CHAPTER

8 Net Present Value and Other Investment Criteria

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- **8-1** Calculate the net present value of a project.
- **8-2** Calculate the internal rate of return of a project and know what to look out for when using the internal rate of return rule.
- **8-3** Calculate the profitability index and use it to choose between projects when funds are limited.
- **8-4** Understand the payback rule and explain why it *doesn't* always make shareholders better off.
- **8-5** Use the net present value rule to analyze three common problems that involve competing projects: (a) when to postpone an investment expenditure, (b) how to choose between projects with unequal lives, and (c) when to replace equipment.

RELATED WEBSITES FOR THIS CHAPTER CAN BE FOUND IN CONNECT.

High-tech businesses often require huge investments. How do companies decide which investments are worth pursuing?

The investment decision, also known as the *capital budgeting decision*, is central to the success of the company. We have already seen that capital investments can sometimes absorb substantial amounts of cash; they also have very long-term consequences. The assets you buy today may determine the business you are in many years hence.

For some investment projects, "substantial" is an understatement. Consider the following examples:

- Verizon spent $23 billion rolling out its fiber-optic network.
- The cost of bringing one new prescription drug to market has been estimated to be as high as $2.6 billion.
- The eventual cost of the Gorgon natural gas project in Western Australia is forecast at $54 billion.
- General Motors' research and development costs for the Chevrolet Volt have been about $1.2 billion.
- Estimated production costs for *The Hobbit* movie trilogy were over $600 million.
- The development costs of the Boeing 787 Dreamliner jet were over $30 billion.

Notice that many of these big capital projects require heavy investment in intangible assets. For example, almost all the cost of drug development is for research and testing. So is much of the cost of developing the electric auto. Any expenditure made in the hope of generating more cash later can be called a *capital investment project*, regardless of whether the cash outlay goes to tangible or intangible assets.

A company's shareholders prefer to be rich rather than poor. Therefore, they want the firm to invest in every project that is worth more than it costs. The difference between a project's value and its cost is termed the *net present value*. Companies can best help their shareholders by investing in projects with a *positive* net present value.

We start this chapter by showing how to calculate the net present value of some simple investment projects. We then examine three other criteria that companies sometimes use to evaluate investments. Often, they compare the expected rate of return offered by a project to the return that their shareholders could earn on equivalent-risk investments in the capital market. They accept only projects that provide a higher return than shareholders could earn for themselves. Generally, this rule will give the same guidance as the net present value rule, but, as we shall see, it presents some pitfalls, especially when choosing between alternative projects. We explore the key pitfalls of the rate of

TABLE 8.3 Capital budgeting techniques used in practice

Investment Criterion	Percentage of Firms That Always or Almost Always Use Criterion	Average Score on 0–4 Scale (0 = never use; 4 = always use)		
		All Firms	Small Firms	Large Firms
Internal rate of return	76	3.1	2.9	3.4
Net present value	75	3.1	2.8	3.4
Payback period	57	2.5	2.7	2.3
Profitability index	12	0.8	0.9	0.8

Source: J. R. Graham and C. R. Harvey, "The Theory and Practice of Corporate Finance: Evidence from the Field," from *Journal of Financial Economics*, Vol. 60, Issue 2–3, May 2001, pp. 187–243. Copyright © 2001 Elsevier Science.

Clearly, NPV is the gold standard. It is designed to tell you whether an investment will increase the value of the firm and by how much it will do so. It is the only rule that can always be used to rank and choose between mutually exclusive investments. The time that you need to be careful when applying the NPV rule occurs when the firm faces capital rationing. In this case, there may not be enough cash to take every project with a positive NPV, and the firm must then rank projects by the profitability index, that is, net present value per dollar invested.

For managers in the field, discounted cash-flow analysis is in fact the dominant tool for project evaluation. Table 8.3 provides a sample of the results of a large survey of chief financial officers (CFOs). Notice that 75% of firms either always or almost always use NPV or IRR to evaluate projects. The dominance of these criteria is even stronger in larger, presumably more sophisticated, firms. Despite the clear advantages of discounted cash-flow methods, however, firms do use other investment criteria to evaluate projects. For example, just over half of corporations always or almost always compute a project's payback period. The profitability index is routinely computed by about 12% of firms.

What explains such wide use of presumably inferior decision rules? To some extent, these rules present rough reality checks on the project. As we noted in Section 8.4, managers might want to consider some simple ways to describe project profitability, even if they present obvious pitfalls. For example, managers talk casually about quick-payback projects in the same way that investors talk about high-P/E stocks. The fact that they talk about payback does not mean that the payback rule governs their decisions. Shortcuts like payback may work for very simple go-or-no-go decisions, but they are dangerous when used to compare projects.

SUMMARY

What is the net present value of an investment, and how do you calculate it? (*LO8-1*)

The **net present value** of a project measures the difference between its value and cost. NPV is, therefore, the amount that the project will add to shareholder wealth. A company maximizes shareholder wealth by accepting all projects that have a positive NPV.

How is the internal rate of return of a project calculated, and what must one look out for when using the internal rate of return rule? (*LO8-2*)

Instead of asking whether a project has a positive NPV, many businesses prefer to ask whether it offers a higher return than shareholders could expect to get by investing in the capital market. Return is usually defined as the discount rate that would result in a zero NPV. This is known as the **internal rate of return,** or **IRR.** The project is attractive if the IRR exceeds the **opportunity cost of capital.**

There are some pitfalls in using the internal rate of return rule. Be careful about using the IRR when (1) you need to choose between two **mutually exclusive projects,** (2) there is more than one change in the sign of the cash flows, or (3) the early cash flows are positive.

How is the profitability index calculated, and how can it be used to choose between projects when funds are limited? (*LO8-3*)

If there is a shortage of capital, companies need to choose projects that offer the highest net present value per dollar of investment. This measure is known as the **profitability index.**

Why doesn't the payback rule always make shareholders better off? (*LO8-4*)

The net present value rule and the rate of return rule both properly reflect the time value of money. But companies sometimes use rules of thumb to judge projects. One is the payback rule, which states that a project is acceptable if you get your money back within a specified period. The payback rule takes no account of any cash flows that arrive after the payback period and fails to discount cash flows within the payback period.

How can the net present value rule be used to analyze three common problems that involve competing projects: when to postpone an investment expenditure, how to choose between projects with unequal lives, and when to replace equipment? (*LO8-5*)

Sometimes, a project may have a positive NPV if undertaken today but an even higher NPV if the investment is delayed. Choose between these alternatives by comparing their NPVs *today.*

When you have to choose between projects with different lives, you should put them on an equal footing by comparing the **equivalent annual annuity** of the two projects. When you are considering whether to replace an aging machine with a new one, you should compare the annual cost of operating the old one with the equivalent annual annuity of the new one.

LISTING OF EQUATIONS

8.1 NPV = PV – required investment

8.2 $$\text{Profitability index} = \frac{\text{net present value}}{\text{initial investment}}$$

8.3 $$\text{Equivalent annual annuity} = \frac{\text{present value of costs}}{\text{annuity factor}}$$

QUESTIONS AND PROBLEMS

connect

Problems 1–9 refer to two projects with the following cash flows:

Year	Project A	Project B
0	–$200	–$200
1	80	100
2	80	100
3	80	100
4	80	

1. **IRR/NPV.** If the opportunity cost of capital is 11%, which of these projects is worth pursuing? (*LO8-1*)
2. **Mutually Exclusive Investments.** Suppose that you can choose only one of these projects. Which would you choose? The discount rate is still 11%. (*LO8-1*)
3. **IRR/NPV.** Which project would you choose if the opportunity cost of capital is 16%? (*LO8-1*)
4. **IRR.** What are the internal rates of return on projects A and B? (*LO8-2*)
5. **Investment Criteria.** In light of your answers to Problems 2, 3, and 4, is the project with the higher IRR the better project? (*LO8-2*)
6. **Profitability Index.** If the opportunity cost of capital is 11%, what is the profitability index for each project? (*LO8-3*)

7. **Profitability Index.** Is the project with the highest profitability index also the one with the highest NPV? (*LO8-3*)
8. **Payback.** What is the payback period of each project? (*LO8-4*)
9. **Investment Criteria.** Is the project with the shortest payback period also the one with the highest NPV? (*LO8-4*)
10. **NPV and IRR.** A project that costs $3,000 to install will provide annual cash flows of $800 for each of the next 6 years. (*LO8-1 and LO8-2*)
    a. What is NPV if the discount rate is 10%?
    b. How high can the discount rate be before you would reject the project?
11. **NPV.** A proposed nuclear power plant will cost $2.2 billion to build and then will produce cash flows of $300 million a year for 15 years. After that period (in year 15), it must be decommissioned at a cost of $900 million. (*LO8-1 and LO8-2*)
    a. What is project NPV if the discount rate is 5%?
    b. What if the discount rate is 18%?
12. **NPV/IRR.** A new computer system will require an initial outlay of $20,000, but it will increase the firm's cash flows by $4,000 a year for each of the next 8 years. (*LO8-1*)
    a. Is the system worth installing if the required rate of return is 9%?
    b. What if the required return is 14%?
    c. How high can the discount rate be before you would reject the project?
13. **NPV/IRR.** Here are the cash flows for a project under consideration: (*LO8-1 and LO8-2*)
    a. Calculate the project's net present value for discount rates of 0, 50%, and 100%.
    b. What is the IRR of the project?

C_0	C_1	C_2
−$6,750	+$4,500	+$18,000

14. **NPV versus IRR.** Here are the cash flows for two mutually exclusive projects: (*LO8-1 and LO8-2*)

Project	C_0	C_1	C_2	C_3
A	−$20,000	+$8,000	+$8,000	+$ 8,000
B	−20,000	0	0	+25,000

    a. At what interest rates would you prefer project A to B? (*Hint:* Try drawing the NPV profile of each project.)
    b. What is the IRR of each project?
15. **NPV/IRR.** Growth Enterprises believes its latest project, which will cost $80,000 to install, will generate a perpetual growing stream of cash flows. Cash flow at the end of the first year will be $5,000, and cash flows in future years are expected to grow indefinitely at an annual rate of 5%. (*LO8-1 and LO8-2*)
    a. If the discount rate for this project is 10%, what is the project NPV?
    b. What is the project IRR?
16. **IRR/NPV.** Consider the following project with an internal rate of return of 13.1%. (*LO8-2*)
    a. Should you accept or reject the project if the discount rate is 12%?
    b. What is project NPV?

Year	Cash Flow
0	+$100
1	−60
2	−60

17. **Multiple IRRs.** Strip Mining Inc. can develop a new mine at an initial cost of $5 million. The mine will provide a cash flow of $30 million in 1 year. The land then must be reclaimed at a cost of $28 million in the second year. (*LO8-2*)

    a. What are the IRRs of this project?
    b. Should the firm develop the mine if the discount rate is 10%?
    c. What if it is 20%?
    d. What if it is 350%?
    e. What if it is 400%?
18. **IRR.** Marielle Machinery Works forecasts the following cash flows on a project under consideration. It uses the internal rate of return rule to accept or reject projects. (*LO8-2*)
    a. What is the project's IRR?
    b. Should this project be accepted if the required return is 12%?

C_0	C_1	C_2	C_3
−$10,000	0	+$7,500	+$8,500

19. **NPV/IRR.** Consider projects A and B: (*LO8-2*)

	Cash Flows (dollars)			
Project	C_0	C_1	C_2	NPV at 10%
A	−30,000	21,000	21,000	+$6,446
B	−50,000	33,000	33,000	+7,273

    a. Calculate IRRs for A and B.
    b. Which project does the IRR rule suggest is best?
    c. Which project is really best?
20. **IRR.** You are offered the chance to participate in a project that produces the following cash flows:

C_0	C_1	C_2
+$5,000	+$4,000	−$11,000

    The internal rate of return is 13.6%. If the opportunity cost of capital is 12%, would you accept the offer? (What is the net present value of the project?) (*LO8-2*)
21. **Multiple IRRs.** Consider the following cash flows: (*LO8-2*)

C_0	C_1	C_2	C_3	C_4
−$22	+$20	+$20	+$20	−$40

    a. Which two of the following rates are the IRRs of this project: 2.5%, 7.2%. 14.3%, 33.7%, 40.0%?
    b. What is project NPV if the discount rate is 5%?
    c. What if it is 20%?
    d. What if it is 40%?
22. **Profitability Index.** What is the profitability index of a project that costs $10,000 and provides cash flows of $3,000 in years 1 and 2 and $5,000 in years 3 and 4? The discount rate is 9%. (*LO8-3*)
23. **Profitability Index.** Consider the following projects: (*LO8-3*)

Project	C_0	C_1	C_2
A	−$2,100	+$2,000	+$1,200
B	−2,100	+1,440	+1,728

    a. Calculate the profitability index for A and B assuming a 22% opportunity cost of capital.
    b. According to the profitability index rule, which project(s) should you accept?

24. **Capital Rationing.** You are a manager with an investment budget of $8 million. You may invest in the following projects. Investment and cash-flow figures are in millions of dollars. (*LO8-3*)

Project	Discount Rate (%)	Investment	Annual Cash Flow	Project Life (years)
A	10	3	1	5
B	12	4	1	8
C	8	5	2	4
D	8	3	1.5	3
E	12	3	1	6

a. Which projects should the manager choose?
b. Which projects will be chosen if there is no capital rationing?

25. **Profitability Index versus NPV.** Consider projects A and B with the following cash flows: (*LO8-3*)

	C_0	C_1	C_2	C_3
A	−$36	+$20	+$20	+$20
B	−50	+25	+25	+25

a. Which project has the higher NPV if the discount rate is 10%?
b. Which has the higher profitability index?
c. Which project is most attractive to a firm that can raise an unlimited amount of funds to pay for its investment projects?
d. Which project is most attractive to a firm that is limited in the funds it can raise?

26. **Investment Criteria.** If you insulate your office for $10,000, you will save $1,000 a year in heating expenses. These savings will last forever. (*LO8-1, LO8-2, and LO8-4*)
a. What is the NPV of the investment when the cost of capital is 8%?
b. What if it is 10%?
c. What is the IRR of the investment?
d. What is the payback period on this investment?

27. **Payback.** A project that costs $2,500 to install will provide annual cash flows of $600 for the next 6 years. (*LO8-4*)
a. The firm accepts projects with payback periods of less than 5 years. What is this project's payback period?
b. Will it be accepted?
c. Should this project be pursued if the discount rate is 2%? (What is its NPV?)
d. What if the discount rate is 12%?
e. Will the firm's decision change as the discount rate changes?

28. **Payback and NPV.** A project has a life of 10 years and a payback period of 10 years. Is the project NPV positive or negative? (*LO8-4*)

29. **Payback and NPV.** Here are the expected cash flows for three projects: (*LO8-4*)

	Cash Flows (dollars)				
Project	Year: 0	1	2	3	4
A	−5,000	+1,000	+1,000	+3,000	0
B	−1,000	0	+1,000	+2,000	+3,000
C	−5,000	+1,000	+1,000	+3,000	+5,000

a. What is the payback period on each of the projects?
b. Given that you wish to use the payback rule with a cutoff period of 2 years, which projects would you accept?

c. If you use a cutoff period of 3 years, which projects would you accept?
d. If the opportunity cost of capital is 10%, which projects have positive NPVs?
e. "Payback gives too much weight to cash flows that occur after the cutoff date." True or false?

30. **Investment Criteria.** A new furnace for your small factory will cost $27,000 a year to install and will require ongoing maintenance expenditures of $1,500 a year. But it is far more fuel-efficient than your old furnace and will reduce your consumption of heating oil by 2,400 gallons per year. Heating oil this year will cost $3 a gallon; the price per gallon is expected to increase by $.50 a year for the next 3 years and then to stabilize for the foreseeable future. The furnace will last for 20 years, at which point it will need to be replaced and will have no salvage value. The discount rate is 8%. (*LO8-1, LO8-2, LO8-4, and LO8-5*)
a. What is the net present value of the investment in the furnace?
b. What is the IRR?
c. What is the payback period?
d. What is the equivalent annual cost of the furnace?
e. What is the equivalent annual savings derived from the furnace?
f. Compare the PV of the difference between the equivalent annual cost and savings to your answer to part (a). Are the two measures the same or is one larger?

31. **Mutually Exclusive Investments.** Here are the cash-flow forecasts for two mutually exclusive projects: (*LO8-5*)

	Cash Flows (dollars)	
Year	Project A	Project B
0	−100	−100
1	30	49
2	50	49
3	70	49

a. Which project would you choose if the opportunity cost of capital is 2%?
b. Which would you choose if the opportunity cost of capital is 12%?

32. **Equivalent Annual Annuity.** A precision lathe costs $10,000 and will cost $20,000 a year to operate and maintain. If the discount rate is 10% and the lathe will last for 5 years, what is the equivalent annual cost of the tool? (*LO8-5*)

33. **Equivalent Annual Annuity.** A firm can lease a truck for 4 years at a cost of $30,000 annually. It can instead buy a truck at a cost of $80,000, with annual maintenance expenses of $10,000. The truck will be sold at the end of 4 years for $20,000. (*LO8-5*)
a. What is the equivalent annual cost of buying and maintaining the truck if the discount rate is 10%?
b. Which is the better option: leasing or buying?

34. **Equivalent Annual Annuity.** Econo-Cool air conditioners cost $300 to purchase, result in electricity bills of $150 per year, and last for 5 years. Luxury Air models cost $500, result in electricity bills of $100 per year, and last for 8 years. The discount rate is 21%. (*LO8-5*)
a. What is the equivalent annual cost of the Econo-Cool model?
b. What is the equivalent annual cost of the Luxury Air model?
c. Which model is more cost-effective?
d. Now you remember that the inflation rate is expected to be 10% per year for the foreseeable future. Redo parts (a) and (b).

35. **Investment Timing.** You can purchase an optical scanner today for $400. The scanner provides benefits worth $60 a year. The expected life of the scanner is 10 years. Scanners are expected to decrease in price by 20% per year. Suppose the discount rate is 10%. (*LO8-5*)
1. Should you purchase the scanner today or wait to purchase?
2. When is the best purchase time?

36. **Replacement Decision.** You are operating an old machine that is expected to produce a cash inflow of $5,000 in each of the next 3 years before it fails. You can replace it now with a new machine that costs $20,000 but is much more efficient and will provide a cash flow of $10,000 a year for 4 years. Should you replace your equipment now? The discount rate is 15%. (*LO8-5*)

37. **Investment Timing.** A classic problem in management of forests is determining when it is most economically advantageous to cut a tree for lumber. When the tree is young, it grows very rapidly. As it ages, its growth slows down. Why is NPV maximized if you cut the tree when its growth rate equals the discount rate? (*LO8-5*)

38. **Replacement Decision.** A forklift will last for only 2 more years. It costs $5,000 a year to maintain. For $20,000 you can buy a new lift that can last for 10 years and should require maintenance costs of only $2,000 a year. (*LO8-5*)
    a. If the discount rate is 4% per year, should you replace the forklift?
    b. What if the discount rate is 12% per year?

SOLUTIONS TO SELF-TEST QUESTIONS

8.1 Even if construction costs are $355,000, NPV is still positive:

$$NPV = PV - \$355{,}000 = \$357{,}143 - \$355{,}000 = \$2{,}143$$

Therefore, the project is still worth pursuing. The project is viable as long as construction costs are less than the PV of the future cash flow, that is, as long as construction costs are less than $357,143. However, if the opportunity cost of capital is 20%, the PV of the $400,000 sales price is lower and NPV is negative:

$$PV = \$400{,}000 \times \frac{1}{1.20} = \$333{,}333$$

$$NPV = PV - \$355{,}000 = -\$21{,}667$$

The present value of the future cash flow is not as high when the opportunity cost of capital is higher. The project would need to provide a higher payoff in order to be viable in the face of the higher opportunity cost of capital.

8.2 The IRR is now about 8.3% because

$$NPV = -\$375{,}000 + \frac{\$25{,}000}{1.083} + \frac{\$25{,}000}{(1.083)^2} + \frac{\$420{,}000}{(1.083)^3} = 0$$

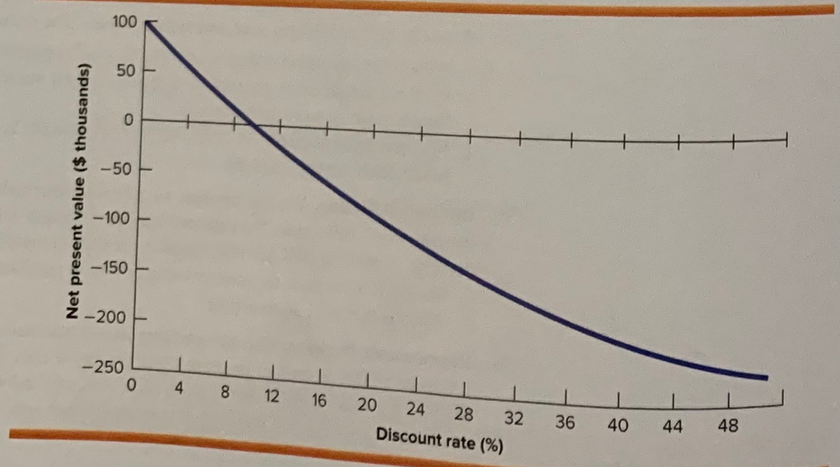
Note in Figure 8.6 that NPV falls to zero as the discount rate reaches 8.3%.

8.3 You want to be rich. The NPV of the long-lived investment is much larger.

$$\text{Short: } NPV = -\$1 + \frac{\$2}{1.07} = +\$.869 \text{ million}$$

$$\text{Long: } NPV = -\$1 + \frac{\$.3}{.07} = +\$3.286 \text{ million}$$

FIGURE 8.6 NPV falls to zero at an interest rate of 8.3%.



8.4 You want to be richer. The second alternative generates greater value at any reasonable discount rate. Other risk-free investments offer 7%. Therefore,

$$NPV = -\$1{,}000 + \frac{\$4{,}000}{1.07} = +\$2{,}738$$

$$NPV = -\$1{,}000{,}000 + \frac{\$1{,}500{,}000}{1.07} = +\$401{,}869$$

8.5 The profitability index gives the correct ranking for the first pair, but the incorrect ranking for the second:

Project	PV	Investment	NPV	Profitability Index (NPV/Investment)
Short	$1,869,159	$1,000,000	$ 869,159	0.869
Long	4,285,714	1,000,000	3,285,714	3.286
Small	3,738	1,000	2,738	2.738
Large	1,401,869	1,000,000	401,869	0.402

8.6 The payback period is $5,000/$660 = 7.6 years. Discounted payback is just over 11 years. Calculate NPV as follows. The present value of a $660 annuity for 20 years at 6% is

$$\text{PV annuity} = \$7{,}570$$

$$NPV = -\$5{,}000 + \$7{,}570 = +\$2{,}570$$

The project should be accepted.

8.7

Year of Purchase	Cost of Computer	PV Savings	NPV at Year of Purchase	NPV Today
0	$50	$70	$20	$20
1	45	66	21	19.1
2	40	62	22	18.2
3	36	58	22	16.5
4	33	54	21	14.3
5	31	50	19	11.8

Purchase the new computer now.

8.8

	Year:	0	1	2	3	PV of Costs
K	Cash flows	$10,000	$1,100	$1,200		$11,992
	Equivalent annual annuity		6,910	6,910		11,992
L	Cash flows	12,000	1,100	1,200	$1,300	14,968
	Equivalent annual annuity		6,019	6,019	6,019	14,968

Machine L is the better buy. However, it's even better to keep the old machine going for 1 more year. That costs $4,300, which is less than L's equivalent annual cost, $6,019.

SOLUTIONS TO SPREADSHEET QUESTIONS

1. NPV is zero at .1256, or 12.56%. This is the office project's IRR.
2. The answer is 54.2. The correct NPV of 57.9 is 7% higher. The Excel function treats the initial cash flow as if it occurs at the end of 1 year, and discounts each successive cash flow by an extra year.

MINICASE

Flowton Products enjoys a steady demand for stainless steel infiltrators used in a number of chemical processes. Revenues from the infiltrator division are $50 million a year, and production costs are $47.5 million. However, the 10 high-precision Munster stamping machines that are used in the production process are coming to the end of their useful life. One possibility is simply to replace each existing machine with a new Munster. These machines would cost $800,000 each and would not involve any additional operating costs. The alternative is to buy 10 centrally controlled Skilboro stampers. Skilboros cost $1.25 million each, but compared with the Munster, they would produce a total saving in operator and material costs of $500,000 a year. Moreover, the Skilboro is sturdily built and would last 10 years, compared with an estimated 7-year life for the Munster.

Analysts in the infiltrator division have produced the accompanying summary table, which shows the forecast total cash flows from the infiltrator business over the life of each machine. Flowton's standard procedures for appraising capital investments involve calculating net present value, internal rate of return, and payback, and these measures are also shown in the table.

As usual, Emily Balsam arrived early at Flowton's head office. She had never regretted joining Flowton. Everything about the place, from the mirror windows to the bell fountain in the atrium, suggested a classy outfit. Ms. Balsam sighed happily and reached for the envelope at the top of her in-tray. It was an analysis from the infiltrator division of the replacement options for the stamper machines. Pinned to the paper was the summary table of cash flows and a note from the CFO, which read, "Emily, I have read through 20 pages of excruciating detail and I still don't know which of these machines we should buy. The NPV calculation seems to indicate that the Skilboro is best, while IRR and payback suggest the opposite. Would you take a look and tell me what we should do and why. You also might check that the calculations are OK."

Can you help Ms. Balsam by writing a memo to the CFO? You need to justify your solution and also to explain why some or all of the measures in the summary table are inappropriate.

	Cash Flows (millions of dollars)				
Year:	0	1–7	8	9	10
Munster					
Investment	–8.0		0	0	0
Revenues		50.0	0	0	0
Costs		47.5	0	0	0
Net cash flow	–8.0	2.5			
NPV at 15%	$2.40 million				
IRR	24.5%				
Payback period	3.2 years				
Skilboro					
Investment	–12.5				
Revenues		50.0	50.0	50.0	50.0
Costs		47.0	47.0	47.0	47.0
Net cash flow	–12.5	3.0	3.0	3.0	3.0
NPV at 15%	$2.56 million				
IRR	20.2%				
Payback period	4.2 years				

APPENDIX

More on the IRR Rule

In Section 8.2, we described several pitfalls that lie in wait for users of the IRR rule. However, there are some tricks that users of the rule may use to circumvent these hazards. In this appendix, we show how you can adapt the IRR rule when you need to choose between competing projects or when there are multiple IRRs.

Using the IRR to Choose between Mutually Exclusive Projects

When you need to choose between mutually exclusive projects, a simple comparison of internal rates of return is liable to lead to poor decisions. We illustrated this by looking at our two office building projects. Your initial proposal involved constructing the office building and then selling it. Under the revised proposal you would construct a more expensive building and rent it out before selling it at the end of 3 years. The cash flows from the two projects were as follows:

	Cash Flows					
	C_0	C_1	C_2	C_3	IRR	NPV at 7%
Initial proposal	–350,000	+400,000			14.29%	+$23,832
Revised proposal	–375,000	+25,000	+25,000	+475,000	12.56%	+$57,942

Although the initial proposal has the higher IRR, it has the lower net present value. If you were misled into choosing the initial rather than the revised proposal, you would have been more than $30,000 poorer.

You can salvage the IRR rule in these cases; you do so by calculating the IRR on the *incremental* cash flows, that is, the difference in cash flows between the two projects. Start with the smaller project, where you plan to invest $350,000 and sell the office building after 1 year. It has an IRR of 14.29%, which is well in excess of the 7% cost of capital. So you know that it is worthwhile. You now ask yourself whether it is worth investing the additional $25,000 and renting out the building for 3 years. Here are the incremental cash flows from doing so, together with their IRR and NPV:

Cash Flows:	C_0	C_1	C_2	C_3	IRR	NPV at 7%
Incremental cash flows	–25,000	–375,000	+25,000	+475,000	11.72%	+$34,110

The IRR on the incremental cash flows is 11.72%. Since this is greater than the opportunity cost of capital, you should prefer the revised proposal.

BEYOND THE PAGE

Calculate the MIRR

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Using the Modified Internal Rate of Return When There Are Multiple IRRs

Whenever there is more than one change in the sign of the cash flows, there is generally more than one internal rate of return and, therefore, no simple IRR rule. Companies sometimes get around this problem by calculating a *modified IRR (MIRR)*, which can then be compared with the cost of capital.

Think back to King Coal's strip-mining project. Its cash flows are as follows:

Cash Flows:	C_0	C_1	C_2	C_3	C_4	C_5
Cash flows ($ millions)	–210	+125	+125	+175	+175	–400

The problem with the IRR rule arises because the cash flow in year 5 is negative. So let us try replacing the last two cash flows with a *single* year 4 cash flow that has the same present value. If the cost of capital is 20%, then we can replace the cash flows in years 4 and 5 with a single cash flow in year 4 of

$$+175 - \frac{400}{1.20} = -158$$

This figure is also negative. So we still have a problem. Therefore, we need to step back a further year and combine the last *three* cash flows into a single year 3 cash flow with the same present value:

$$+175 + \frac{175}{1.20} - \frac{400}{1.20^2} = +43$$

This value is positive, so if we use it in place of the last three cash flows, we will have only one change of sign. Now we can compute IRR using the *modified* cash-flow sequence:

Year:	0	1	2	3	4	5
Cash flows ($ millions)	−210	+125	+125	+43	—	—

The modified IRR (MIRR) is the discount rate at which the net present value of these cash flows is zero:

$$-210 + \frac{125}{1 + \text{MIRR}} + \frac{125}{(1 + \text{MIRR})^2} + \frac{43}{(1 + \text{MIRR})^3} = 0$$

We solve to find that MIRR = .22, or 22%, which is greater than the cost of capital of 20%. If the modified IRR is greater than the cost of capital, then the project must have a positive NPV.