loan discount fee and a prepayment penalty are charged on the loan. To illustrate, we consider both the effects of the 3 percent loan discount and a 3 percent prepayment penalty on the outstanding loan balance for the $60,000, 30-year loan with a contract interest rate of 12 percent used in the preceding section. We assume the loan is the effective interest rate to the borrower (yield to the lender). To solve for the yield, mortgage funds actually disbursed in this case will be $60,000 minus the origination fee of $1,800, or $58,200. Taking the loan discount fee into account, we want to find the discount rate which, when used to discount the series of monthly payments of $617.17 plus the outstanding loan balance of $58,598.16 and the prepayment penalty of $1,758 (3% of $58,598.16), or a total of $60,356, will result in a present value equal to the amount of funds actually disbursed, $58,200.

Using a financial calculator, with a 3 percent origination fee, early payment in the fifth year, and a 3 percent prepayment penalty, we see that the effective yield on the loan will increase to about 13.25 percent.

Solution:

$n = 5 \times 12 = 60$

$PMT = -\$617.17$

$PV = \$58,200$

$FV = -\$60,356$

Solve for i (monthly) = 1.10425%

Solve for i (annually) = (1.10425% × 12) = 13.25%

Function:

$i\ (n, PMT, FV, PV)$

In this case, the *APR* will still be disclosed at 12.375 percent, which reflects the loan discount only, not the prepayment penalty, and assumes the loan is repaid at the end of 30 years. The actual yield computed here of 13.25 percent is a marked difference from both the loan contract rate of 12 percent and the disclosed *APR* of 12.375 percent.

Charging Fees to Achieve Yield, or When "Pricing" FRMs

In the preceding examples, we have developed the notion of the effective borrowing costs and yield from a given set of loan terms. However, we should consider how these are determined by lenders when "pricing" a loan. As we discussed earlier in the chapter, lenders generally have alternatives in which they can invest funds. Hence, they will determine available yields on those alternatives for given maturities and weigh those yields and risks against yields and risks on mortgage loans. Similarly, competitive lending terms established by other lenders establish yields that managers must consider when establishing loan terms. By continually monitoring alternatives and competitive conditions, management establishes loan offer terms for various categories of loans, given established underwriting and credit standards for borrowers. Hence, a set of terms designed to achieve a competitive yield on categories of loans representing various ratios of loan-to-property value (70% loans, 80% loans, etc.) are established for borrowers who are acceptable risks. These terms are then revised as competitive conditions change.

If, based on competitive yields available on alternative investments of equal risk, managers of a lending institution believe that a 13 percent yield is competitive on 80 percent mortgages with terms of 30 years and expected repayment periods of 10 years, how can they set terms on all loans made in the 80 percent category to ensure a 13 percent yield? Obviously, one way is to price all loans being originated at a contract rate of 13 percent. However, management may also consider pricing loans at 12 percent interest and charging either loan fees or prepayment penalties or both to achieve the required yield. Why would lenders do this? Because (1) they have fixed origination costs to recover and (2) competitors may still be originating loans at a contract rate of 12 percent.

To illustrate how fees for all loans in a specific category can be set, we consider the following solution:

STEP 1: Solve for payment

Solution:

Function: $PMT\,(PV, i, n, FV)$

$PV = -1$

$i = 12\%/12 = 1\%$ or .01

$n = 30 \times 12 = 360$

$FV = 0$

Solve for $PMT = .010286$

STEP 2: Solve for loan balance EOY_{10}

Solution:

Function: $FV\,(PV, i, n, PMT)$

$PV = -1$

$n = 120$

$i = 12\%/12$

$PMT = .010286$

Solve for $PMT = .934180$

STEP 3: Solve for PV, $i = 13\%$

Solution:

Function: $PV\,(FV, i, n, PMT)$

$FV = .934180$

$i = 13\%12$

$n = 120$

$PMT = .010286$

Solve for $PV = .9453$

The result $PV = .9453$ means that the net disbursement at loan closing should be 94.53 percent, or 94.5 percent (rounded), of the loan amount. Therefore, if the loan is priced by offering terms of 12 percent interest and a 5.5 percent origination fee (100% − 94.5%) and the loan is repaid at the end of 10 years (EOY_{10}), management will have its 13 percent yield.

Financing Loan Fees and Other Closing Costs

In some situations, the lender may also agree to provide financing for loan fees. In our example, this would mean that the loan amount would be $61,800 or the sum of the $60,000 loan amount and the loan fees of $1,800. The borrower would not be required to pay loan fees of $1,800 at closing, however monthly payment would be higher. For example payments would be

$$\begin{aligned} PV &= -61{,}800 \\ I &= 1\% \text{ or } .01 \\ N &= 360 \\ FV &= 0 \\ \text{Solve for PMT} &= \$635.68 \end{aligned}$$

Function:
PMT (PV, i, n, FV)

This compares to our previous payment of $617.17 if the former pays $1,800 at closing. Borrowers may prefer to make higher monthly payments in order to reduce the amounts due at closing.

W should point out that the APR will now be based on the higher loan amount and monthly payments also will be higher. In our example based on a 30 year loan we have

$$\begin{aligned} PV &= -60{,}000 \\ I &= 1\% \text{ or } .01 \\ N &= 360 \\ FV &= 0 \\ Pmt &= 635.68 \\ \text{Solve for } i &= 12.40\% \end{aligned}$$

Function:
i (PV, Pmt, n, FV)

Note that when the loan fees are financed, the APR will be 12.40% as compared to 12.41% when the loan fees are paid at closing. In our example, financing loan fees has a very small impact on the APR. However, we should point out that if the loan fees were higher there could be a more significant difference. The lower APR comes about because the fees paid by the borrower at closing are being pushed into slightly higher payments further into the future which, in this case, benefits the borrower and therefore lowers the APR.

The reader should also remember that when loan fees are financed, finding loan balances, and effective borrowing costs will also change as the loan amount is now $60,000 and monthly payments are $635.68.

Other FRM Loan Patterns—Declining Payments and Constant Amortization Rates

The **constant amortizing mortgage (CAM)** loan pattern represents yet another variation that may be considered in loan structuring. Payments on CAMs are determined first by computing a constant amount of each monthly payment to be applied to principal or monthly amortization. Interest is then computed on the monthly loan balance and added to the monthly amount of amortization. The total monthly payment is determined by adding the constant amount of monthly amortization to interest on the outstanding loan balance. Consider the following example of a CAM loan. A loan was made for $60,000 for a 30-year term at 12 percent (annual rate compounded monthly); payments were to be made monthly and were to consist of *both* interest and amortization (or reduction of principal), so that the loan would be repaid at the end of 30 years. However, amortization is determined by dividing the number of months or the term of the loan (360) into the loan amount ($60,000), resulting in a reduction of principal of $166.67 per month. Interest would be computed on the outstanding loan balance and then added to amortization to determine the monthly payment. An illustration of the payment pattern and loan balance is shown in Exhibit 4–9.

The computation in Exhibit 4–9 shows that the initial monthly payment of $766.67 includes amortization of $166.67, plus interest computed on the outstanding loan balance. The total monthly payment would decline each month by a constant amount, or $1.67 (.01 × $166.67). The loan payment and the balance patterns are shown in Exhibit 4–10. It should be kept in mind that in spite of the declining monthly payment pattern, the yield in the CAM remains at 12 percent interest. Like the fully amortizing, partially amortizing, interest only, and negative amortizing loans, the CAM is also a fixed interest rate mortgage loan. Its payments and balance pattern are simply structured differently than the loans in our previous examples.

The constant amortizing payment pattern is considered to be a very conservative loan structure because it places primary emphasis on the amortization of the loan. Nonetheless, it demonstrates yet another alternative that may be of value when structuring a loan. Or perhaps, it may be combined with another loan amortization pattern. It also demonstrates the range of alternative payment and amortization patterns available within the fixed interest rate class of mortgage loans.

Amortization Schedules and Callable Loans

A final variation in loan structuring to be included in this section is a commonly used provision in which an amortization schedule, if specified in the note, is different from the maturity date. For example, two parties may agree that a $100,000 fully amortizing loan will be made at 12 percent interest with monthly payments calculated based on a 30 year *amortization schedule.* However, both parties agree that the loan *will be,* or may be, **callable** at the lender's option, at the end of 10 years. In this case, payments can be calculated as follows:

STEP 1: Determine payment

Solution:

$PV = -\$100,000$

$i = 12\%/12 = .01$

$n = 30 \times 12 = 360$

$FV = 0$

Solve for $PMT = \$1{,}028.61$

Function:

$PMT\ (i, n, PV, FV)$

STEP 2: Determine balance

Solution:

$PV = -\$100,000$

$PMT = \$1{,}028.61$

$i = 12\%/12 = .01$

$n = 12 \times 10 = 120$

Solve for $PV = \$93{,}418.59$

Function:

$FV\ (PV, PMT, i, n)$

EXHIBIT 4–9
Monthly Payments and Loan Balance (Constant Amortizing Loan)

(1) Month	(2) Opening Balance ×	(3) Interest (.12 ÷ 12)	(4) Amortization	(3)+(4) Monthly Payment	(2)–(4) Ending Balance
1	$60,000.00	$600.00	$166.67	$766.67	$59,833.33
2	59,833.33	598.33	166.67	765.00	59,666.66
3	59,666.66	596.67	166.67	763.34	59,499.99
4	59,499.99	595.00	166.67	761.67	59,333.32
5	59,333.32	593.33	166.67	760.00	59,166.65
6	59,166.65	591.67	166.67	758.34	58,999.98
.	.	.	.	.	.
.	.	.	.	.	.
.	.	.	.	.	.
360	166.67	1.67	166.67	168.34	–0–

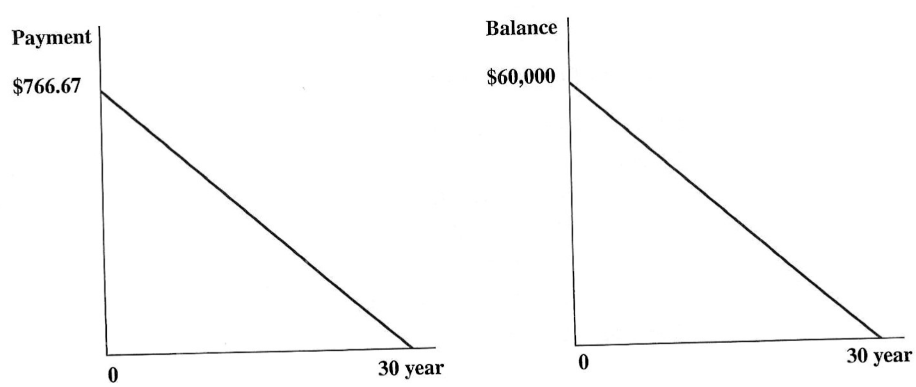
If the loan is called at the end of 10 years, the amount due would be the balance at that time, or $93,418.59 (reader should verify). This loan structure is used by borrowers and lenders (1) to keep monthly loan payments lower than they would be if a fully amortizing loan was made for 10 years, (2) to keep the loan maturity relatively short at 10 years, and (3) to achieve some loan amortization even though the loan is paid after 10 years.

By now, the reader should be aware that depending on the objectives of the borrower and lender, loans can be *structured* to achieve many results. **Loan structuring** is usually described as a process of adjusting loan terms to calibrate loan payments, loan balances, amortization rates, and so on, to achieve desired results. Such results usually include lower monthly payments than would be available under a fully amortizing structure. However, it could also be that another objective is sought, such as having a desired loan balance at maturity.

"Reverse Mortgages"

Reverse mortgage loans[10] represent a very different type of mortgage loan. Instead of receiving the full loan proceeds at closing, the loan amount in a reverse mortgage is "taken down" as irregular periodic payments until such payments and accrued interest reach the agreed-upon loan amount. In real estate construction and development, these "take-downs" are also referred to as "draws." In recent years, reverse mortgages have also become important to the home-owning population as they approach retirement and seek ways to supplement their retirement income. For example, assume that a household owns a residential property worth $500,000 today. They would like to use the value of the property to supplement their retirement income with a reverse mortgage. A lender agrees to make a loan in an

EXHIBIT 4–10
Declining Loan Payment and Balance Patterns (Constant Amortizing Loan)



amount not to exceed $250,000 for a period of 10 years. However, instead of giving the borrower cash in the amount of $250,000, the lender agrees to let the borrower *take down the loan in monthly installments* over the life of the mortgage. The lender will charge an interest rate of 10 percent on the loan. What will be the maximum monthly payments that the lender will make to the borrower under these terms? We can solve for this as follows:

Solution:	Function:
$FV = -\$250{,}000$	$PMT\,(FV, i, n, PV)$
$i = 10\%/12$	
$n = 120$	
$PV = 0$	
Solve for $PMT = \$1{,}220.44$	

Looking at this example, note that the borrower can supplement his income by drawing $1,220.44 from the lender each month for 120 months, at the end of which time the lender will be owed a total of $250,000. If the property has retained its value of $500,000, the homeowner will have $250,000 in equity at the end of 10 years. If the borrower makes no more draws, the loan balance will continue to increase at 10 percent until the property is sold or the owner dies.

Exhibit 4–11 shows the monthly payment pattern and loan balances for our reverse mortgage example. To determine reverse mortgage loan balances, we reverse the procedure used in our previous FRM examples; that is, we solve for *future value (FV)* and not present value. As shown in the exhibit, instead of the declining loan balances that are characteristic of fully amortizing mortgage loans, reverse mortgage balances *increase* over time. For example, after three years our reverse mortgage loan balance would be

Solution:	Function:
$i = 10\%/12$	$FV\,(i, n, PV, PMT)$
$PMT = \$1220.44$	
$n = 36$	
$PV = 0$	
Solve for $FV = \$50{,}992.21$	

EXHIBIT 4–11
Reverse Mortgage Draw and Balance Patterns

Year	Draw per Month	Balance EOY
1	$1,220.44	$ 15,335.52
2	1,220.44	32,276.87
3	1,220.44	50,992.21
4	1,220.44	71,667.28
5	1,220.44	94,505.30
6	1,220.44	119,738.97
7	1,220.44	147,612.73
8	1,220.44	178,405.23
9	1,220.44	212,422.11
10	1,220.44	250,000.00

[10] These are also known as home equity conversation mortgages (HECMs).

Concept Box 4.3 Reverse Mortgages

With the baby boomers advancing in age and many having a substantial amount of equity in residential housing, reverse mortgages, also known as home equity conversion mortgages (HECMs), have been developed. These loans are designed to help seniors "unlock" home equity by borrowing against property value, thereby supplementing their retirement income. Most mortgage loans are *federally insured* and require borrowers to consult with an approved reverse mortgage counselor. Borrowers should be aware of the following general qualifications:

- Applicant must be 62 years of age or older.
- The borrower must have title to the home and the home must be the borrower's primary residence.
- Proceeds from a RM may be used to repay any existing mortgage loans on a property at the time of closing.
- Borrowers can generally qualify as follows:
  - —age 62, 50% of value, 10 years or expected life
  - —age 72, 60% of value, 10 years or expected life
  - —age 82, 70% of value, 10 years or expected life
- Draw downs may be based on a "term option" such as 10 years, or based on a "tenure option" which is based on life expectancy at the time of closing. Because loans are federally insured in either case, the borrower continues to receive monthly payments even if the loan balance eventually exceeds the market value of the residence. This could occur if the borrower outlives his expected life at closing or if a decline in the market value of the property occurs.

  As is the case with other mortgage loans, reverse mortgages usually require certain up-front costs, including loan fees, mortgage insurance, loan servicing fees, an appraisal, and other closing costs.
- Draw-downs generally may be taken in one lump sum at closing, or in monthly installments, or through a line of credit. The latter loan pattern gives the borrower the option of making draw-downs when desired. In this event, the interest rate is floating and interest is charged at prevailing rates on the monthly outstanding balance.
- At origination, interest rates may be fixed or floating (called adjustable rate mortgages, or ARMs--to be discussed in the next chapter).
- Borrowers must continue to maintain the residence, pay property taxes and hazard insurance.
- The sum of all draw-downs plus accrued interest does not have to be paid back unless the homeowner permanently leaves the home or dies. In the latter event, title to the property would be obtained by the borrower's heirs. Typically, the property would then be sold and the loan balance repaid (just as would be the case without the reverse mortgage).
- Amounts received from a reverse mortgage are not subject to federal income taxes.
- All interest accrued under a reverse mortgage is not deductible when determining federal income taxes until actually paid. This is usually when the loan is paid off in full.

Useful websites:

www.aarp.org/money/personal/reverse_mortgages
www.reversemortgage.org
www.hud.gov/buying/rvrsmort.cfm

The balance at the end of any year can be determined by changing the values for n and resolving for future value.

Conclusion

In this chapter, we discussed various approaches to pricing and structuring fixed interest rate mortgage loans. We saw that the price or interest rate on the loan depends on a number of factors, including various types of risk that affect mortgage lenders. It is important to keep these risk factors in mind in future chapters as we consider alternative mortgage instruments which are often designed in ways that alter risk characteristics. Although the focus of this chapter has been on residential mortgages, the concepts and calculations are equally important for commercial mortgages, which also are discussed in later chapters. We will find that the riskiness of the mortgage is also a factor in the risk and expected rate of return for investors in real estate income properties.

Key Terms

Useful Websites

www.nahb.org—National Association of Home Buyers—Provides industry news, new home listings, and remodeling information.

www.bankrate.com—Provides mortgage rate information from numerous national lenders.

www.freddiemac.com/pmms/pmms30.htm—This is a good site for finding fixed rates and points for 30-year mortgages.

Questions

1. What are the major differences between the four CPM loans discussed in this chapter? What are the advantages to borrowers and risks to lenders for each? What elements do each of the loans have in common?
2. Define *amortization.* List the five types discussed in this chapter.
3. Why do the monthly payments in the beginning months of a CPM loan contain a higher proportion of interest than principal repayment?
4. What are loan closing costs? How can they be categorized?
5. In the absence of loan fees, does repaying a loan early ever affect the actual or true interest cost to the borrower?
6. Why do lenders charge origination fees and loan discount fees?
7. What is the connection between the Truth-in-Lending Act and the annual percentage rate (*APR*)?
8. What is the effective borrowing cost?
9. What is meant by a nominal rate of interest on a mortgage loan?
10. What is the accrual rate and payment rate on a mortgage loan? What happens when the two are equal? What happens when the accrual rate exceeds the payment rate? What if the payment rate exceeds the accrual rate?
11. An expected inflation premium is said to be part of the interest rate. What does this mean?
12. A mortgage loan is made to Mr. Jones for $30,000 at 10 percent interest for 20 years. If Mr. Jones has a choice between either a fully amortizing CPM or a CAM, which one would result in his paying a greater amount of total interest over the life of the mortgage? Would one of these mortgages be likely to have a higher interest rate than the other? Explain your answer.
13. What is negative amortization?
14. What is partial amortization?

Problems

1. A borrower obtains a fully amortizing CPM loan for $125,000 at 11 percent interest for 10 years. What will be the monthly payment on the loan? If this loan had a maturity of 30 years, what would be the monthly payment?
2. A fully amortizing mortgage loan is made for $80,000 at 6 percent interest for 25 years. Payments are to be made monthly. Calculate:
   a. Monthly payments.
   b. Interest and principal payments during month 1.
   c. Total principal and total interest paid over 25 years.
   d. The outstanding loan balance if the loan is repaid at the end of year 10.
   e. Total monthly interest and principal payments through year 10.
   f. What would the breakdown of interest and principal be during month 50?
3. A fully amortizing mortgage loan is made for $100,000 at 6 percent interest for 30 years. Determine payments for each of the periods *a–d* below if interest is accrued
   a. Monthly.
   b. Quarterly.
   c. Annually.
   d. Weekly.
4. Regarding Problem 3, how much total interest and principal would be paid over the entire 30-year life of the mortgage in each case? Which payment pattern would have the greatest total amount of interest over the 30-year term of the loan? Why?
5. A fully amortizing mortgage loan is made for $100,000 at 6 percent interest for 20 years.
   a. Calculate the monthly payment for a CPM loan.
   b. What will the *total* of payments be for the entire 20-year period? Of this total, how much will be the interest?
   c. Assume the loan is repaid at the end of eight years. What will be the outstanding balance? How much total interest will have been collected by then?
   d. The borrower now chooses to reduce the loan balance by $5,000 at the end of year 8.
      (1) What will be the new loan maturity assuming that loan payments are not reduced?
      (2) Assume the loan maturity will not be reduced. What will the new payments be?
6. A 30-year fully amortizing mortgage loan was made 10 years ago for $75,000 at 6 percent interest. The borrower would like to prepay the mortgage balance by $10,000.
   a. Assuming he can reduce his monthly mortgage payments, what is the new mortgage payment?
   b. Assuming the loan maturity is shortened and using the original monthly payments, what is the new loan maturity?
7. A fully amortizing mortgage is made for $100,000 at 6.5 percent interest. If the monthly payments are $1,000 per month, when will the loan be repaid?
8. A fully amortizing mortgage is made for $80,000 for 25 years. Total monthly payments will be $900 per month. What is the interest rate on the loan?
9. A partially amortizing mortgage is made for $60,000 for a term of 10 years. The borrower and lender agree that a balance of $20,000 will remain and be repaid as a lump sum at that time.
   a. If the interest rate is 7 percent, what must monthly payments be over the 10-year period?
   b. If the borrower chooses to repay the loan after five years instead of at the end of year 10, what must the loan balance be?
10. An interest-only mortgage is made for $80,000 at 10 percent interest for 10 years. The lender and borrower agree that monthly payments will be constant and will require *no* loan amortization.
    a. What will the monthly payments be?
    b. What will be the loan balance after five years?
    c. If the loan is repaid after five years, what will be the yield to the lender?
    d. Instead of being repaid after five years, what will be the yield if the loan is repaid after 10 years?

11. A partially amortizing loan for $90,000 for 10 years is made at 6 percent interest. The lender and borrower agree that payments will be monthly and that a balance of $20,000 will remain and be repaid at the end of year 10. Assuming 2 points are charged by the lender, what will be the yield if the loan is repaid at the end of year 10? What must the loan balance be if it is repaid after year 4? What will be the yield to the lender if the loan is repaid at the end of year 4?

12. A loan for $50,000 is made for 10 years at 8 percent interest and *no monthly payments* are scheduled.

    a. How much will be due at the end of 10 years?

    b. What will be the yield to the lender if it is repaid after eight years? (Assume monthly compounding.)

    c. If 1 point is charged in (*b*) what will be the yield to the lender?

13. John wants to buy a property for $105,000 and wants an 80 percent loan for $84,000. A lender indicates that a fully amortizing loan can be obtained for 30 years (360 months) at 8 percent interest; however, a loan fee of $3,500 will also be necessary for John to obtain the loan.

    a. How much will the lender actually disburse?

    b. What is the APR for the borrower, assuming that the mortgage is paid off after 30 years (full term)?

    c. If John pays off the loan after five years, what is the effective interest rate? Why is it different from the effective interest rate in (*b*)?

    d. Assume the lender also imposes a prepayment penalty of 2 percent of the outstanding loan balance if the loan is repaid within eight years of closing. If John repays the loan after five years with the prepayment penalty, what is the effective interest rate?

14. Our borrower, John, in problem 13 wants to "roll in" or finance the loan fee of $3,500 into the loan amount which would make the loan $87,500. Answer parts (a) through (d) from problem 13 assuming that the lender agrees to allow the loan fees to be included in the loan amount.

15. A lender is considering what terms to allow on a loan. Current market terms are 9 percent interest for 25 years for a fully amortizing loan. The borrower, Rich, has requested a loan of $100,000. The lender believes that extra credit analysis and careful loan control will have to be exercised because Rich has never borrowed such a large sum before. In addition, the lender expects that market rates will move upward very soon, perhaps even before the loan is closed. To be on the safe side, the lender decides to extend to Rich a CPM loan commitment for $95,000 at 9 percent interest for 25 years; however, the lender wants to charge a loan origination fee to make the mortgage loan yield 10 percent. What origination fee should the lender charge? What fee should be charged if it is expected that the loan will be repaid after 10 years?

16. A borrower is faced with choosing between two loans. Loan A is available for $75,000 at 6 percent interest for 30 years, with 6 points to be included in closing costs. Loan B would be made for the same amount, but for 7 percent interest for 30 years, with 2 points to be included in the closing costs. Both loans will be fully amortizing.

    a. If the loan is repaid after 20 years, which loan would be the better choice?

    b. If the loan is repaid after five years, which loan is the better choice?

17. A reverse mortgage is made with a balance not to exceed $300,000 on a property now valued at $700,000. The loan calls for monthly payments to be made to the borrower for 120 months at an interest rate of 11 percent.

    a. What will the monthly payments be?

    b. What will be the loan balance at the end of year 3?

    c. Assume that the borrower must have monthly draws of $2,000 for the first 50 months of the loan. Remaining draws from months 51 to 120 must be determined so that the $300,000 maximum is not exceeded in month 120. What will draws by the borrower be during months 51 to 120?

18. A borrower and a lender agree on a $200,000 loan at 10 percent interest. An amortization schedule of 25 years has been agreed on; however, the lender has the option to "call" the loan after five years. If called, how much will have to be paid by the borrower at the end of five years?

19. A fully amortizing CAM loan is made for $125,000 at 11 percent interest for 20 years.
    a. What will be the payments and balances for the first six months?
    b. What would payments be for a CPM loan?
    c. If both loans were repaid at the end of year 5, would the lender earn a higher rate of interest on either loan?

20. A $50,000 interest-only mortgage loan is made for 30 years at a nominal interest rate of 6 percent. Interest is to be accrued daily, but payments are to be made monthly. Assume 30 days each month.
    a. What will the monthly payments be on such a loan?
    b. What will the loan balance be at the end of 30 years?
    b. What is the effective annual rate on this loan?

21. **Comprehensive Review Problem:** A mortgage loan in the amount of $100,000 is made at 12 percent interest for 20 years. Payments are to be monthly in each part of this problem.
    a. What will monthly payments be if
       (1) The loan is fully amortizing?
       (2) It is partially amortizing and a balloon payment of $50,000 is scheduled at the end of year 20?
       (3) It is a nonamortizing, or "interest-only" loan?
       (4) It is a negative amortizing loan and the loan balance will be $150,000 at the end of year 20?
    b. What will the loan balance be at the end of year 5 under parts *a* (1) through *a* (4)?
    c. What would be the interest portion of the payment scheduled for payment at the end of month 61 for each case (1) through (4) above?
    d. Assume that the lender charges 3 points to close the loans in parts *a* (1) through *a* (4). What would be the *APR* for each?
    e. Assuming that 3 points are paid at closing and the loan is prepaid at the end of year 5, what will be the effective rate of interest for each loan in parts *a* (1) through *a* (4) ?
    f. Assume conditions in *a* (1) except that payments will be "interest only" for the first three years (36 months). If the loan is to fully amortize over the remaining 17 years, what must the monthly payments be from year 4 through year 20?
    g. Refer to *a* (4) above, where the borrower and lender agree that the loan balance of $150,000 will be payable at the end of year 20:
       (1) How much total interest will be paid from all payments? How much total amortization will be paid?
       (2) What will be the loan balance at the end of year 3?
       (3) If the loan is repaid at the end of year 3, what will be the effective rate of interest?
       (4) If the lender charges 4 points to make this loan, what will the effective rate of interest be if the loan is repaid at the end of year 3?

22. **Excel.** Refer to the "Ch4 Eff Cost" tab in the Excel Workbook provided on the website. Suppose that another loan is available that is an 11 percent interest rate with 6 points. What is the effective cost of this loan compared to the original example on the template?

23. **Excel.** Refer to the "Ch4 GPM" tab in the Excel Workbook provided on the website. How would the loan balance at the end of year 7 change if the payments increase by 5 percent each year instead of 7.5 percent?

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Appendix

Inflation, Mortgage Pricing, and Payment Structuring

The fully amortizing, constant payment mortgage has been the most widely used mortgage instrument in the United States for some time. In more recent times, particularly during the 1970s and early 1980s, inflation and its effect on this "standard" mortgage instrument have caused problems for both lenders and borrowers. Because of these problems, a number of different mortgage instruments have been proposed as alternatives to the standard mortgage instrument. In this section, we outline the problems that inflation has brought for both borrowers and lenders who have relied on the standard mortgage instrument. Also included is a detailed description of the graduated payment mortgage. This mortgage is also a fixed interest rate mortgage and may be used in place of the constant payment mortgage, particularly during periods of rising interest rates.

Effects on Lenders and Borrowers

How does inflation relate to mortgage lending and cause difficulty for lenders and borrowers desiring to make constant payment loans with fixed interest rates? The answer to this question can be easily illustrated. Let us assume initially that a $60,000 loan is made at a time when no inflation is expected. The loan is expected to be outstanding for a 30-year period. Because there is no inflation, an inflation premium (f) is not required; hence, the lender will earn a return equivalent to the riskless interest rate (r), plus a premium for risk (p) over the period of the loan.[1] We assume that the interest rate charged under such assumptions would be 4 percent, representing a 3 percent real rate of interest and a risk premium of 1 percent over the period of the loan. Assuming a constant payment, fixed interest rate loan made in an inflationless environment, the lender would collect constant payments of approximately $286 per month, based on the loan constant for 4 percent and 30 years. This amount is shown in Exhibit 4A–1 as a straight line (*RP*) over the life of the loan and represents the series of *constant real payments* necessary to earn the lender a 3 percent fixed real return plus a 1 percent risk premium each year that the loan is outstanding.

Now assume that the same loan is made in an inflationary environment, where a 6 percent rate of inflation is expected to prevail during each year that the loan is outstanding. The interest rate on the mortgage loan would now have to increase to approximately 10 percent for the lender to earn the same real return. This includes the base rate of 4 percent earned when no inflation was expected, plus an inflation premium of 6 percent.[2] Given that the standard mortgage instrument is to be used, the lender must now collect approximately $527 a month (rounded). This new payment pattern is shown in Exhibit 4A–1 as the horizontal line labeled *NP*, representing a constant series of nominal payments received over the term of the loan. Hence, included in the series of nominal payments are amounts that will provide the lender with a 4 percent basic rate of interest representing a real return and risk premium, plus a 6 percent inflation premium over the 30-year loan term.

In our example, an expected inflation rate of 6 percent caused an 84 percent rise in the monthly mortgage payments from $286 to $527, or $241 per month. Why is there such a significant increase in these monthly payments? The reason can be easily seen by again examining curve *NPD* in Exhibit 4A–1. This curve represents the real value of the monthly payments that the lender will receive over the 30-year loan period. It is determined by "deflating" the $527 nominal monthly payments by the rate of inflation.[3] The *NPD* curve is important because the lender, realizing that inflation is going to occur, expects that the constant stream of $527 payments to be received over time will be worth less and less because of lost purchasing power. Hence, to receive the full 10 percent interest necessary to leave enough for a 4 percent real return and risk premium over the life of the loan, more "real dollars" must be collected in the *early* years of the loan (payments collected toward the *end* of the life of the mortgage will be worth much less in purchasing power).

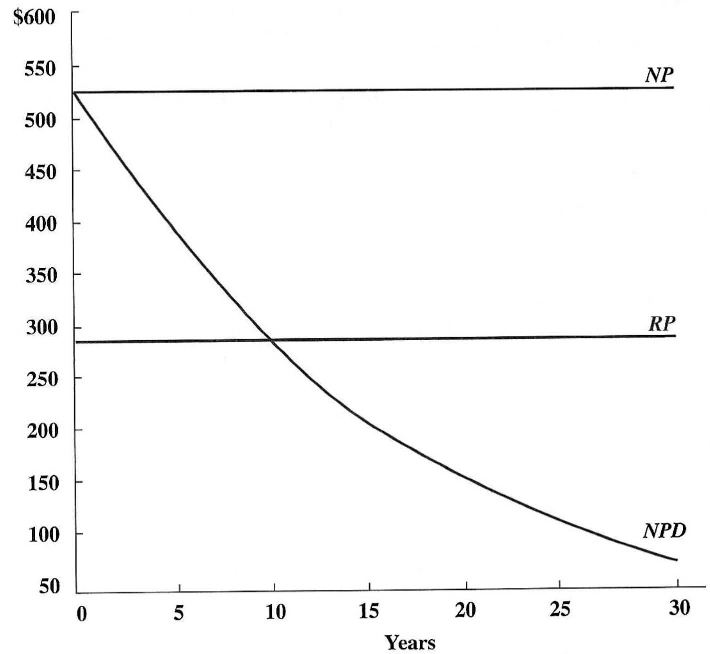
To illustrate, let us examine the deflated or real value of the $527 payments collected each month, as represented by the curve *NPD*. Note that for about the first 10 years of the loan life, the real value of these payments is greater than those for the 4 percent loan. However, after 10 years, the real value of these payments falls below the payments required on the 4 percent loan. However, even though the two payment streams differ, the real value of the nominal payment

[1] Actually the interest rate charged will be related to the expected repayment period that may occur before maturity. However, this will not alter the concept being illustrated. The figures chosen here are arbitrary. Some studies indicate that the real rate of interest has historically been in the 1 to 3 percent range and risk premium on mortgages in the 2 to 3 percent range.

[2] The nominal interest rate would actually be $(1 + .04)(1 + .06) - 1$, or 10.24 percent. However, as indicated earlier, we use 10 percent to simplify calculations.

[3] Deflating an income stream is done by computing the monthly inflation factor $.06 \div 12$, or $.005$, and multiplying $\$527(1 \div 1.005)^1$ in the first month, $\$527(1 \div 1.005)^2$ in the second month, and so on, until the end of year 30.

EXHIBIT 4A–1
*IRR*Real and Nominal Values of Mortgage Payments



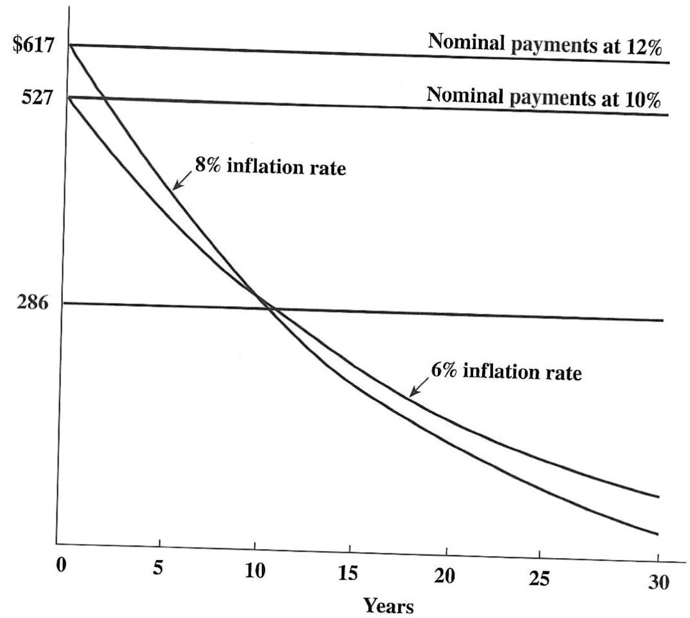
stream is equal to the required real payments at 4 percent, or $NPD = RP$. This means that from the stream of nominal $527 monthly receipts, the lender will ultimately earn the same real value as a stream of $286 payments or 4 percent on investment after deflating the nominal payments by the inflation rate. However, in order to earn the same real interest rate, the real value of the payment stream (*NPD*) must be greater than *RP* in the early years, since it will fall below *RP* in the later years. This relationship is referred to as *tilting* the real payment stream in the early years to make up for the loss in purchasing power in later years.

This **tilt effect** also has a considerable impact on the borrower. Recall that with no inflation the borrower faced a $286 payment; however, with inflation a $527 monthly payment is necessary. When the loan is first originated, the difference in the two payments is about $241 per month and represents an additional amount of real or current dollars that the borrower must allocate from current real *income* to meet mortgage payments.

Over time, this burden moderates. For example, by the end of the first year, the real value of the $527 payments deflated by the 6 percent rate of inflation would be about $497 per month and the borrower's real income will have increased by 3 percent, or by the real rate of growth in the economy. At that time, the borrower will have more real income to pay declining real mortgage payments. The important point is that even though the borrower's income is increasing in both real and nominal terms each year, it is not enough to offset the tilt effect in the early years of a loan. From this analysis, it becomes apparent from Exhibit 4A–1 why it is so difficult for first-time home buyers to qualify for constant payment, fixed interest rate loans during periods of rising inflation. With the general rate of inflation and growth in the economy, borrower incomes will grow gradually or on a year-by-year basis. However, as expected inflation increases, lenders must build estimates of the full increase into current interest rates "up front," or *when the loan is made.* This causes a dramatic increase in required real monthly payments relative to the borrower's current real income.

One final observation about the tilt effect is that, as the rate of inflation increases, the tilt effect increases. In Exhibit 4A–2, we show the effect of an increase in inflation from 6 percent in our previous example to 8 percent per year. Note that nominal monthly payments increase from $527 to $617 per month, the latter figure based on an increase in the mortgage interest rate to 12 percent. The impact of the tilt effect on a constant payment loan when inflation is expected to be 8 percent can be seen relative to the effect when inflation was expected to be 6 percent. Note that when the $617 monthly payments are deflated at 8 percent (*NPD* @ 8%) for inflation, the burden of the real payments to be made by the borrower increases relative to the real payments required when inflation was 6 percent in the early years of the loan. The curve corresponding to monthly payments deflated at 8 percent indicates that the real value of monthly payments on the 12 percent mortgage exceeds the real value of payments on the 10 percent mortgage for about the first 10 years of the loan term. This is true even though the lender will

EXHIBIT 4A–2
Relationship between Real and Nominal Payments at Various Rates of Inflation



earn a 4 percent real return on both mortgages after inflation. Further, if we again assume that the "average" borrower's real income will increase by 3 percent, regardless of the rate of inflation, as inflation increases from 6 percent to 8 percent, it is clear that the borrower will have to allocate even more current real income to mortgage payments. This indicates that in the early years of the mortgage, the burden of the tilt effect on borrowers increases as the rate of inflation increases. This increased burden is due solely to (1) the nature of the mortgage instrument, that is, a constant payment, fixed interest rate mortgage and (2) the rate of inflation. Further, the tilt effect makes it even more difficult for borrowers to qualify for loans based on their current income and make payments from current income. To partially overcome the tilt effect, lenders have designed a mortgage loan that retains a fixed rate of interest but includes a series of stepped-up payments that are lower in the earlier years, thereby better matching borrowers' incomes, and then rising over time. These loans are known as graduated payment mortgages (GPMs).

The Graduated Payment Mortgage (GPM)

In an attempt to deal with the problem of inflation and its impact on mortgage interest rates and monthly payments, lenders have instituted new mortgage instruments. One such instrument is the **graduated payment mortgage (GPM).** The objective of a GPM is to provide for a series of mortgage payments that are *lower* in the initial years of the loan than they would be with a standard mortgage loan. GPM payments then gradually increase at a predetermined rate as borrower incomes are expected to rise over time. The payment pattern thus offsets the tilt effect to some extent, reducing the burden faced by households when meeting mortgage payments from current income in an inflationary environment.[4]

An example of the payment pattern for the graduated payment mortgage is illustrated in Exhibit 4A–3. The exhibit contains information on how payments should be structured for the 30-year, $60,000, fully amortizing loan used in our previous examples. GPMs can have a number of plans allowing for differences in initial payment levels, rates of graduation, and graduation periods. Exhibit 4A–3 contains information on one of the more popular payment plans in use today. This plan allows for a 7.5 percent rate of graduation in monthly payments over five years, after which time the payments level off for the remaining 25 years. Computing initial payments on a mortgage of this kind is complex and is illustrated at the end of this appendix.[5]

Looking at the information contained in Exhibit 4A–3, we see that for a standard constant payment mortgage (CPM) loan of $60,000 originated at 12 percent for 30 years, the required constant monthly payments would be $617.17. A GPM loan made for the same amount and interest rate, where the monthly payments are increased (graduated) at the end of each year at a predetermined rate of 7.5 percent,

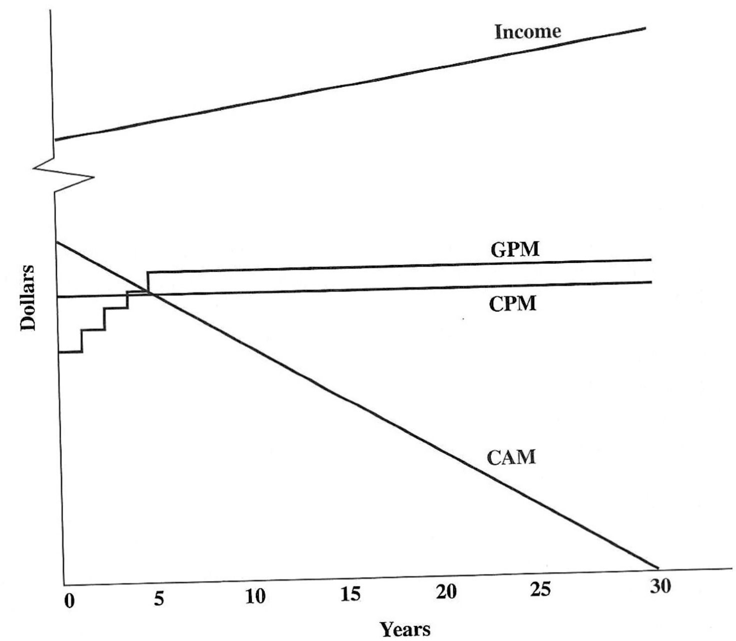
[4] The Federal Housing Administration initiated the first widely accepted graduated payment plan under its Section 245 program. For more detail, see the *HUD Handbook's* various issues.

[5] Although we discuss GPMs relative to single family lending, this type of loan also could be used to structure debt financing for income-producing properties where mortgage loan payments are designed to match income growth from rents collected over time.

EXHIBIT 4A–3
Comparison of GPM Payments and Constant Payments ($60,000, 30-Year Maturity, Fully Amortizing, at Various Interest Rates)

	Interest Rate				
	10%	11%	12%	13%	14%
Constant Payments	$526.54	$571.39	$617.17	$663.72	$710.94
GPM Payments Graduated (7.5% Annually)					
1	$400.22	$436.96	$474.83	$513.71	$553.51
2	430.24	469.73	510.44	552.24	595.03
3	462.51	504.96	548.72	593.66	639.65
4	497.19	542.83	589.87	638.18	687.63
5	534.48	583.55	634.11	686.04	739.20
6–30	574.57	627.31	681.67	737.50	794.64

EXHIBIT 4A–4
Comparison of Mortgage Payment Patterns (Loan Amount = $60,000, Maturity = 30 Years, Interest 12% GPM Add: 7.5% Graduation Rate, 5 years)



begins with an initial payment of approximately $474.83. This initial payment will then increase by 7.5 percent per year to an amount equal to $681.67 at the beginning of year 6 and will remain constant from that point until the end of year 30. Compared with $617.17 in the constant payment mortgage, GPM payments are initially lower by $142.34 in the first year. The difference becomes smaller over time. The graduated payment level reaches approximately the same payment under the standard mortgage between the fourth and fifth years after origination. GPM payments exceed constant payments by $64.50 ($681.67 – $617.17) beginning in year 6. GPM payments then remain at the $681.67 level for the remaining 25 years of the loan term.

Exhibit 4A–4 provides a comparison of payment patterns for a graduated payment mortgage (GPM), a constant payment mortgage (CPM), and a constant amortizing mortgage (CAM). GPM payments are based on the 7.5 percent graduation plan. All three loans are assumed to be originated for $60,000 at 12 percent interest for 30 years. Note that the GPM is below that of the CPM for approximately five years, at which point the GPM payments begin to exceed CPM payments. The reason for this pattern should be obvious. Under either payment plan, the yield to the lender must be an annual rate of 12 percent compounded monthly, assuming no origination fees, penalties, and so on. Therefore, because the GPM payments are below those of the CPM in the early years, GPM payments must eventually exceed the level payment on the CPM loan to "make up" for the lower payments on the GPM in the early years. Hence, if the borrower chooses the GPM in our example, the payments will exceed those of a standard CPM mortgage from years 6 to 30.

The advantages of the GPM program are obvious from the borrower's standpoint. The initial payment level under the GPM plan shown in Exhibit 4A–4 is significantly lower than under the CPM plan. Further, in the early years, GPM payments correspond more closely to increases in borrower's income. Hence, the burden of the tilt effect requiring borrowers to allocate more current real dollars for mortgage payments from current real income in an inflationary environment is reduced somewhat with the GPM. Based on this analysis, it is easy to conclude that the GPM significantly reduces monthly payments for borrowers in the early years of the mortgage loan, corresponds more closely to increases in borrower income, and therefore may increase the demand for mortgage credit by borrowers.

When judged relative to the CAM, the CPM and GPM clearly provide for initial payments that are far below payments required for the CAM with the same terms. It is important to stress that higher rates of inflation have caused a modification in mortgage instruments over time. Even though all three mortgage instruments provide the same yield (12%), changes in mortgage payments have clearly been structured to reduce initial payments. This has been done with the expectation that growth in real incomes and expected inflation will extend into the future, resulting in sufficiently high borrower incomes to repay the debt while reducing initial payments sufficiently to reduce the payment burden at the time of loan origination.

Outstanding Loan Balances: GPMs

Because the initial loan payments under GPM plans are usually lower than payments necessary to cover the monthly interest, the outstanding loan balance under the GPM will *increase* during the initial years of the loan. It will remain higher than that of the standard CPM mortgage until full repayment occurs at maturity. A comparison of loan balances for a GPM and a standard mortgage, based on the 12 percent, $60,000, 30-year terms used in our previous example, is shown in Exhibit 4A–5.

Exhibit 4A–6 indicates that the mortgage balance with the GPM *increases* until approximately year 5, when it begins to decline until it reaches zero in year 30. Hence, if a borrower sold this property during the first four years after making a GPM loan, more would be owed than originally borrowed. The loan balance increases during the first four years after origination because the initial GPM payments are lower than

EXHIBIT 4A–5
Loan Balances for Graduated Payment and Constant Payment Mortgages as Compared with Expected Property Value

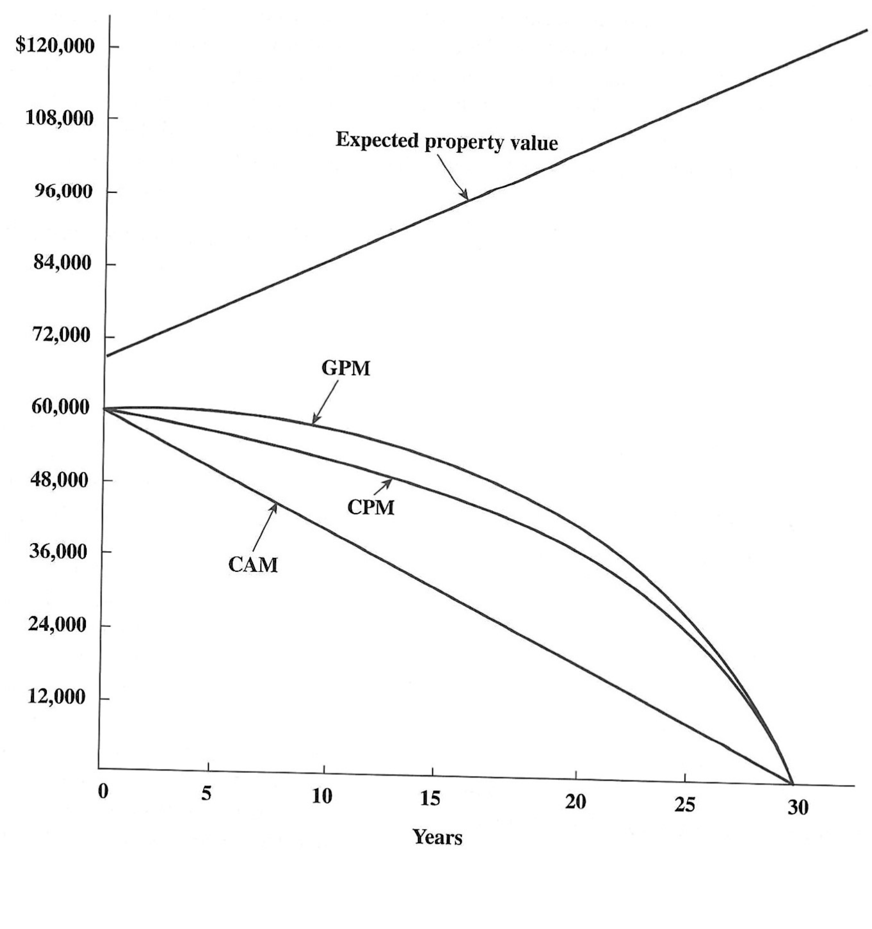


EXHIBIT 4A–6
Determining Loan Balance on a GPM ($60,000 Loan, 12%, 30 Years, 7.5 % Rate of Graduation)

Year	Beginning Balance	Required Monthly Interest Payment	GPM Payment	First Month Loan Amortization	Change in Balance	Ending Balance
1	$60,000.00	$600.00	$474.83	$125.17	$1,587.47	$61,587.47
2	61,587.47	615.87	510.44	105.43	1,337.12	62,924.59
3	62,924.59	629.25	548.72	80.53	1,021.32	63,945.91
4	63,945.91	639.46	589.87	49.59	628.93	64,574.84
5	64,574.84	645.75	634.11	11.64	147.62	64,722.46*
6	64,722.46	647.22	681.67	(34.45)	(436.91)	64,285.55

*Maximum balance. During the sixth year, the payments ($681.67) will exceed the required interest ($647.22), and loan amortization will begin.

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the monthly interest requirements at 12 percent. Therefore, no amortization of principal occurs until payments increase in later periods. To illustrate, in our previous example, the interest requirements under a GPM after the first month of origination would be $60,000 × (.12 ÷ 12), or $600.00. The GPM payments during the first year of the loan are only $474.83, which are less than the monthly interest requirement of $600.00. The difference, or $125.17, must be added to the initial loan balance of $60,000, as if that difference represented an additional amount *borrowed* each month. This $125.17 monthly difference is **negative amortization.** Further, this shortfall in interest must also accumulate interest at the rate of 12 percent compounded monthly. Hence, during the first year, $125.17 per month plus monthly compound interest must be added to the $60,000 loan balance. This process amounts to compounding a monthly annuity of $125.17 at 12 percent per month and adding that result to the initial loan balance to determine the balance at year-end. The amount added to the loan balance at the end of the year will be the future value (*FV*) of $125.17 per month, compounded monthly at 12 percent or $125.17(12.682503) = $1,587.47.

Solution:

$n = 12$
$i = 12\% \div 12 = 1\%$
$PMT = \$474.83$
$PV = -\$60{,}000$
Solve for FV = $61,587.47 (loan balance)

Alternate Solution:

$n = 12$
$i = 12\% \div 12 = 1\%$
$PMT = \$125.17$
$PV = 0$ (addition to annual loan balance)
Solve for FV = −$1,587.47

The importance of the increasing GPM loan balance and negative amortization can be seen in relationship to the property value also (see Exhibit 4A–5). It is important to note the "margin of safety," or difference between property value and loan balance. This margin is much *lower* when a GPM loan balance is compared with that of a CPM. This makes a GPM loan riskier to the lender than a CPM, because more consideration must be given to *future* market values of real estate and *future* borrower income. For example, let us assume that the GPM borrower decides to sell a property after five years. When compared with the CPM, the lender will have received relatively lower monthly payments up to that point. Further, because of negative amortization, the proceeds from sale of the property must be great enough to repay the loan balance that has increased relative to the original amount borrowed. In short, with a GPM, the lender must now be more concerned about trends in real estate values because resale value will constitute a more important source of funds for loan repayment.

GPM Mortgage Loans and Effective Borrowing Costs

A closing note in this appendix considers the question of the effective interest rate and GPMs. In the absence of origination fees and prepayment penalties, the yield on GPMs, like yields on CAMs and CPMs, is equal to the contract rate of interest as specified in the note because the GPM, like the CPM, is a fixed interest rate mortgage. This is true whether or not the GPM loan, like CAM and CPM loans, is repaid before maturity. However, to the extent points or origination fees are charged, the effective yield on a GPM will be *greater* than the contract rate of interest, and it will increase the earlier the loan is repaid. When computing yields on GPMs originated with points, the same procedure should be followed as described with the standard CPM; that is, the interest rate making the stream of GPM payments equal to the funds disbursed after deducting financing fees is the effective cost of the loan. Where origination fees are charged on GPMs, the authors have computed results that are very close to those computed for standard mortgage loans with the same terms and origination fees. This is true regardless of the loan amount or rate of graduation on the GPM. In the GPM discussed above, for example, if 3 points are charged and the loan is repaid after five years, the effective rate would be about 12.78 percent, compared with 12.82 on a CPM with the same terms.[6] Is a borrower better off or worse off with a GPM or a CPM loan? Generally, if a standard loan and a GPM are originated at the same rate of interest and have the same fees, there will be little, if any, difference in their effective costs. However, because the graduated payment pattern reduces the tilt effect, the borrower is definitely better off with a GPM *if it can be obtained at the same interest rate* as the standard mortgage.

Would a GPM generally be available at the same interest rate as a standard CPM mortgage? It would appear that because of the additional risk taken by the lender—in the form of an increasing loan balance due to negative amortization in the early years of the loan and lower initial monthly cash flows received from reduced payments—the GPM lender would require a *higher risk premium* than the CPM lender. Hence, all things being equal, a slightly higher interest rate may be required on a GPM than on a CPM. This would tend to neutralize some of the positive features of the GPM compared with the CPM.

Graduated Payment Mortgages—Further Extensions

As explained in the chapter, the mechanics of determining monthly payment streams and loan balance are relatively straightforward for FRMs. However, when designing a GPM structure, the reader should be aware that the rate of graduation, number of years during which payments will graduate, term, and interest rate will vary, depending on the goals of the borrower and lender and the loan market that it is being designed to serve.

Perhaps, the most complex problem associated with a GPM is *establishing the initial monthly payment.* For example, in Exhibit 4A–3 monthly payments for various GPMs are illustrated. Each group of monthly payments was assumed to increase at the rate of 7.5 percent beginning in year 2. However, the reader may be wondering how the initial payment is determined and, perhaps more importantly, how one may go about designing a structure for payment patterns requiring a graduated or stepped up repayment schedule. For example, in the case of our 12 percent GPM mortgage, we note that the initial payment would be $474.83. How was that payment determined?

To answer this question, we provide what appears to be a difficult solution. Upon closer examination, however, we will see that it is an application of the present value formulas that we have learned. It is important to recall that for the GPM, as was the case with the CAM, the present value of all payments discounted at the contract rate of interest will equal the initial loan amount. This concept is very important and must be kept in mind as we work through the problem at hand. What follows is a general formula for determining the initial monthly payment for a GPM:

$$
\begin{aligned}
PV = & \left[MP_1 \cdot \sum_{t=1}^{12} \frac{1}{(1+i/12)^t}\right] \\
& + \left[MP_1(1+g)^1 \cdot \sum_{t=1}^{12} \frac{1}{(1+i/12)^t} \cdot \frac{1}{(1+i/12)^{12}}\right] \\
& + \left[MP_1(1+g)^2 \cdot \sum_{t=1}^{12} \frac{1}{(1+i/12)^t} \cdot \frac{1}{(1+i/12)^{24}}\right] \\
& + \left[MP_1(1+g)^3 \cdot \sum_{t=1}^{12} \frac{1}{(1+i/12)^t} \cdot \frac{1}{(1+i/12)^{36}}\right] \\
& + \left[MP_1(1+g)^4 \cdot \sum_{t=1}^{12} \frac{1}{(1+i/12)^t} \cdot \frac{1}{(1+i/12)^{48}}\right] \\
& + \left[MP_1(1+g)^5 \cdot \sum_{t=1}^{300} \frac{1}{(1+i/12)^t} \cdot \frac{1}{(1+i/12)^{60}}\right]
\end{aligned}
$$

where

PV = loan amount

i = contract interest rate

MP_1 = monthly payments during year 1

g = rate of graduation in the monthly payment

While the computation appears to be complex, a relatively simple solution for MP_1 is obtainable for our $60,000, 12 percent, 30-year GPM with a graduation rate of 7.5 percent. Note that the expressions containing the Σ's are simply the interest factors for the present value of an annuity presented in Chapters 3 and 4. The terms $1 \div (1 + i/12)^{12}$, $1 \div (1 + i/12)^{24}$, and so on are simply the factors also discussed in Chapters 3 and 4. These factors correspond to various 12-month intervals during which monthly payments will be greater than the previous 12-month period. However, in any given year, monthly payments (unknown) will remain constant during that year. Hence, what we have in our base case example are six different groups of unknown monthly annuities, which, when discounted by the contract rate of

[6] Computations for the effective interest rate on GPMs are much more difficult than those for the CPM because the amount disbursed must be set equal to a series of seven "grouped cash flows" or annuities, representing different payments for 12 periods in each of the six years, with the final annuity payment covering years 6 to 30. Similarly, when finding loan balances, we may use for GPMs the same procedure demonstrated for CPMs; that is, the remaining payment streams would be discounted at the contract rate of interest and the present value would be determined. However, determining loan balances on GPMs may involve discounting a series of one or more annuities spanning many different 12-month intervals if any remaining CPM payments differ. For an illustration of how to calculate payments for GPMs see the following section.

EXHIBIT 4A–7
Worksheet for Solving for Initial *GPM* Payments

www.mhhe.com/bf16e

(1) Payment Period	(2) Payment	(3) Graduated Payment Factor	(4) Present Value of 12 Payments Received at End of Month Each Year	(5) Present Value of (Col. 4) Received at End of Each Year	(6) (3 × 4 × 5)
MP_1	$= MP_1(1.0)$	1.0	11.255077	—	11.255077
MP_2	$= MP_1(1 + .075)$	1.075000	11.255077	.887449	10.737430
MP_3	$= MP_1(1 + .075)^2$	1.155625	11.255077	.787566	10.243594
MP_4	$= MP_1(1 + .075)^3$	1.242297	11.255077	.698925	9.772473
MP_5	$= MP_1(1 + .075)^4$	1.335469	11.255077	.620260	9.323008
MP_{6-30}	$= MP_1(1 + .075)^5$	1.435629	94.946551	.550450	75.030751
				Total	126.362333

interest on the mortgage (i), must equal the initial amount of the loan. This process is usually referred to as discounting *grouped cash flows* and is a problem encountered frequently in real estate finance.

Essentially, our problem involves finding MP_1, which is the only unknown. We know the loan amount ($60,000), the monthly interest rate ($i \div 12 = .01$), and the term of the loan (360 months). Further, we know that MP in years 2, 3, 4, 5, and 6 will be equal to MP_1, increased by $(1 + g)^1, (1 + g)^2, (1 + g)^3, (1 + g)^4$, and $(1 + g)^5$, respectively, where $g = .075$. Given this information, MP_1 can be found by assembling the information as shown in Exhibit 4A–7.

Looking at Exhibit 4A–7, we see that column 1 corresponds to the payments for years 1 to 30. Column 2 merely indicates that payments during each year will be increased at the rate of 7.5 percent per year, which is equivalent to compounding MP_1 (unknown) by 1.075 for each year's set of payments *beginning* in year 2. In other words, we are solving for payments in year 1 which will remain the same for 12 months, and then increase by 1.075 beginning in year 2. Hence, column 3 is simply the compound interest factor for the rate of graduation (7.5 percent) applied to MP_1.

In Exhibit 4A–7, column 4 contains the present value of 12 payments received at the end of the month each year at 12 percent. This factor is, in effect, being used to discount the six different series of monthly payments *within* the interval during which they will occur. For example, in each of the first five years, 12 monthly payments will be received and must be discounted for that 12-month interval, hence the factor 11.255077. From years 6 to 30, 300 payments will be received and must be discounted for that interval, hence the factor 94.946551.

Column 5 contains the present value of column(4) received at the end of each year, which must be used to discount each series of monthly annuities back to time period zero or present value. In other words, column 4 discounts the 12 monthly payments *within* the 12-month interval. Column 5 is necessary because each series of grouped payments is not received all at once; instead, the series received during the second year has a lower present value than the series received in the first year. Hence, each series must be discounted again by the column (5) factor for one year, the third year must be discounted for two years, and so on.

Finally, column 6 is simply the product of columns 3, 4, and 5. Note that these factors are additive because we have been able to express each series of payments (MP_2, MP_3, MP_4, MP_5 and MP_{6-30}) in terms of MP_1 because we know that each succeeding period's payment will increase by the same rate of graduation $(1 + g)$. Careful inspection of the equation shows that the compounding and discount rates in columns (4) and (5) and $1 + g$ may be factored, multiplied, and added. This is in essence what we have done in Exhibit 4A–7. Hence, the equation reduces to

$$MP_1(126.362333) = \$60,000$$
$$MP = \$474.83$$

Because we know that MP_2 will be 1.075 times greater than MP, we have we have $474.83(1.075) or $510.44, and so on. The reader may now complete the calculations and verify the payments in the 12 percent column in Exhibit 4A–3.

This formula and procedure have widespread application in real estate finance whenever one is faced with a series of payments which are scheduled to increase after given time intervals at any specified rate of increase.[7] The student is also encouraged to think about how the schedule and formula may change if different rates of graduation over different periods of time are desired.

[7] This procedure can be programmed into many financial calculators and spreadsheets. An explanation can usually be found in the manual accompanying calculators and spreadsheets under graduated payment mortgages and/or discounting grouped cash flows.