Discussion Questions and Problems

Discussion Questions

7-1 Discuss the similarities and differences between minimization and maximization problems using the graphical solution approaches of LP.

7-2 It is important to understand the assumptions underlying the use of any quantitative analysis model. What are the assumptions and requirements for an LP model to be formulated and used?

7-3 It has been said that each LP problem that has a feasible region has an infinite number of solutions. Explain.

7-4 You have just formulated a maximization LP problem and are preparing to solve it graphically. What criteria should you consider in deciding whether it would be easier to solve the problem by the corner point method or the isoprofit line approach?

7-5 Under what condition is it possible for an LP problem to have more than one optimal solution?

7-6 Develop your own set of constraint equations and inequalities and use them to illustrate graphically each of the following conditions:

(a) an unbounded problem
(b) an infeasible problem
(c) a problem containing redundant constraints

7-7 The production manager of a large Cincinnati manufacturing firm once made the statement, "I would like to use LP, but it's a technique that operates under conditions of certainty. My plant doesn't have that certainty; it's a world of uncertainty. So LP can't be used here." Do you think this statement has any merit? Explain why the manager may have said it.

7-8 The mathematical relationships that follow were formulated by an operations research analyst at the Smith–Lawton Chemical Company. Which ones are invalid for use in an LP problem, and why?

$$
\begin{array}{lrl}
\text{Maximize profit} = & 4X_1 + 3X_1X_2 + 8X_2 + 5X_3 & \\
\text{subject to} & 2X_1 + X_2 + 2X_3 & \le 50 \\
 & X_1 - 4X_2 & \ge 6 \\
 & 1.5X_1^2 + 6X_2 + 3X_3 & \ge 21 \\
 & 19X_2 - 0.35X_3 & = 17 \\
 & 5X_1 + 4X_2 + 3\sqrt{X_3} & \le 80 \\
 & -X_1 - X_2 + X_3 & = 5
\end{array}
$$

7-9 Discuss the role of sensitivity analysis in LP. Under what circumstances is it needed, and under what conditions do you think it is not necessary?

7-10 A linear program has the objective of maximizing profit $= 12X + 8Y$. The maximum profit is \$8,000. Using a computer we find the upper bound for profit on X is 20 and the lower bound is 9. Discuss the changes to the optimal solution (the values of the variables and the profit) that would occur if the profit on X were increased to \$15. How would the optimal solution change if the profit on X were increased to \$25?

7-11 A linear program has a maximum profit of \$600. One constraint in this problem is $4X + 2Y \le 80$. Using a computer we find the dual price for this constraint is 3, and there is a lower bound of 75 and an upper bound of 100. Explain what this means.

7-12 Develop your own original LP problem with two constraints and two real variables.

(a) Explain the meaning of the numbers on the right-hand side of each of your constraints.
(b) Explain the significance of the technological coefficients.
(c) Solve your problem graphically to find the optimal solution.
(d) Illustrate graphically the effect of increasing the contribution rate of your first variable (X_1) by 50% over the value you first assigned it. Does this change the optimal solution?

7-13 Explain how a change in a technological coefficient can affect a problem's optimal solution. How can a change in resource availability affect a solution?

Problems

• 7-14 The Electrocomp Corporation manufactures two electrical products: air conditioners and large fans. The assembly process for each is similar in that both require a certain amount of wiring and drilling. Each air conditioner takes 3 hours of wiring and 2 hours of drilling. Each fan must go through 2 hours of wiring and 1 hour of drilling. During the next production period, 240 hours of wiring time are available and up to 140 hours of drilling time may be used. Each air conditioner sold yields a profit of \$25. Each fan assembled may be sold for a \$15 profit. Formulate and solve this LP production mix situation to find the best combination of air conditioners and fans that yields the highest profit. Use the corner point graphical approach.

• 7-15 Electrocomp's management realizes that it forgot to include two critical constraints (see Problem 7-14). In particular, management decides that there should be a minimum number of air conditioners produced

ISBN 1-269-73132-7

Note: means the problem may be solved with QM for Windows; means the problem may be solved with Excel; and means the problem may be solved with QM for Windows and/or Excel.

in order to fulfill a contract. Also, due to an oversupply of fans in the preceding period, a limit should be placed on the total number of fans produced.

(a) If Electrocomp decides that at least 20 air conditioners should be produced but no more than 80 fans should be produced, what would be the optimal solution? How much slack or surplus is there for each of the four constraints?

(b) If Electrocomp decides that at least 30 air conditioners should be produced but no more than 50 fans should be produced, what would be the optimal solution? How much slack or surplus is there for each of the four constraints at the optimal solution?

7-16 A candidate for mayor in a small town has allocated $40,000 for last-minute advertising in the days preceding the election. Two types of ads will be used: radio and television. Each radio ad costs $200 and reaches an estimated 3,000 people. Each television ad costs $500 and reaches an estimated 7,000 people. In planning the advertising campaign, the campaign manager would like to reach as many people as possible, but she has stipulated that at least 10 ads of each type must be used. Also, the number of radio ads must be at least as great as the number of television ads. How many ads of each type should be used? How many people will this reach?

7-17 The Outdoor Furniture Corporation manufactures two products, benches and picnic tables, for use in yards and parks. The firm has two main resources: its carpenters (labor force) and a supply of redwood for use in the furniture. During the next production cycle, 1,200 hours of labor are available under a union agreement. The firm also has a stock of 3,500 board feet of good-quality redwood. Each bench that Outdoor Furniture produces requires 4 labor hours and 10 board feet of redwood; each picnic table takes 6 labor hours and 35 board feet of redwood. Completed benches will yield a profit of $9 each, and tables will result in a profit of $20 each. How many benches and tables should Outdoor Furniture produce to obtain the largest possible profit? Use the graphical LP approach.

7-18 The dean of the Western College of Business must plan the school's course offerings for the fall semester. Student demands make it necessary to offer at least 30 undergraduate and 20 graduate courses in the term. Faculty contracts also dictate that at least 60 courses be offered in total. Each undergraduate course taught costs the college an average of $2,500 in faculty wages, and each graduate course costs $3,000. How many undergraduate and graduate courses should be taught in the fall so that total faculty salaries are kept to a minimum?

7-19 MSA Computer Corporation manufactures two models of minicomputers, the Alpha 4 and the Beta 5. The firm employs five technicians, working 160 hours each per month, on its assembly line. Management insists that full employment (i.e., *all* 160 hours of time) be maintained for each worker during next month's operations. It requires 20 labor hours to assemble each Alpha 4 computer and 25 labor hours to assemble each Beta 5 model. MSA wants to see at least 10 Alpha 4s and at least 15 Beta 5s produced during the production period. Alpha 4s generate $1,200 profit per unit, and Beta 5s yield $1,800 each. Determine the most profitable number of each model of minicomputer to produce during the coming month.

7-20 A winner of the Texas Lotto has decided to invest $50,000 per year in the stock market. Under consideration are stocks for a petrochemical firm and a public utility. Although a long-range goal is to get the highest possible return, some consideration is given to the risk involved with the stocks. A risk index on a scale of 1–10 (with 10 being the most risky) is assigned to each of the two stocks. The total risk of the portfolio is found by multiplying the risk of each stock by the dollars invested in that stock.

The following table provides a summary of the return and risk:

STOCK	ESTIMATED RETURN	RISK INDEX
Petrochemical	12%	9
Utility	6%	4

The investor would like to maximize the return on the investment, but the average risk index of the investment should not be higher than 6. How much should be invested in each stock? What is the average risk for this investment? What is the estimated return for this investment?

7-21 Referring to the Texas Lotto situation in Problem 7-20, suppose the investor has changed his attitude about the investment and wishes to give greater emphasis to the risk of the investment. Now the investor wishes to minimize the risk of the investment as long as a return of at least 8% is generated. Formulate this as an LP problem and find the optimal solution. How much should be invested in each stock? What is the average risk for this investment? What is the estimated return for this investment?

7-22 Solve the following LP problem using the corner point graphical method. At the optimal solution, calculate the slack for each constraint:

$$\begin{aligned} \text{Maximize profit} &= 4X + 4Y \\ \text{subject to} \quad & 3X + 5Y \le 150 \\ & X - 2Y \le 10 \\ & 5X + 3Y \le 150 \\ & X, Y \ge 0 \end{aligned}$$

ISBN 1-269-73132-7

7-23 Consider this LP formulation:

$$\begin{aligned} \text{Minimize cost} &= \$X + 2Y \\ \text{subject to} \quad X + 3Y &\geq 90 \\ 8X + 2Y &\geq 160 \\ 3X + 2Y &\geq 120 \\ Y &\leq 70 \\ X, Y &\geq 0 \end{aligned}$$

Graphically illustrate the feasible region and apply the isocost line procedure to indicate which corner point produces the optimal solution. What is the cost of this solution?

7-24 The stock brokerage firm of Blank, Leibowitz, and Weinberger has analyzed and recommended two stocks to an investors' club of college professors. The professors were interested in factors such as short-term growth, intermediate growth, and dividend rates. These data on each stock are as follows:

	STOCK ($)	
FACTOR	LOUISIANA GAS AND POWER	TRIMEX INSULATION COMPANY
Short-term growth potential, per dollar invested	0.36	0.24
Intermediate growth potential (over next three years), per dollar invested	1.67	1.50
Dividend rate potential	4%	8%

Each member of the club has an investment goal of (1) an appreciation of no less than $720 in the short term, (2) an appreciation of at least $5,000 in the next three years, and (3) a dividend income of at least $200 per year. What is the smallest investment that a professor can make to meet these three goals?

7-25 Woofer Pet Foods produces a low-calorie dog food for overweight dogs. This product is made from beef products and grain. Each pound of beef costs $0.90, and each pound of grain costs $0.60. A pound of the dog food must contain at least 9 units of Vitamin 1 and 10 units of Vitamin 2. A pound of beef contains 10 units of Vitamin 1 and 12 units of Vitamin 2. A pound of grain contains 6 units of Vitamin 1 and 9 units of Vitamin 2. Formulate this as an LP problem to minimize the cost of the dog food. How many pounds of beef and grain should be included in each pound of dog food? What is the cost and vitamin content of the final product?

7-26 The seasonal yield of olives in a Piraeus, Greece, vineyard is greatly influenced by a process of branch pruning. If olive trees are pruned every two weeks, output is increased. The pruning process, however, requires considerably more labor than permitting the olives to grow on their own and results in a smaller size olive. It also, though, permits olive trees to be spaced closer together. The yield of 1 barrel of olives by pruning requires 5 hours of labor and 1 acre of land. The production of a barrel of olives by the normal process requires only 2 labor hours but takes 2 acres of land. An olive grower has 250 hours of labor available and a total of 150 acres for growing. Because of the olive size difference, a barrel of olives produced on pruned trees sells for $20, whereas a barrel of regular olives has a market price of $30. The grower has determined that because of uncertain demand, no more than 40 barrels of pruned olives should be produced. Use graphical LP to find

(a) the maximum possible profit.
(b) the best combination of barrels of pruned and regular olives.
(c) the number of acres that the olive grower should devote to each growing process.

7-27 Consider the following four LP formulations. Using a graphical approach, determine

(a) which formulation has more than one optimal solution.
(b) which formulation is unbounded.
(c) which formulation has no feasible solution.
(d) which formulation is correct as is.

Formulation 1

$$\begin{aligned} \text{Maximize } & 10X_1 + 10X_2 \\ \text{subject to } & 2X_1 \leq 10 \\ & 2X_1 + 4X_2 \leq 16 \\ & 4X_2 \leq 8 \\ & X_1 = 6 \end{aligned}$$

Formulation 2

$$\begin{aligned} \text{Maximize } & X_1 + 2X_2 \\ \text{subject to } & X_1 \leq 1 \\ & 2X_2 \leq 2 \\ & X_1 + 2X_2 \leq 2 \end{aligned}$$

Formulation 3

$$\begin{aligned} \text{Maximize } & 3X_1 + 2X_2 \\ \text{subject to } & X_1 + X_2 \geq 5 \\ & X_1 \geq 2 \\ & 2X_2 \geq 8 \end{aligned}$$

Formulation 4

$$\begin{aligned} \text{Maximize } & 3X_1 + 3X_2 \\ \text{subject to } & 4X_1 + 6X_2 \leq 48 \\ & 4X_1 + 2X_2 \leq 12 \\ & 3X_2 \geq 3 \\ & 2X_1 \geq 2 \end{aligned}$$

7-28 Graph the following LP problem and indicate the optimal solution point:

$$\begin{aligned} \text{Maximize profit} &= \$3X + \$2Y \\ \text{subject to} \quad 2X + Y &\leq 150 \\ 2X + 3Y &\leq 300 \end{aligned}$$

(a) Does the optimal solution change if the profit per unit of X changes to $4.50?
(b) What happens if the profit function should have been $3X + $3Y?

ISBN 1-269-73132-7