Net Present Value and Other Investment Rules 7

OPENING CASE

In February 2016, Ford announced that it would spend about $1 billion to build a new plant in Sal Luis Potosí, Mexico, to compete with Toyota's Prius. This followed Ford's announcement the previous year that it would invest $2.5 billion in two new plants in Mexico to manufacture engines and transmissions. Ford was not alone in new building in the auto industry. Rival GM was expected to announce that it would spend $5 billion in new plants in Mexico as well.

Ford's new plants are examples of capital budgeting decisions. Decisions such as this one, with a price tag of $3.5 billion, are obviously major undertakings, and the risks and rewards must be carefully weighed. In this chapter, we discuss the basic tools used in making such decisions.

In Chapter 1, we saw that increasing the value of the stock in a company is the goal of financial management. Thus, what we need to know is how to tell whether a particular investment will achieve that or not. This chapter considers a variety of techniques that are used in practice for this purpose. More importantly, it shows how many of these techniques can be misleading, and it explains why the net present value approach is the right one.

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7.1 WHY USE NET PRESENT VALUE?

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This chapter, as well as the next two, focuses on *capital budgeting*, the decision-making process for accepting or rejecting projects. This chapter develops the basic capital budgeting methods, leaving much of the practical application to Chapters 8 and 9. But we don't have to develop these methods from scratch. In Chapter 4, we pointed out that a dollar received in the future is worth less than a dollar received today. The reason, of course, is that today's dollar can be reinvested, yielding a greater amount in the future. And we showed in Chapter 4 that the exact worth of a dollar to be received in the future is its present value. Furthermore, Section 4.1 suggested calculating the *net present value* of any project. That is, the section suggested calculating the difference between the sum of the present values of the project's future cash flows and the initial cost of the project.

The net present value (NPV) method is the first one to be considered in this chapter. We begin by reviewing the approach with a simple example. Next, we ask why the method leads to good decisions.

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EXAMPLE 7.1

Net Present Value

The Alpha Corporation is considering investing in a riskless project costing $100. The project receives $107 in one year and has no other cash flows. The discount rate is 6 percent.

The NPV of the project can easily be calculated as:

$$\$.94 = -\$100 + \frac{\$107}{1.06}$$

From Chapter 4, we know that the project should be accepted since its NPV is positive. Had the NPV of the project been negative, as would have been the case with an interest rate greater than 7 percent, the project should be rejected.

The basic investment rule can be generalized to:

Accept a project if the NPV is greater than zero.

Reject a project if the NPV is less than zero.

We refer to this as the **NPV rule.**

Now why does the NPV rule lead to good decisions? Consider the following two strategies available to the managers of Alpha Corporation:

1. Use $100 of corporate cash to invest in the project. The $107 will be paid as a dividend in one year.
2. Forgo the project and pay the $100 of corporate cash as a dividend today.

If Strategy 2 is employed, the stockholder might deposit the dividend in his bank for one year. With an interest rate of 6 percent, Strategy 2 would produce cash of $106 (= $100 × 1.06) at the end of the year. The stockholder would prefer Strategy 1, since Strategy 2 produces less than $107 at the end of the year.

Thus, our basic point is:

Accepting positive NPV projects benefits the stockholders.

You can get a freeware NPV calculator at www.wheatworks.com.

How do we interpret the exact NPV of $.94? This is the increase in the value of the firm from the project. For example, imagine that the firm today has productive assets worth $V and has $100 of cash. If the firm forgoes the project, the value of the firm today would simply be:

$$\$V + \$100$$

If the firm accepts the project, the firm will receive $107 in one year but will have no cash today. Thus, the firm's value today would be:

$$\$V + \frac{\$107}{1.06}$$

The difference between the above equations is just $.94, the present value of Example 7.1. Thus:

The value of the firm rises by the NPV of the project.

Note that the value of the firm is merely the sum of the values of the different projects, divisions, or other entities within the firm. This property, called **value additivity**, is quite important. It implies that the contribution of any project to a firm's value is simply the NPV of the project. As we will see later, alternative methods discussed in this chapter do not generally have this nice property.

One detail remains. We assumed that the project was riskless, a rather implausible assumption. Future cash flows of real-world projects are invariably risky. In other words, cash flows can only be estimated, rather than known. Imagine that the managers of Alpha *expect* the cash flow of the project to be $107 next year. That is, the cash flow could be higher, say $117, or lower, say $97. With this slight change, the project is risky. Suppose the project is about as risky as the stock market as a whole, where the expected return this year is, say, 10 percent. Then 10 percent becomes the discount rate, implying that the NPV of the project would be:

$$-\$2.73 = -\$100 + \frac{\$107}{1.10}$$

Since the NPV is negative, the project should be rejected. This makes sense because a stockholder of Alpha receiving a $100 dividend today could invest it in the stock market, expecting a 10 percent return. Why accept a project with the same risk as the market but with an expected return of only 7 percent?

Conceptually, the discount rate on a risky project is the return that one can expect to earn on a financial asset of comparable risk. This discount rate is often referred to as an *opportunity cost*, since corporate investment in the project takes away the stockholder's opportunity to invest the same cash in a financial asset. The calculation is by no means impossible. While we forgo the calculation in this chapter, we present it in Chapter 12 of the text.

Having shown that NPV is a sensible approach, how can we tell whether alternative methods are as good as NPV? The key to NPV is its three attributes:

1. *NPV Uses Cash Flows*. Cash flows from a project can be used for other corporate purposes (e.g., dividend payments, other capital budgeting projects, or payments of corporate interest). By contrast, earnings are an artificial construct. While earnings are useful to accountants, they should not be used in capital budgeting because they do not represent cash.
2. *NPV Uses All the Cash Flows of the Project*. Other approaches ignore cash flows beyond a particular date; beware of these approaches.
3. *NPV Discounts the Cash Flows Properly*. Other approaches may ignore the time value of money when handling cash flows. Beware of these approaches as well.

Calculating NPVs by hand can be tedious. A nearby *Spreadsheet Techniques* box shows how to do it the easy way and also illustrates an important caveat.

7.2 THE PAYBACK PERIOD METHOD

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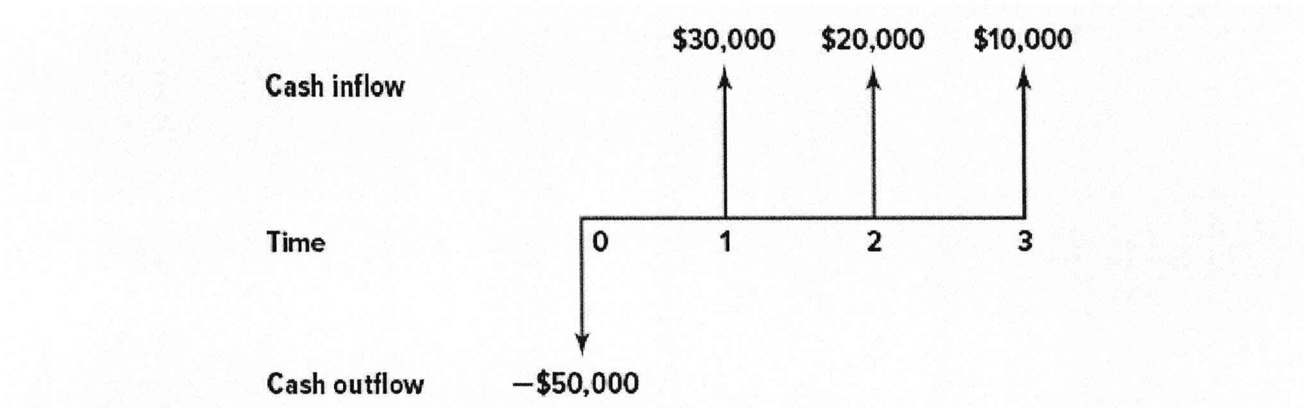
Defining the Rule

One of the most popular alternatives to NPV is **payback**. Here is how payback works: Consider a project with an initial investment of −$50,000. Cash flows are $30,000, $20,000, and $10,000 in the first three years, respectively. These flows are illustrated in Figure 7.1. A useful way of writing down investments like the preceding is with the notation:

(−$50,000, $30,000, $20, 000, $10,000)

FIGURE 7.1
Cash Flows of an Investment Project



SPREADSHEET TECHNIQUES

Calculating NPVs with a Spreadsheet

Spreadsheets are commonly used to calculate NPVs. Examining the use of spreadsheets in this context also allows us to issue an important warning. Consider the following:

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate net present values**							
3								
4	A project's cost is $10,000. The cash flows are $2,000 per year for the first two years,							
5	$4,000 per year for the next two, and $5,000 in the last year. The discount rate is							
6	10 percent; what's the NPV?							
7								
8		Year	Cash flow					
9		0	-$10,000	Discount rate =		10%		
10		1	2,000					
11		2	2,000		**NPV =**	**$2,102.72**	(*wrong* answer)	
12		3	4,000		**NPV =**	**$2,312.99**	(*right* answer)	
13		4	4,000					
14		5	5,000					
15								
16	The formula entered in cell F11 is =NPV(F9, C9:C14). However, this gives the wrong answer because the							
17	NPV function actually calculates present values, not *net* present values.							
18								
19	The formula entered in cell F12 is =NPV(F9, C10:C14) + C9. This gives the right answer because the							
20	NPV function is used to calculate the present value of the cash flows and then the initial cost is							
21	subtracted to calculate the answer. Notice that we added cell C9 because it is already negative.							

In our spreadsheet example, notice that we have provided two answers. The first answer is wrong even though we used the spreadsheet's NPV formula. What happened is that the "NPV" function in our spreadsheet is actually a PV function; unfortunately, one of the original spreadsheet programs many years ago got the definition wrong, and subsequent spreadsheets have copied it! Our second answer shows how to use the formula properly.

The example here illustrates the danger of blindly using calculators or computers without understanding what is going on; we shudder to think of how many capital budgeting decisions in the real world are based on incorrect use of this particular function.

The minus sign in front of the $50,000 reminds us that this is a cash outflow for the investor, and the commas between the different numbers indicate that they are received—or if they are cash outflows, that they are paid out—at different times. In this example we are assuming that the cash flows occur one year apart, with the first one occurring the moment we decide to take on the investment.

The firm receives cash flows of $30,000 and $20,000 in the first two years, which add up to the $50,000 original investment. This means that the firm has recovered its investment within two years. In this case two years is the *payback period* of the investment.

The payback period rule for making investment decisions is simple and potentially informative. The payback tells us when the cash outflow of an investment is "paid back" by cash inflows. If a particular cutoff date, say two years, is selected, all investment projects that have payback periods of two years or less are accepted and all of those that pay off in more than two years—if at all—are rejected.

Problems with the Payback Method

There are at least three problems with payback. To illustrate the first two problems, we consider the three projects in Table 7.1. All three projects have the same three-year payback period, so they should all be equally attractive—right?

TABLE 7.1 Expected Cash Flows for Projects *A* through *C* ($)

YEAR	*A*	*B*	*C*
0	−$100	−$100	−$100
1	20	50	50
2	30	30	30
3	50	20	20
4	60	60	100
Payback period (years)	3	3	3

Actually, they are not equally attractive, as can be seen by a comparison of different *pairs* of projects. To illustrate the payback period problems, consider Table 7.1. Suppose the expected return on comparable risky projects is 10 percent. Then we would use a discount rate of 10 percent for these projects. If so, the NPV would be $21.5, $26.3, and $53.6 for A, B and C, respectively. When using the payback period, these projects are equal to one another (i.e., they each have a payback period of three years). However, when considering all cash flows, B has a higher NPV than A because of the timing of cash flows within the payback period. And C has the highest NPV because of the $100 cash flow after the payback period.

PROBLEM 1: TIMING OF CASH FLOWS WITHIN THE PAYBACK PERIOD Let us compare Project *A* with Project *B*. In Years 1 through 3, the cash flows of Project *A* rise from $20 to $50, while the cash flows of Project *B* fall from $50 to $20. Because the large cash flow of $50 comes earlier with Project *B*, its net present value must be higher. Nevertheless, we saw above that the payback periods of the two projects are identical. Thus, a problem with the payback method is that it does not consider the timing of the cash flows within the payback period. This example shows that the payback method is inferior to NPV because, as we pointed out earlier, the NPV method *discounts the cash flows properly.*

PROBLEM 2: PAYMENTS AFTER THE PAYBACK PERIOD Now consider Projects *B* and *C*, which have identical cash flows within the payback period. However, Project *C* is clearly preferred because it has a cash flow of $60,000 in the fourth year. Thus, another problem with the payback method is that it ignores all cash flows occurring after the payback period. Because of the short-term orientation of the payback method, some valuable long-term projects are likely to be rejected. The NPV method does not have this flaw since, as we pointed out earlier, this method *uses all the cash flows of the project.*

PROBLEM 3: ARBITRARY STANDARD FOR PAYBACK PERIOD We do not need to refer to Table 7.1 when considering a third problem with the payback method. Capital markets help us estimate the discount rate used in the NPV method. The riskless rate, perhaps proxied by the yield on a Treasury instrument, would be the appropriate rate for a riskless investment. Later chapters of this textbook show how to use historical returns in the capital markets in order to estimate the discount rate for a risky project. However, there is no comparable guide for choosing the payback cutoff date, so the choice is somewhat arbitrary.

Managerial Perspective

The payback method is often used by large, sophisticated companies when making relatively small decisions. The decision to build a small warehouse, for example, or to pay for a tune-up for a truck is the sort of decision that is often made by lower-level management. Typically, a manager might reason that a tune-up would cost, say, $200, and if it saved $120 each year in reduced fuel costs, it would pay for itself in less than two years. On such a basis the decision would be made.

Although the treasurer of the company might not have made the decision in the same way, the company endorses such decision making. Why would upper management condone or even encourage such retrograde activity in its employees? One answer would be that it is easy to make decisions using payback. Multiply the tune-up decision into 50 such decisions a month, and the appeal of this simple method becomes clearer.

The payback method also has some desirable features for managerial control. Just as important as the investment decision itself is the company's ability to evaluate the manager's decision-making ability. Under the NPV method, a long time may pass before one decides whether or not a decision was correct. With the payback method we may know in just a few years whether the manager's assessment of the cash flows was correct.

It has also been suggested that firms with good investment opportunities but no available cash may justifiably use payback. For example, the payback method could be used by small, privately held firms with good growth prospects but limited access to the capital markets. Quick cash recovery enhances the reinvestment possibilities for such firms.

Finally, practitioners often argue that standard academic criticisms of payback overstate any real-world problems with the method. For example, textbooks typically make fun of payback by positing a project with low cash inflows in the early years but a huge cash inflow right after the payback cutoff date. This project is likely to be rejected under the payback method, though its acceptance would, in truth, benefit the firm. Project *C* in our Table 7.1 is an example of such a project. Practitioners point out that the pattern of cash flows in these textbook examples is much too stylized to mirror the real world. In fact, a number of executives have told us that, for the overwhelming majority of real-world projects, both payback and NPV lead to the same decision. In addition, these executives indicate that, if an investment like Project *C* were encountered in the real world, decision makers would almost certainly make *ad hoc* adjustments to the payback rule so that the project would be accepted.

Notwithstanding all of the preceding rationale, it is not surprising to discover that as the decisions grow in importance, which is to say when firms look at bigger projects, NPV becomes the order of the day. When questions of controlling and evaluating the manager become less important than making the right investment decision, payback is used less frequently. For big-ticket decisions, such as whether or not to buy a machine, build a factory, or acquire a company, the payback method is seldom used.

Summary of Payback

The payback method differs from NPV and is therefore conceptually wrong. With its arbitrary cutoff date and its blindness to cash flows after that date, it can lead to some flagrantly foolish decisions if it is used too literally. Nevertheless, because of its simplicity, as well as its other advantages mentioned above, companies often use it as a screen for making the myriad minor investment decisions they continually face.

Although this means that you should be wary of trying to change approaches such as the payback method when you encounter them in companies, you should probably be careful not to accept the sloppy financial thinking they represent. After this course, you would do your company a disservice if you used payback instead of NPV when you had a choice.

7.3 THE DISCOUNTED PAYBACK PERIOD METHOD

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Aware of the pitfalls of payback, some decision makers use a variant called the **discounted payback period method**. Under this approach, we first discount the cash flows. Then we ask how long it takes for the discounted cash flows to equal the initial investment.

For example, suppose that the discount rate is 10 percent and the cash flows for a project are given by:

(−\$100, \$50, \$50, \$20)

This investment has a payback period of two years, because the investment is paid back in that time.

To compute the project's discounted payback period, we first discount each of the cash flows at the 10 percent rate. These discounted cash flows are:

$$[-\$100, \$50/1.1, \$50/(1.1)^2, \$20/(1.1)^3] = (-\$100, \$45.45, \$41.32, \$15.03)$$

The discounted payback period of the original investment is simply the payback period for these discounted cash flows. The payback period for the discounted cash flows is slightly less than three years since the discounted cash flows over the three years are

\$101.80 (= \$45.45 + 41.32 + 15.03). As long as the cash flows are positive, the discounted payback period will never be smaller than the payback period, because discounting reduces the value of the cash flows.

At first glance, discounted payback may seem like an attractive alternative, but on closer inspection we see that it has some of the same major flaws as payback. Like payback, discounted payback first requires us to make a somewhat magical choice of an arbitrary cutoff period, and then it ignores all of the cash flows after that date.

If we have already gone to the trouble of discounting the cash flows, any small appeal to simplicity or to managerial control that payback may have has been lost. We might just as well add up all the discounted cash flows and use NPV to make the decision. Although discounted payback looks a bit like NPV, it is just a poor compromise between the payback method and NPV.

7.4 THE AVERAGE ACCOUNTING RETURN METHOD

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Defining the Rule

Another attractive, but fatally flawed, approach to capital budgeting is the **average accounting return**. The average accounting return is the average project earnings after taxes and depreciation, divided by the average book value of the investment during its life. In spite of its flaws, the average accounting return method is worth examining because it is used frequently in the real world.

EXAMPLE 7.2

Average Accounting Return

Consider a company that is evaluating whether to buy a store in a new mall. The purchase price is \$500,000. We will assume that the store has an estimated life of five years and will need to be completely scrapped or rebuilt at the end of that time. The projected yearly sales and expense figures are shown in Table 7.2.

TABLE 7.2 Projected Yearly Revenue and Costs for Average Accounting Return

	YEAR 1	YEAR 2	YEAR 3	YEAR 4	YEAR 5
Revenue	\$433,333	\$450,000	\$266,667	\$200,000	\$133,333
Expenses	200,000	150,000	100,000	100,000	100,000
Before-tax cash flow	233,333	300,000	166,667	100,000	33,333
Depreciation	100,000	100,000	100,000	100,000	100,000

Earnings before taxes	133,333	200,000	66,667	0	- 66,667
Taxes (t_c = .25)*	33,333	50,000	16,667	0	- 16,667
Net income	$100,000	$150,000	$ 50,000	$ 0	-$ 50,000

$$\text{Average net income} = \frac{(\$100{,}000 + 150{,}000 + 50{,}000 + 0 - 50{,}000)}{5} = \$50{,}000$$

$$\text{Average investment} = \frac{(\$500{,}000 + 400{,}000 + 300{,}000 + 200{,}000 + 100{,}000 + 0)}{6} = \$250{,}000$$

$$\text{AAR} = \frac{\$50{,}000}{\$250{,}000} = 20\%$$

* Corporate tax rate = t_c. The tax rebate in Year 5 of −$16,667 occurs if the rest of the firm is profitable. Here, the loss in the project reduces the taxes of the entire firm.

It is worth examining Table 7.2 carefully. In fact, the first step in any project assessment is a careful look at projected cash flows. First-year sales for the store are estimated to be $433,333. Before-tax cash flow will be $233,333. Sales are expected to rise and expenses are expected to fall in the second year, resulting in a before-tax cash flow of $300,000.

Competition from other stores and the loss in novelty will reduce before-tax cash flow to $166,667, $100,000, and $33,333, respectively, in the next three years.

To compute the average accounting return (AAR) on the project, we divide the average net income by the average amount invested. This can be done in three steps.

STEP 1: DETERMINING AVERAGE NET INCOME Net income in any year is net cash flow minus depreciation and taxes. Depreciation is *not* a cash outflow.[1] Rather, it is a charge reflecting the fact that the investment in the store becomes less valuable every year.

We assume the project has a useful life of five years, at which time it will be worthless. Because the initial investment is $500,000 and because it will be worthless in five years, we assume that it loses value at the rate of $100,000 each year. This steady loss in value of $100,000 is called *straight-line depreciation*. We subtract both depreciation and taxes from before-tax cash flow to derive net income, as shown in Table 7.2. Net income is $100,000 in the first year, $150,000 in Year 2, $50,000 in Year 3, $0 in Year 4, and −$50,000 in the last year. The average net income over the life of the project is therefore:

Average Net Income:

$$[\$100{,}000 + 150{,}000 + 50{,}000 + 0 + (-50{,}000)]/5 = \$50{,}000$$

STEP 2: DETERMINING AVERAGE INVESTMENT We stated earlier that, due to depreciation, the investment in the store becomes less valuable every year. Because depreciation is $100,000 per year, the value at the end of Year 0 is $500,000, the value at the end of Year 1 is $400,000, and so on. What is the average value of the investment over the life of the investment?

The mechanical calculation is:

Average Investment:

$$(\$500{,}000 + 400{,}000 + 300{,}000 + 200{,}000 + 100{,}000 + 0)/6 = \$250{,}000$$

We divide by 6 and not 5, because $500,000 is what the investment is worth at the beginning of the five years and $0 is what it is worth at the beginning of the sixth year. In other words, there are six terms in the parentheses of the average investment equation.

STEP 3: DETERMINING AAR The average accounting return is simply:

$$\text{AAR} = \frac{\$50{,}000}{\$250{,}000} = 20\%$$

If the firm had a targeted accounting rate of return greater than 20 percent, the project would be rejected, and if its targeted return were less than 20 percent, it would be accepted.

Analyzing the Average Accounting Return Method

By now you should be able to see what is wrong with the AAR method.

The most important flaw with AAR is that it does not work with the right raw materials. It uses the net income and book value of the investment, both of which come from the accounting books. Accounting numbers are somewhat arbitrary. For example, certain cash outflows, such as the cost of a building, are depreciated under current accounting rules. Other flows, such as maintenance, are expensed. In real-world situations, the decision to depreciate or expense an item involves judgment. Thus, the basic inputs of the AAR method, income and average investment, are affected by the accountant's judgment. Conversely, the NPV method *uses cash flows*. Accounting judgments do not affect cash flow.

Second, AAR takes no account of timing. In the previous example, the AAR would have been the same if the $100,000 net income in the first year had occurred in the last year. However, delaying an inflow for five years would have lowered the NPV of the investment. As mentioned earlier in this chapter, the NPV approach *discounts properly*.

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Third, just as payback requires an arbitrary choice of the cutoff date, the AAR method offers no guidance on what the right targeted rate of return should be. It could be the discount rate in the market. But then again, because the AAR method is not the same as the present value method, it is not obvious that this would be the right choice.

Given these problems, is the AAR method employed in practice? Like the payback method, the AAR method (and variations of it) is frequently used as a "backup" to discounted cash flow methods. Perhaps this is so because it is easy to calculate and uses accounting numbers readily available from the firm's accounting system. In addition, both stockholders and the media pay a lot of attention to the overall profitability of a firm. Thus, some managers may feel pressured to select projects that are profitable in the near term, even if the projects come up short in terms of NPV. These managers may focus on the AAR of individual projects more than they should.

7.5 THE INTERNAL RATE OF RETURN

Now we come to the most important alternative to the NPV method, the internal rate of return, universally known as the IRR. The IRR is about as close as you can get to the NPV without actually being the NPV. The basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. That number does not depend on the various interest rates prevailing in the capital markets. That is why it is called the internal rate of return; the number is internal or intrinsic to the project and does not depend on anything except the cash flows of the project.

For example, consider the simple project (−\$100, \$110) in Figure 7.2. For a given rate, the net present value of this project can be described as:

$$\text{NPV} = -\$100 + \frac{\$110}{1 + R}$$

where R is the discount rate. What must the discount rate be to make the NPV of the project equal to zero?

We begin by using an arbitrary discount rate of 8 percent, which yields:

$$\$1.85 = -\$100 + \frac{\$110}{1.08}$$

Since the NPV in this equation is positive, we now try a higher discount rate, say, 12 percent. This yields:

$$-\$1.79 = -\$100 + \frac{\$110}{1.12}$$

Since the NPV in the equation above is negative, we lower the discount rate to, say, 10 percent. This yields:

$$0 = -\$100 + \frac{\$110}{1.10}$$

FIGURE 7.2
Cash Flows for a Simple Project

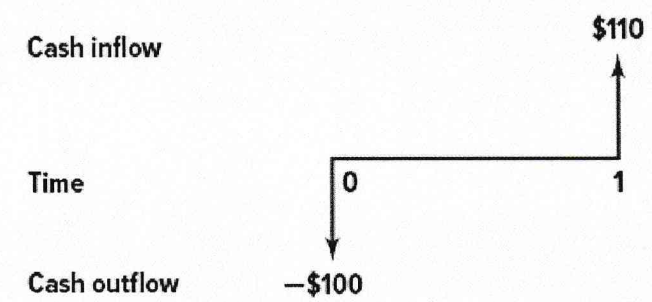
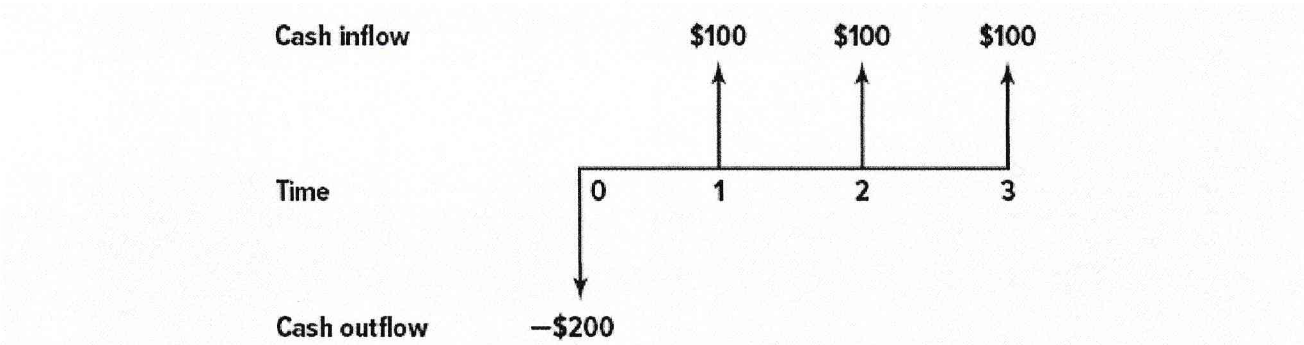


FIGURE 7.3
Cash Flows for a More Complex Project



This trial-and-error procedure tells us that the NPV of the project is zero when R equals 10 percent.[2] Thus, we say that 10 percent is the project's **internal rate of return (IRR)**. In general, the IRR is the rate that causes the NPV of the project to be zero. The implication of this exercise is very simple. The firm should be equally willing to accept or reject the project if the discount rate is 10 percent. The firm should accept the project if the discount rate is below 10 percent. The firm should reject the project if the discount rate is above 10 percent.

The general investment rule is clear:

Accept the project if IRR is greater than the discount rate. Reject the project if IRR is less than the discount rate.

We refer to this as the **basic IRR rule**. Now we can try the more complicated example (−$200, $100, $100, $100) in Figure 7.3.

As we did previously, let's use trial and error to calculate the internal rate of return. We try 20 percent and 30 percent, yielding:

DISCOUNT RATE	NPV
20%	$10.65
30	−18.39

After much more trial and error, we find that the NPV of the project is zero when the discount rate is 23.38 percent. Thus, the IRR is 23.38 percent. With a 20 percent discount rate, the NPV is positive and we would accept it. However, if the discount rate were 30 percent, we would reject it.

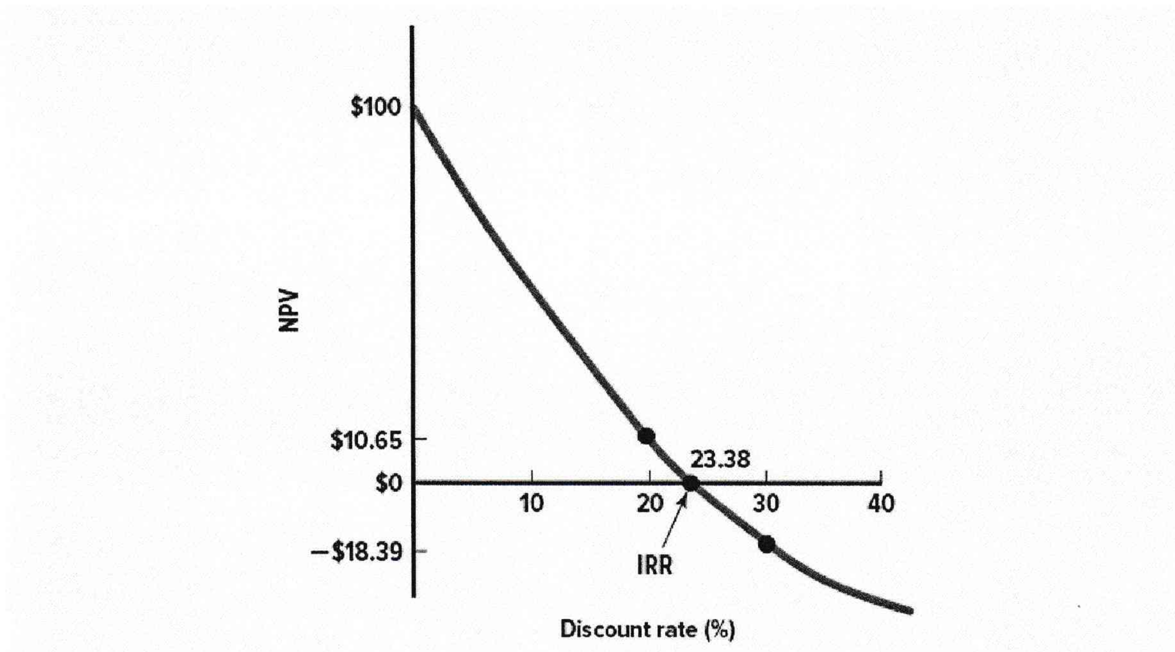
Algebraically, IRR is the unknown in the following equation:[3]

$$0 = -\$200 + \frac{\$100}{1 + \text{IRR}} + \frac{\$100}{(1 + \text{IRR})^2} + \frac{\$100}{(1 + \text{IRR})^3}$$

Figure 7.4 illustrates what the IRR of a project means. The figure plots the NPV as a function of the discount rate. The curve crosses the horizontal axis at the IRR of 23.38 percent because this is where the NPV equals zero.

It should also be clear that the NPV is positive for discount rates below the IRR and negative for discount rates above the IRR. This means that if we accept projects like this one when the discount rate is less than the IRR, we will be accepting positive NPV projects. Thus, the IRR rule coincides exactly with the NPV rule.

FIGURE 7.4
Net Present Value (NPV) and Discount Rates for a More Complex Project



If this were all there were to it, the IRR rule would always coincide with the NPV rule. This would be a wonderful discovery because it would mean that just by computing the IRR for a project we would be able to tell where it ranks among all of the projects we are considering. For example, if the IRR rule really works, a project with an IRR of 20 percent will always be at least as good as one with an IRR of 15 percent.

But the world of finance is not so kind. Unfortunately, the IRR rule and the NPV rule are the same only for examples like the ones above. Several problems with the IRR approach occur in more complicated situations. In the real world, spreadsheets are used to avoid boring trial-and-error calculations. A nearby *Spreadsheet Techniques* box shows how.

Calculating IRRs with a Spreadsheet

SPREADSHEET TECHNIQUES

Because IRRs are so tedious to calculate by hand, financial calculators and, especially, spreadsheets are generally used. The procedures used by various financial calculators are too different for us to illustrate here, so we will focus on using a spreadsheet. As the following example illustrates, using a spreadsheet is very easy.

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate internal rates of return**							
3								
4	Suppose we have a four-year project that costs $500. The cash flows over the four-year life will be							
5	$100, $200, $300, and $400. What is the IRR?							
6								
7		Year	Cash flow					
8		0	-$500					
9		1	100		**IRR =**	27.3%		
10		2	200					
11		3	300					
12		4	400					
13								
14								
15	The formula entered in cell F9 is =IRR(C8:C12). Notice that the Year 0 cash flow has a negative							
16	sign representing the initial cost of the project.							
17								

7.6 PROBLEMS WITH THE IRR APPROACH

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Definition of Independent and Mutually Exclusive Projects

An **independent project** is one whose acceptance or rejection is independent of the acceptance or rejection of other projects. For example, imagine that McDonald's is considering putting a hamburger outlet on a remote island. Acceptance or rejection of this unit is likely to be unrelated to the acceptance or rejection of any other restaurant in its system. The remoteness of the outlet in question ensures that it will not pull sales away from other outlets.

Now consider the other extreme, **mutually exclusive investments**. What does it mean for two projects, *A* and B, to be mutually exclusive? You can accept *A* or you can accept *B* or you can reject both of them, but you cannot accept both of them. For example, *A* might be a decision to build an apartment house on a corner lot that you own, and *B* might be a decision to build a movie theater on the same lot.

We now present two general problems with the IRR approach that affect both independent and mutually exclusive projects. Then we deal with two problems affecting mutually exclusive projects only.

Two General Problems Affecting Both Independent and Mutually Exclusive Projects

We begin our discussion with Project *A*, which has the following cash flows:

(−\$100, \$130)

The IRR for Project *A* is 30 percent. Table 7.3 provides other relevant information on the project. The relationship between NPV and the discount rate is shown for this project in Figure 7.5. As you can see, the NPV declines as the discount rate rises.

PROBLEM 1: INVESTING OR FINANCING Now consider Project *B*, with cash flows of

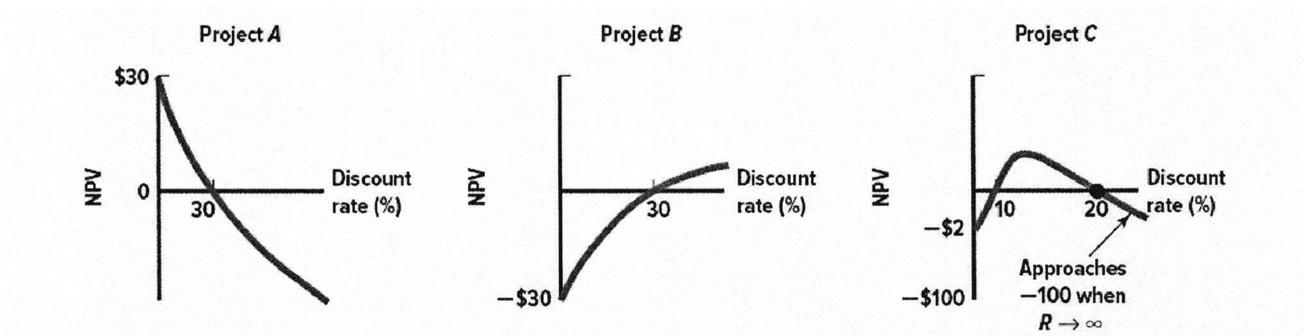
\$100, −\$130

These cash flows are exactly the reverse of the flows for Project *A*. In Project B, the firm receives funds first and then pays out funds later. While unusual, projects of this type do exist. For example, consider a corporation conducting a seminar where the participants pay in advance. Because large expenses are frequently incurred at the seminar date, cash inflows precede cash outflows.

TABLE 7.3 The Internal Rate of Return and Net Present Value

	Project *A*			**Project *B***			**Project *C***		
DATES:	0	1	2	0	1	2	0	1	2
Cash flows	−\$100	\$130		\$100	−\$130		−\$100	\$230	−\$132
IRR		30%			30%		10%	and	20%
NPV @10%		\$18.2			−\$18.2			\$0	
Accept if market rate		<30%			>30%		>10%	but	<20%
Financing or investing		Investing			Financing			Mixture	

FIGURE 7.5
Net Present Value and Discount Rates for Projects *A*, *B* and *C*



Consider our trial-and-error method to calculate IRR:

$$-\$4 = +\$100 - \frac{\$130}{1.25}$$

$$\$0 = +\$100 - \frac{\$130}{1.30}$$

$$\$3.70 = +\$100 - \frac{\$130}{1.35}$$

As with Project A, the internal rate of return is 30 percent. However, notice that the net present value is *negative* when the discount rate is *below* 30 percent. Conversely, the net present value is positive when the discount rate is above 30 percent. The decision rule is exactly the opposite of our previous result. For this type of a project, the rule is:

Accept the project when IRR is less than the discount rate. Reject the project when IRR is greater than the discount rate.

This unusual decision rule follows from the graph of Project *B* in Figure 7.5. The curve is upward sloping, implying that NPV is *positively* related to the discount rate.

The graph makes intuitive sense. Suppose that the firm wants to obtain $100 immediately. It can either (1) accept Project *B* or (2) borrow $100 from a bank. Thus, the project is actually a substitute for borrowing. In fact, because the IRR is 30 percent, taking on Project *B* is tantamount to borrowing at 30 percent. If the firm can borrow from a bank at, say, only 25 percent, it should reject the project. However, if a firm can only borrow from a bank at, say, 35 percent, it should accept the project. Thus, Project *B* will be accepted if and only if the discount rate is *above* the IRR.[4]

This should be contrasted with Project *A*. If the firm has $100 of cash to invest, it can either (1) accept Project *A* or (2) lend $100 to the bank. The project is actually a substitute for lending. In fact, because the IRR is 30 percent, taking on Project *A* is tantamount to lending at 30 percent. The firm should accept Project *A* if the lending rate is below 30 percent. Conversely, the firm should reject Project *A* if the lending rate is above 30 percent.

Because the firm initially pays out money with Project *A* but initially receives money with Project *B*, we refer to Project *A* as an *investing type project* and Project *B* as a *financing type project*. Investing type projects are the norm. Since the IRR rule is reversed for financing type projects, be careful when using it with this type of project.

PROBLEM 2: MULTIPLE RATES OF RETURN Suppose the cash flows from a project are:

(−$100, $230, −$132)

Because this project has a negative cash flow, a positive cash flow, and another negative cash flow, we say that the project's cash flows exhibit two changes of signs, or "flip-flops." While this pattern of cash flows might look a bit strange at first, many projects require outflows of cash after receiving some inflows. An example would be a strip-mining project. The first stage in such a project is the initial investment in excavating the mine. Profits from operating the mine are received in the second stage. The third stage involves a further investment to reclaim the land and satisfy the requirements of environmental protection legislation. Cash flows are negative at this stage.

Projects financed by lease arrangements may produce a similar pattern of cash flows. Leases often provide substantial tax subsidies, generating cash inflows after an initial investment. However, these subsidies decline over time, frequently leading to negative cash flows in later years. (The details of leasing will be discussed in a later chapter.)

It is easy to verify that this project has not one but two IRRs, 10 percent and 20 percent.[5] In a case like this, the IRR does not make any sense. What IRR are we to use, 10 percent or 20 percent? Because there is no good reason to use one over the other, IRR simply cannot be used here.

Why does this project have multiple rates of return? Project *C* generates multiple internal rates of return because both an inflow and an outflow occur after the initial investment. In general, these flip-flops or changes in sign produce multiple IRRs. In theory, a cash flow stream with *K* changes in sign can have up to *K* sensible internal rates of return (IRRs above −100 percent). Therefore, since Project *C* has two changes in sign, it can have as many as two IRRs. As we pointed out, projects whose cash flows change sign repeatedly can occur in the real world.

NPV RULE Of course, we should not be too worried about multiple rates of return. After all, we can always fall back on the NPV rule. Figure 7.5 plots the NPV of Project *C* (−$100, $230, −$132) as a function of the discount rate. As the figure shows, the NPV is zero at both 10 percent and 20 percent and negative outside the range. Thus, the NPV rule tells us to accept the project if the appropriate discount rate is between 10 percent and 20 percent. The project should be rejected if the discount rate lies outside of this range.

MODIFIED IRR As an alternative to NPV, we now introduce the **modified IRR (MIRR)** method, which handles the multiple IRR problem by combining cash flows until only one change in sign remains. To see how it works, consider Project *C* again. With a discount rate of, say, 14 percent, the value of the last cash flow, −$132, is:

$$-\$132/1.14 = -\$115.79$$

as of Date 1. Since $230 is already received at that time, the "adjusted" cash flow at Date 1 is $114.21 (= $230 − 115.79). Thus, the MIRR approach produces the following two cash flows for the project:

$$(-\$100, \$114.21)$$

Note that, by discounting and then combining cash flows, we are left with only one change in sign. The IRR rule can now be applied. The IRR of these two cash flows is 14.21 percent, implying that the project should be accepted given our assumed discount rate of 14 percent.

Of course, Project *C* is relatively simple to begin with, since it has only three cash flows and two changes in sign. However, the same procedure can easily be applied to more complex projects; that is, just keep discounting and combining the later cash flows until only one change of sign remains.

While this adjustment does correct for multiple IRRs, it appears, at least to us, to violate the "spirit" of the IRR approach. As stated earlier, the basic rationale behind the IRR method is that it provides a single number summarizing the merits of a project. That number does not depend on the discount rate. In fact, that is why it is called the internal rate of return; the number is *internal*, or intrinsic, to the project and does not depend on anything except the cash flows of the project. By contrast, MIRR is clearly a function of the discount rate. However, a firm using this adjustment will avoid the multiple IRR problem, just as a firm using the NPV rule will avoid it.

THE GUARANTEE AGAINST MULTIPLE IRRs If the first cash flow of a project is negative—because it is the initial investment—and if all of the remaining flows are positive, there can be only a single, unique IRR, no matter how many periods the project lasts. This is easy to understand by using the concept of the time value of money. For example, it is simple to verify that Project *A* in Table 7.3 has an IRR of 30 percent, because using a 30 percent discount rate gives:

$$\begin{aligned} \text{NPV} &= -\$100 + \$130/(1.3) \\ &= 0 \end{aligned}$$

How do we know that this is the only IRR? Suppose we were to try a discount rate greater than 30 percent. In computing the NPV, changing the discount rate does not change the value of the initial cash flow of −$100 because that cash flow is not discounted. But raising the discount rate can lower the present value of the future cash flows. In other words, because the NPV is zero at 30 percent, any increase in the rate will push the NPV into the negative range. Similarly, if we try a discount rate of less than 30 percent, the overall NPV of the project will be positive. Though this example has only one positive flow, the above reasoning still implies a single, unique IRR if there are many inflows (but no outflows) after the initial investment.

If the initial cash flow is positive—and if all of the remaining flows are negative— there can only be a single, unique IRR. This result follows from reasoning similar to that above. Both these cases have only one change of sign or flip-flop in the cash flows. Thus, we are safe from multiple IRRs whenever there is only one sign change in the cash flows.

GENERAL RULES The following chart summarizes our rules:

FLOWS	NUMBER OF IRRs	IRR CRITERION	NPV CRITERION
First cash flow is negative and all remaining cash flows are positive.	1	Accept if IRR > *R* Reject if IRR < *R*	Accept if NPV > 0 Reject if NPV < 0
First cash flow is positive and all remaining cash flows are negative.	1	Accept if IRR < *R* Reject if IRR > *R*	Accept if NPV > 0 Reject if NPV < 0
Some cash flows after first are positive and some cash flows after first are negative.	May be more than 1	No valid IRR	Accept if NPV > 0 Reject if NPV < 0

Note that the NPV criterion is the same for each of the three cases. In other words, NPV analysis is always appropriate. Conversely, the IRR can be used only in certain cases. When it comes to NPV, the preacher's words, "You just can't lose with the stuff I use," clearly apply.

Problems Specific to Mutually Exclusive Projects

As mentioned earlier, two or more projects are mutually exclusive if the firm can, at most, accept only one of them. We now present two problems dealing with the application of the IRR approach to mutually exclusive projects. These two problems are quite similar, though logically distinct.

THE SCALE PROBLEM A professor we know motivates class discussions on this topic with the statement: "Students, I am prepared to let one of you choose between two mutually exclusive 'business' propositions. Opportunity 1—You give me $1 now and I'll give you $1.50 back at the end of the class period. Opportunity 2—You give me $10 and I'll give you $11 back at the end of the class period. You can only choose one of the two opportunities. And you cannot choose either opportunity more than once. I'll pick the first volunteer."

Which would you choose? The correct answer is Opportunity 2.[6] To see this, look at the following chart:

	CASH FLOW AT BEGINNING OF CLASS	CASH FLOW AT END OF CLASS (90 MINUTES LATER)	NPV[7]	IRR
Opportunity 1	−$ 1	+$ 1.50	$ .50	50%
Opportunity 2	−10	+ 11.00	1.00	10

As we have stressed earlier in the text, one should choose the opportunity with the highest NPV. This is Opportunity 2 in the example. Or, as one of the professor's students explained it: "I'm bigger than the professor, so I know I'll get my money back. And I have $10 in my pocket right now so I can choose either opportunity. At the end of the class, I'll be able to play two rounds of my favorite video game with Opportunity 2 and still have my original investment, safe and sound.[8] The profit on Opportunity 1 buys only one round."

This business proposition illustrates a defect with the internal rate of return criterion. The basic IRR rule indicates the selection of Opportunity 1, because the IRR is 50 percent. The IRR is only 10 percent for Opportunity 2.

Where does IRR go wrong? The problem with IRR is that it ignores issues of *scale*. While Opportunity 1 has a greater IRR, the investment is much smaller. In other words, the high percentage return on Opportunity 1 is more than offset by the ability to earn at least a decent return[9] on a much bigger investment under Opportunity 2. page 211

Since IRR seems to be misguided here, can we adjust or correct it? We illustrate how in the next example.

EXAMPLE 7.3

NPV versus IRR

Stanley Jaffe and Sherry Lansing have just purchased the rights to *Corporate Finance: The Motion Picture*. They will produce this major motion picture on either a small budget or a big budget. The estimated cash flows are

	CASH FLOW AT DATE 0	CASH FLOW AT DATE 1	NPV @25%	IRR
Small budget	−$10 million	$40 million	$22 million	300%
Large budget	− 25 million	65 million	27 million	160

Because of high risk, a 25 percent discount rate is considered appropriate. Sherry wants to adopt the large budget because the NPV is higher. Stanley wants to adopt the small budget because the IRR is higher. Who is right?

For the reasons espoused in the classroom example above, NPV is correct. Hence, Sherry is right. However, Stanley is very stubborn where IRR is concerned. How can Sherry justify the large budget to Stanley using the IRR approach?

This is where *incremental IRR* comes in. Sherry calculates the incremental cash flows from choosing the large budget instead of the small budget as

	CASH FLOW AT DATE 0 (IN $ MILLIONS)	CASH FLOW AT DATE 1 (IN $ MILLIONS)
Incremental cash flows from choosing large budget instead of small budget	−$25 − (−10) = −$15	$65 − 40 = $25

This chart shows that the incremental cash flows are −$15 million at Date 0 and $25 million at Date 1. Sherry calculates incremental IRR as:

Formula for Calculating the Incremental IRR:

$$0 = -\$15 \text{ million} + \frac{\$25 \text{ million}}{1 + \text{IRR}}$$

IRR equals 66.67 percent in this equation, implying that the **incremental IRR** is 66.67 percent. Incremental IRR is the IRR on the incremental investment from choosing the large project instead of the small project.

In addition, we can calculate the NPV of the incremental cash flows:

NPV of Incremental Cash Flows:

$$-\$15 \text{ million} + \frac{\$25 \text{ million}}{1.25} = \$5 \text{ million}$$

We know the small-budget picture would be acceptable as an independent project since its NPV is positive. We want to know whether it is beneficial to invest an additional $15 million in order to make the large-budget picture instead of the small-budget picture. In other words, is it beneficial to invest an additional $15 million in order to receive an additional $25 million next year?

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First, the above calculations show the NPV on the incremental investment to be positive. Second, the incremental IRR of 66.67 percent is higher than the discount rate of 25 percent. For both reasons, the incremental investment can be justified. Hence, the large-budget movie should be made. The second reason is what Stanley needed to hear to be convinced.

In review, we can handle this example (or any mutually exclusive example) in one of three ways:

1. *Compare the NPVs of the two choices.* The NPV of the large-budget picture is greater than the NPV of the small-budget picture. That is, $27 million is greater than $22 million.
2. *Calculate the incremental NPV from making the large-budget picture instead of the small-budget picture.* Because the incremental NPV equals $5 million, we choose the large-budget picture.
3. *Compare the incremental IRR to the discount rate.* Because the incremental IRR is 66.67 percent and the discount rate is 25 percent, we take the large-budget picture.

All three approaches always give the same decision. However, we must *not* compare the IRRs of the two pictures. If we did, we would make the wrong choice. That is, we would accept the small-budget picture.

While students frequently think that problems of scale are relatively unimportant, the truth is just the opposite. A well-known chef on TV often says, “I don’t know about your flour, but the flour I buy don’t come seasoned.” The same thing applies to capital budgeting. No real-world project comes in one clear-cut size. Many times, the firm has to *determine* the best size for the project. The movie budget of $25 million is not fixed in stone. Perhaps an extra $1 million to hire a bigger star or to film at a better location will increase the movie’s gross. Similarly, an industrial firm must decide whether it wants a warehouse of, say, 500,000 square feet or 600,000 square feet. And, earlier in the chapter, we imagined McDonald’s opening an outlet on a remote island. If it does this, it must decide how big the outlet should be. For almost any project, someone in the firm has to decide on its size, implying that problems of scale abound in the real world.

One final note here. Students often ask which project should be subtracted from the other in calculating incremental flows. Notice that we are subtracting the smaller project’s cash flows from the bigger project’s cash flows. This leaves an *outflow* at Date 0. We then use the basic IRR rule on the incremental flows.[10]

THE TIMING PROBLEM Next we illustrate another, but quite similar, problem with the IRR approach when evaluating mutually exclusive projects.

EXAMPLE 7.4

Mutually Exclusive Investments

Suppose that the Kaufold Corporation has two alternative uses for a warehouse. It can store toxic waste containers (Investment *A*) or electronic equipment (Investment *B*). The cash flows are as follows:

	Cash Flow at Year				NPV			
YEAR:	0	1	2	3	@0%	@10%	@15%	**IRR**
Investment *A*	−$10,000	$10,000	$1,000	$ 1,000	$2,000	$669	$109	16.04%
Investment *B*	− 10,000	1,000	1,000	12,000	4,000	751	− 484	12.94

high percentage return on Opportunity 1 is more than offset by the ability to earn at least a decent return[9] on page 211 a much bigger investment under Opportunity 2.

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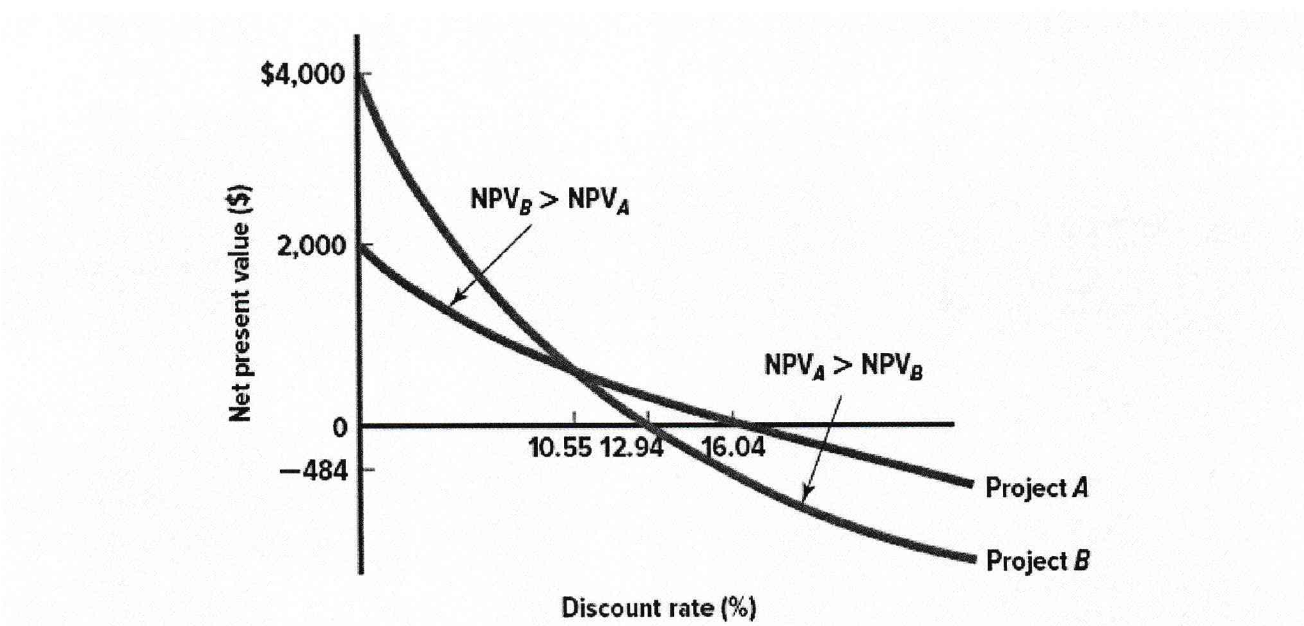
Investment *B*	− 10,000	1,000	1,000	12,000	4,000	751	− 484	12.94

We find that the NPV of Investment *B* is higher with low discount rates, and the NPV of Investment *A* is higher with high discount rates. This is not surprising if you look closely at the cash flow patterns. The cash flows of *A* occur early, whereas the cash flows of *B* occur later. If we assume a high discount rate, we favor Investment *A* because we are implicitly assuming that the early cash flow (for example, $10,000 in Year 1) can be reinvested at that rate. Because most of Investment *B*'s cash flows occur in Year 3, *B*'s value is relatively high with low discount rates.

The patterns of cash flow for both projects appear in Figure 7.6. Project *A* has an NPV of $2,000 at a discount rate of zero. This is calculated by simply adding up the cash flows without discounting them. Project *B* has an NPV of $4,000 at the zero rate. However, the NPV of Project *B* declines more rapidly as the discount rate increases than does the NPV of Project *A*. As we mentioned above, this occurs because the cash flows of *B* occur later. Both projects have the same NPV at a discount rate of 10.55 percent. The IRR for a project is the rate at which the NPV equals zero. Because the NPV of *B* declines more rapidly, *B* actually has a lower IRR.

FIGURE 7.6
Net Present Value and the Internal Rate of Return for Mutually Exclusive Projects



As with the movie example presented earlier, we can select the better project with one of three different methods:

1. *Compare the NPVs of the Two Projects.* Figure 7.6 aids our decision. If the discount rate is below 10.55 percent, one should choose Project *B* because *B* has a higher NPV. If the rate is above 10.55 percent, one should choose Project *A* because *A* has a higher NPV.
2. *Compare the Incremental IRR to the Discount Rate.* The above method employed NPV. Another way of determining that *B* is a better project is to subtract the cash flows of *A* from the cash flows of *B* and then to calculate the IRR. This is the incremental IRR approach we spoke of earlier.

   The incremental cash flows are:

NPV OF INCREMENTAL CASH FLOWS

YEAR:	0	1	2	3	INCREMENTAL IRR	@0%	@10%	@15%
B–A	0	–$9,000	0	$11,000	10.55%	$2,000	$83	–$593

This chart shows that the incremental IRR is 10.55 percent. In other words, the NPV on the incremental investment is zero when the discount rate is

10.55 percent. Thus, if the relevant discount rate is below 10.55 percent, Project *B* is preferred to Project *A*. If the relevant discount rate is above 10.55 percent, Project *A* is preferred to Project *B*.[11]

3. *Calculate the NPV on the Incremental Cash Flows*. Finally, one could calculate the NPV on the incremental cash flows. The chart that appears with the previous method displays these NPVs. We find that the incremental NPV is positive when the discount rate is either 0 percent or 10 percent. The incremental NPV is negative if the discount rate is 15 percent. If the NPV is positive on the incremental cash flows, one should choose *B*. If the NPV is negative, one should choose *A*.

In summary, the same decision is reached whether one (*a*) compares the NPVs of the two projects, (*b*) compares the incremental IRR to the relevant discount rate, or (*c*) examines the NPV of the incremental cash flows. However, as mentioned earlier, one should *not* compare the IRR of Project *A* with the IRR of Project *B*.

We suggested earlier that one should subtract the cash flows of the smaller project from the cash flows of the bigger project. What do we do here since the two projects have the same initial investment? Our suggestion in this case is to perform the subtraction so that the *first* nonzero cash flow is negative. In the Kaufold Corp. example, we achieved this by subtracting *A* from *B*. In this way, we can still use the basic IRR rule for evaluating cash flows.

The preceding examples illustrate problems with the IRR approach in evaluating mutually exclusive projects. Both the professor-student example and the motion picture example illustrate the problem that arises when mutually exclusive projects have different initial investments. The Kaufold Corp. example illustrates the problem that arises when mutually exclusive projects have different cash flow timing. When working with mutually exclusive projects, it is not necessary to determine whether it is the scale problem or the timing problem that exists. Very likely both occur in many real-world situations. Instead, the practitioner should simply use either an incremental IRR or an NPV approach.

Redeeming Qualities of IRR

IRR probably survives because it fills a need that NPV does not. People seem to want a rule that summarizes the information about a project in a single rate of return. This single rate provides people with a simple way of discussing projects. For example, one manager in a firm might say to another, "Remodeling the north wing has a 20 percent IRR."

To their credit, however, companies that employ the IRR approach seem to understand its deficiencies. For example, companies frequently restrict managerial projections of cash flows to be negative at the beginning and strictly positive later. In these cases, the IRR approach and the NPV approach are very often compatible. Perhaps, then, the ability of the IRR approach to capture a complex investment project in a single number and the ease of communicating that number explain the survival of the IRR.

A Test

To test your knowledge, consider the following two statements:

1. You must know the discount rate to compute the NPV of a project, but you compute the IRR without referring to the discount rate.
2. Hence, the IRR rule is easier to apply than the NPV rule because you don't use the discount rate when applying IRR.

The first statement is true. The discount rate is needed to *compute* NPV. The IRR is *computed* by solving for the rate where the NPV is zero. No mention is made of the