6 Interest Rates and Bond Valuation

Generally, when you make an investment, you expect that you will get back more money in the future than you invested today. But in December 2017, this wasn't the case for many bond investors. The yield on a five-year German government bond was about negative .20 percent, and the yields on two-year and five-year Japanese government bonds were negative .14 percent and negative .09 percent, respectively. In fact, in 2016, the amount of debt worldwide that had a negative yield reached a record $13.4 trillion! And negative yields were not restricted to government bonds, as at one point the yield on a bond issued by chocolate maker Nestlé was negative as well.

So what happened? Central banks were in a race to the bottom, lowering interest rates in an attempt to improve their domestic economies.

This chapter takes what we have learned about the time value of money and shows how it can be used to value one of the most common of all financial assets, a bond. It then discusses bond features, bond types, and the operation of the bond market.

What we will see is that bond prices depend critically on interest rates, so we will go on to discuss some very fundamental issues regarding interest rates. Clearly, interest rates are important to everybody because they underlie what businesses of all types—small and large—must pay to borrow money.

LEARNING OBJECTIVES

After studying this chapter, you should be able to:

- **LO 1** Identify important bond features and types of bonds.
- **LO 2** Describe bond values and why they fluctuate.
- **LO 3** Discuss bond ratings and what they mean.
- **LO 4** Evaluate the impact of inflation on interest rates.
- **LO 5** Explain the term structure of interest rates and the determinants of bond yields.

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6.1 BONDS AND BOND VALUATION

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When a corporation (or government) wishes to borrow money from the public on a long-term basis, it usually does so by issuing, or selling, debt securities that are generically called bonds. In this section, we describe the various features of corporate bonds and some of the terminology associated with bonds. We then discuss the cash flows associated with a bond and how bonds can be valued using our discounted cash flow procedure.

Bond Features and Prices

As we mentioned in our previous chapter, a bond is normally an interest-only loan, meaning that the borrower will pay the interest every period, but none of the principal will be repaid until the end of the loan. Suppose the Beck Corporation wants to borrow $1,000 for 30 years. The interest rate on similar debt issued by similar corporations is 12 percent. Beck will thus pay $.12 \times \$1{,}000 = \$120$ in interest every year for 30 years. At the end of 30 years, Beck will repay the $1,000. As this example suggests, a bond is a fairly simple financing arrangement. There is, however, a rich jargon associated with bonds, so we will use this example to define some of the more important terms.

In our example, the $120 regular interest payments that Beck promises to make are called the bond's **coupons**. Because the coupon is constant and paid every year, the type of bond we are describing is sometimes called a *level coupon bond*. The amount that will be repaid at the end of the loan is called the bond's **face value** or **par value**. As in our example, this par value is usually $1,000 for corporate bonds, and a bond that sells for its par value is called a *par value bond*. Government bonds frequently have much larger face, or par, values. Finally, the annual coupon divided by the face value is called the **coupon rate** on the bond; in this case, because $\$120/\$1{,}000 = .12$, or 12 percent, the bond has a 12 percent coupon rate.

The number of years until the face value is paid is called the bond's time to **maturity**. A corporate bond will frequently have a maturity of 30 years when it is originally issued, but this varies. Once the bond has been issued, the number of years to maturity declines as time goes by.

coupon
The stated interest payment made on a bond.

face value
The principal amount of a bond that is repaid at the end of the term. Also *par value*.

par value
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coupon rate
The annual coupon divided by the face value of a bond.

Bond Values and Yields

As time passes, interest rates change in the marketplace. The cash flows from a bond, however, stay the same. As a result, the value of the bond will fluctuate. When interest rates rise, the present value of the bond's remaining cash flows declines, and the bond is worth less. When interest rates fall, the bond is worth more.

To determine the value of a bond at a particular point in time, we need to know the

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coupon rate
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maturity
Specified date on which the principal amount of a bond is paid.

yield to maturity (YTM)
The rate required in the market on a bond.

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To determine the value of a bond at a particular point in time, we need to know the number of periods remaining until maturity, the face value, the coupon, and the market interest rate for bonds with similar features. This interest rate required in the market on a bond is called the bond's **yield to maturity (YTM)**. This rate is sometimes called the bond's *yield* for short. Given all this information, we can calculate the present value of the cash flows as an estimate of the bond's current market value.

For example, suppose the Xanth (pronounced "zanth") Co. were to issue a bond with 10 years to maturity. The Xanth bond has an annual coupon of $80. Similar bonds have a yield to maturity of 8 percent. Based on our preceding discussion, the Xanth bond will pay $80 per year for the next 10 years in coupon interest. In 10 years, Xanth will pay $1,000 to the owner of the bond. The cash flows from the bond are shown in Figure 6.1. What would this bond sell for?

As illustrated in Figure 6.1, the Xanth bond's cash flows have an annuity component (the coupons) and a lump sum (the face value paid at maturity). We thus estimate the market value of the bond by calculating the present value of these two components separately

FIGURE 6.1 Cash flows for Xanth Co. bond

Cash flows											
Year	0	1	2	3	4	5	6	7	8	9	10
Coupon		$80	$80	$80	$80	$80	$80	$80	$80	$80	$ 80
Face Value											1,000
		$80	$80	$80	$80	$80	$80	$80	$80	$80	$1,080

As shown, the Xanth bond has an annual coupon of $80 and a face, or par, value of $1,000 paid at maturity in 10 years.

and adding the results together. First, at the going rate of 8 percent, the present value of the $1,000 paid in 10 years is:

$$\text{Present value} = \$1{,}000/1.08^{10} = \$1{,}000/2.1589 = \$463.19$$

Second, the bond offers $80 per year for 10 years; the present value of this annuity stream is:

$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.08^{10})/.08 \\ &= \$80 \times (1 - 1/2.1589)/.08 \\ &= \$80 \times 6.7101 \\ &= \$536.81\end{aligned}$$

We can now add the values for the two parts together to get the bond's value:

$$\text{Total bond value} = \$463.19 + 536.81 = \$1{,}000$$

This bond sells for exactly its face value. This is not a coincidence. The going interest rate in the market is 8 percent. Considered as an interest-only loan, what interest rate does this bond have? With an $80 coupon, this bond pays exactly 8 percent interest only when it sells for $1,000.

To illustrate what happens as interest rates change, suppose that a year has gone by. The Xanth bond now has 9 years to maturity. If the interest rate in the market has risen to 10 percent, what will the bond be worth? To find out, we repeat the present value calculations with 9 years instead of 10, and a 10 percent yield instead of an 8 percent yield. First, the present value of the $1,000 paid in 9 years at 10 percent is:

$$\text{Present value} = \$1{,}000/1.10^{9} = \$1{,}000/2.3579 = \$424.10$$

Second, the bond now offers $80 per year for nine years; the present value of this annuity stream at 10 percent is:

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$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.10^{9})/.10 \\ &= \$80 \times (1 - 1/2.3579)/.10 \\ &= \$80 \times 5.7590 \\ &= \$460.72\end{aligned}$$

We can now add the values for the two parts together to get the bond's value:

Total bond value = $\$424.10 + 460.72 = \$884.82$

Therefore, the bond should sell for about $885. In the vernacular, we say that this bond, with its 8 percent coupon, is priced to yield 10 percent at $885.

The Xanth Co. bond now sells for less than its $1,000 face value. Why? The market interest rate is 10 percent. Considered as an interest-only loan of $1,000, this bond only pays 8 percent, its coupon rate. Because this bond pays less than the going rate, investors are only

A good bond site to visit is www.bloomberg.com/markets/rates-bonds, which has loads of useful information.

willing to lend something less than the \$1,000 promised repayment. Because the bond sells for less than face value, it is said to be a *discount bond*.

The only way to get the interest rate up to 10 percent is to lower the price to less than \$1,000 so that the purchaser, in effect, has a built-in gain. For the Xanth bond, the price of \$885 is \$115 less than the face value, so an investor who purchased and kept the bond would get \$80 per year and would have a \$115 gain at maturity as well. This gain compensates the lender for the below-market coupon rate.

Another way to see why the bond is discounted by \$115 is to note that the \$80 coupon is \$20 below the coupon on a newly issued par value bond, based on current market conditions. The bond would be worth \$1,000 only if it had a coupon of \$100 per year. In a sense, an investor who buys and keeps the bond gives up \$20 per year for nine years. At 10 percent, this annuity stream is worth:

$$\begin{aligned}\text{Annuity present value} &= \$20 \times (1 - 1/1.10^9)/.10 \\ &= \$20 \times 5.7590 \\ &= \$115.18\end{aligned}$$

This is the amount of the discount.

What would the Xanth bond sell for if interest rates had dropped by 2 percent instead of rising by 2 percent? As you might guess, the bond would sell for more than \$1,000. Such a bond is said to sell at a *premium* and is called a *premium bond*.

This case is the opposite of that of a discount bond. The Xanth bond has a coupon rate of 8 percent when the market rate is now only 6 percent. Investors are willing to pay a premium to get this extra coupon amount. In this case, the relevant discount rate is 6 percent, and there are nine years remaining. The present value of the \$1,000 face amount is:

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$$\text{Present value} = \$1{,}000/1.06^9 = \$1{,}000/1.6895 = \$591.90$$

The present value of the coupon stream is:

$$\begin{aligned}\text{Annuity present value} &= \$80 \times (1 - 1/1.06^9)/.06\\ &= \$80 \times (1 - 1/1.6895)/.06\\ &= \$80 \times 6.8017\\ &= \$544.14\end{aligned}$$

We can now add the values for the two parts together to get the bond's value:

$$\text{Total bond value} = \$591.90 + 544.14 = \$1{,}136.03$$

The total bond value is therefore about $136 in excess of par value. Once again, we can verify this amount by noting that the coupon is now $20 too high, based on current market conditions. The present value of $20 per year for nine years at 6 percent is:

$$\begin{aligned}\text{Annuity present value} &= \$20 \times (1 - 1/1.06^9)/.06\\ &= \$20 \times 6.8017\\ &= \$136.03\end{aligned}$$

This is as we calculated.

Based on our examples, we can now write the general expression for the value of a bond. If a bond has (1) a face value of F paid at maturity, (2) a coupon of C paid per period, (3) t periods to maturity, and (4) a yield of r per period, its value is:

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$$\text{Bond value} = C \times [1 - 1/(1 + r)^t]/r + F/(1 + r)^t \quad [6.1]$$

Bond value = Present value of the coupons + Present value of the face amount

EXAMPLE 6.1 Semiannual Coupons

In practice, bonds issued in the United States usually make coupon payments twice a year. So, if an ordinary bond has a coupon rate of 14 percent, then the owner will get a total of $140 per year, but this $140 will come in two payments of $70 each. Suppose we are examining such a bond. The yield to maturity is quoted at 16 percent.

Bond yields are quoted like APRs; the quoted rate is equal to the actual rate per period multiplied by the number of periods. In this case, with a 16 percent quoted yield and semiannual payments, the true yield is 8 percent per six months. The bond matures in seven years. What is the bond's price? What is the effective annual yield on this bond?

Based on our discussion, we know that the bond will sell at a discount because it has a coupon rate of 7 percent every six months when the market requires 8 percent every six months. So, if our answer is equal to or exceeds $1,000, we know that we have made a mistake.

To get the exact price, we first calculate the present value of the bond's face value of $1,000 paid in seven years. This seven-year period has 14 periods of six months each. At 8 percent per period, the value is:

$$\text{Present value} = \$1{,}000/1.08^{14} = \$1{,}000/2.9372 = \$340.46$$

The coupons can be viewed as a 14-period annuity of $70 per period. At an 8 percent discount rate, the present value of such an annuity is:

$$\begin{aligned}\text{Annuity present value} &= \$70 \times (1 - 1/1.08^{14})/.08\\ &= \$70 \times (1 - .3405)/.08\\ &= \$70 \times 8.2442\\ &= \$577.10\end{aligned}$$

The total present value gives us what the bond should sell for:

$$\text{Total present value} = \$340.46 + 577.10 = \$917.56$$

To calculate the effective yield on this bond, note that 8 percent every six months is equivalent to:

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To calculate the effective yield on this bond, note that 8 percent every six months is equivalent to:

Effective annual rate = $(1 + .08)^2 - 1 = .1664$, or 16.64%

The effective yield, therefore, is 16.64 percent.

As we have illustrated in this section, bond prices and interest rates always move in opposite directions. When interest rates rise, a bond's value, like any other present value, will decline. Similarly, when interest rates fall, bond values rise. Even if we are considering a bond that is riskless in the sense that the borrower is certain to make all the payments, there is still risk in owning a bond. We discuss this next.

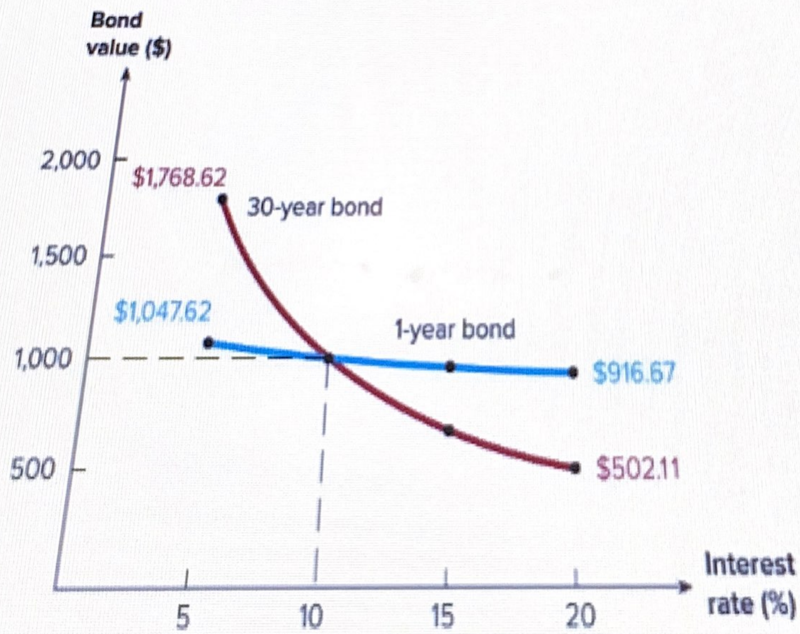
Online bond calculators and interest rate information are available at money.cnn.com/data/bonds and www.bankrate.com.

Interest Rate Risk

The risk that arises for bond owners from fluctuating interest rates is called *interest rate risk*. How much interest rate risk a bond has depends on how sensitive its price is to interest rate changes. This sensitivity directly depends on two things: the time to maturity and the coupon rate. As we will see momentarily, you should keep the following in mind when looking at a bond:

1. All other things being equal, the longer the time to maturity, the greater the interest rate risk.
2. All other things being equal, the lower the coupon rate, the greater the interest rate risk.

FIGURE 6.2 **Interest rate risk and time to maturity**



Value of a Bond with a 10 Percent Coupon Rate for Different Interest Rates and Maturities

	Time to Maturity	
Interest Rate	1 Year	30 Years
5%	$1,047.62	$1,768.62
10	1,000.00	1,000.00
15	956.52	671.70
20	916.67	502.11

20	916.67	502.11

We illustrate the first of these two points in Figure 6.2. As shown, we compute and plot prices under different interest rate scenarios for 10 percent coupon bonds with maturities of 1 year and 30 years. Notice how the slope of the line connecting the prices is much steeper for the 30-year maturity bond than it is for the 1-year maturity bond. This steepness tells us that a relatively small change in interest rates will lead to a substantial change in the bond's value. In comparison, the one-year bond's price is relatively insensitive to interest rate changes.

Intuitively, we can see that the reason that longer-term bonds have greater interest rate sensitivity is that a large portion of a bond's value comes from the $1,000 face amount. The present value of this amount isn't greatly affected by a small change in interest rates if the amount is to be received in one year. Even a small change in the interest rate, however, once it is compounded for 30 years, can have a significant effect on the present value. As a result, the present value of the face amount will be much more volatile with a longer-term bond.

Visit www.investorguide.com to learn more about bonds.

The other thing to know about interest rate risk is that, like most things in finance and economics, it increases at a decreasing rate. In other words, if we compared a 10-year bond to a 1-year bond, we would see that the 10-year bond has much greater interest rate risk. However, if you were to compare a 20-year bond to a 30-year bond, you would find that the 30-year bond has somewhat greater interest rate risk because it has a longer maturity, but the difference in the risk would be fairly small.

The reason that bonds with lower coupons have greater interest rate risk is easy to understand. As we discussed earlier, the value of a bond depends on the present value of its coupons and the present value of the face amount. If two bonds with different coupon rates have the same maturity, then the value of the one with the lower coupon is proportionately more dependent on the face amount to be received at maturity. As a result, all other things being equal, its value will fluctuate more as interest rates change. Put another way, the bond with the higher coupon has a larger cash flow early in its life, so its value is less sensitive to changes in the discount rate.

Bonds are usually not issued with maturities longer than 30 years. However, low interest rates have led to the issuance of bonds with much longer maturities. In the 1990s, Walt Disney issued "Sleeping Beauty" bonds with a 100-year maturity. Similarly, BellSouth (now known as AT&T), Coca-Cola, and Dutch banking giant ABN AMRO all issued bonds with 100-year maturities. These companies wanted to lock in the historically low interest rates for a *long* time. The current record holder for corporations appears to be Republic National Bank, which sold bonds with 1,000 years to maturity. Before these fairly recent issues, it appears the last time 100-year bonds were issued was in May 1954 by the Chicago and Eastern Railroad. And low interest rates in recent years have led to more 100-year bonds. For example, in 2017, Argentina joined Mexico, Ireland, and Belgium when it issued $2.75 billion in 100-year bonds. What made Argentina's bond sale unique was that the country had defaulted three times in the past 23 years.

We can illustrate the effect of interest rate risk using a 100-year BellSouth issue. The following table provides some basic information on this issue, along with its prices on December 31, 1995; May 6, 2008; and February 1, 2018.

Coupon	Price on	Price on	Percentage Change in Price	Price on	Percentage Change in Price

Maturity	Coupon Rate	Price on 12/31/95	Price on 5/6/08	Percentage Change in Price 1995–2008	Price on 2/1/18	Percentage Change in Price 2008–2018
2095	7.00%	$1,000.00	$1,008.40	+.84%	$1,164.21	+15.5%

Several things emerge from this table. First, interest rates apparently fell slightly between December 31, 1995, and May 6, 2008 (why?). After that, however, they fell even more (why?). The bond's price first gained .84 percent and then gained an additional 15.5 percent. These swings illustrate that longer-term bonds have significant interest rate risk.

Finding the Yield to Maturity: More Trial and Error

Frequently, we will know a bond's price, coupon rate, and maturity date, but not its yield to maturity. Suppose we are interested in a six-year, 8 percent coupon bond. The coupons are paid annually. A broker quotes a price of $955.14. What is the yield on this bond?

We've seen that the price of a bond can be written as the sum of its annuity and lump-sum components. Knowing that there is an $80 coupon for six years and a $1,000 face value, we can say that the price is:

$$\$955.14 = \$80 \times [1 - 1/(1 + r)^6]/r + \$1{,}000/(1 + r)^6$$

where r is the unknown discount rate, or yield to maturity. We have one equation here and one unknown, but we cannot solve for r explicitly. The only way to find the answer is to use trial and error (or, better yet, a spreadsheet or financial calculator).

This problem is essentially identical to the one we examined in the last chapter when we tried to find the unknown interest rate on an annuity. However, finding the rate (or yield) on a bond is even more complicated because of the $1,000 face amount

We can speed up the trial-and-error process by using what we know about bond prices and yields. In this case, the bond has an \$80 coupon and is selling at a discount. We thus know that the yield is greater than 8 percent. If we compute the price at 10 percent:

$$\begin{aligned}\text{Bond value} &= \$80 \times (1 - 1/1.10^6)/.10 + \$1{,}000/1.10^6 \\ &= \$80 \times 4.3553 + \$1{,}000/1.7716 \\ &= \$912.89\end{aligned}$$

Current market rates are available at www.bankrate.com.

At 10 percent, the value we calculate is lower than the actual price, so 10 percent is too high. The true yield must be somewhere between 8 and 10 percent. At this point, it's "plug and chug" to find the answer. You would probably want to try 9 percent next. If you did, you would see that this is, in fact, the bond's yield to maturity.

current yield
A bond's coupon payment divided by its closing price.

A bond's yield to maturity should not be confused with its **current yield**, which is a bond's annual coupon divided by its price. In the example we just worked, the bond's annual coupon was \$80 and its price was \$955.14. Given these numbers, we see that the current yield is \$80/\$955.14 = .0838, or 8.38 percent, which is less than the yield to maturity of 9 percent. The reason the current yield is too low is that it only considers the coupon portion of your return; it doesn't consider the built-in gain from the price discount. For a premium bond, the reverse is true, meaning that current yield would be higher because it ignores the built-in loss.

Our discussion of bond valuation is summarized in Table 6.1.

EXAMPLE 6.2 Current Events

built-in loss.

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EXAMPLE 6.2 Current Events

A bond has a quoted price of $1,080.42. It has a face value of $1,000, a semiannual coupon of $30, and a maturity of five years. What is its current yield? What is its yield to maturity? Which is bigger? Why?

Notice that this bond makes semiannual payments of $30, so the annual payment is $60. The current yield is thus $60/$1,080.42 = .0555, or 5.55 percent. To calculate the yield to maturity, refer back to Example 6.1. Now, in this case, the bond pays $30 every six months and it has 10 six-month periods until maturity. So, we need to find *r* as follows:

$$\$1,080.42 = \$30 \times [1 - 1/(1 + r)^{10}]/r + \$1,000/(1 + r)^{10}$$

After some trial and error, we find that *r* is equal to 2.1 percent. But the tricky part is that this 2.1 percent is the yield per *six months*. We have to double it to get the yield to maturity, so the yield to maturity is 4.2 percent, which is less than the current yield. The reason is that the current yield ignores the built-in loss of the premium between now and maturity.

TABLE 6.1

Summary of bond valuation

I. **Finding the value of a bond**
Bond value $= C \times [1 - 1/(1 + r)^t]/r + F/(1 + r)^t$
where:
C = Coupon paid each period
r = Rate per period
t = Number of periods
F = Bond's face value

II. **Finding the yield on a bond**
Given a bond value, coupon, time to maturity, and face value, it is possible to find the implicit discount rate, or yield to maturity, by trial and error only. To do this, try different discount rates

HOW TO CALCULATE BOND PRICES AND YIELDS USING A FINANCIAL CALCULATOR

CALCULATOR HINTS

Many financial calculators have fairly sophisticated built-in bond valuation routines. However, these vary quite a lot in implementation, and not all financial calculators have them. As a result, we will illustrate a simple way to handle bond problems that will work on just about any financial calculator.

To begin, of course, we first remember to clear out the calculator! Next, for Example 6.3, we have two bonds to consider, both with 12 years to maturity. The first one sells for $935.08 and has a 10 percent coupon rate. To find its yield, we can do the following:

Enter	12		100	−935.08	1,000
	N	I/Y	PMT	PV	FV
Solve for		11			

Notice that here we have entered both a future value of $1,000, representing the bond's face value, and a payment of 10 percent of $1,000, or $100, per year, representing the bond's annual coupon. Also notice that we have a negative sign on the bond's price, which we have entered as the present value.

For the second bond, we now know that the relevant yield is 11 percent. It has a 12 percent coupon and 12 years to maturity, so what's the price? To answer, we enter the relevant values and solve for the present value of the bond's cash flows:

Enter	12	11	120		1,000
	N	I/Y	PMT	PV	FV
Solve for				−1,064.92	

(continued)

HOW TO CALCULATE BOND PRICES AND YIELDS USING A FINANCIAL CALCULATOR

CALCULATOR HINTS

Many financial calculators have fairly sophisticated built-in bond valuation routines. However, these vary quite a lot in implementation, and not all financial calculators have them. As a result, we will illustrate a simple way to handle bond problems that will work on just about any financial calculator.

To begin, of course, we first remember to clear out the calculator! Next, for Example 6.3, we have two bonds to consider, both with 12 years to maturity. The first one sells for $935.08 and has a 10 percent coupon rate. To find its yield, we can do the following:

	N	I/Y	PMT	PV	FV
Enter	12		100	−935.08	1,000
Solve for		11			

Notice that here we have entered both a future value of $1,000, representing the bond's face value, and a payment of 10 percent of $1,000, or $100, per year, representing the bond's annual coupon. Also notice that we have a negative sign on the bond's price, which we have entered as the present value.

For the second bond, we now know that the relevant yield is 11 percent. It has a 12 percent coupon and 12 years to maturity, so what's the price? To answer, we enter the relevant values and solve for the present value of the bond's cash flows:

	N	I/Y	PMT	PV	FV
Enter	12	11	120		1,000
Solve for				−1,064.92	

(continued)

There is an important detail that comes up here. Suppose we have a bond with a price of $902.29, 10 years to maturity, and a coupon rate of 6 percent. As we mentioned earlier, most bonds actually make semiannual payments. Assuming that this is the case for the bond here, what's the bond's yield to maturity? To answer, we need to enter the relevant numbers like this:

Enter	20		30	−902.29	1,000
	N	I/Y	PMT	PV	FV
Solve for		3.7			

Notice that we entered $30 as the payment because the bond actually makes payments of $30 every six months. Similarly, we entered 20 for N because there are actually 20 six-month periods. When we solve for the yield, we get 3.7 percent, but the tricky thing to remember is that this is the yield per six months, so we have to double it to get the right answer: 2 × 3.7% = 7.4 percent, which would be the bond's reported yield.

SPREADSHEET STRATEGIES

HOW TO CALCULATE BOND PRICES AND YIELDS USING A SPREADSHEET

Like financial calculators, most spreadsheets have fairly elaborate routines available for calculating bond values and yields; many of these routines involve details that we have not discussed. However, setting up a simple spreadsheet to calculate prices or yields is straightforward, as our next two spreadsheets show:

SPREADSHEET STRATEGIES

HOW TO CALCULATE BOND PRICES AND YIELDS USING A SPREADSHEET

Like financial calculators, most spreadsheets have fairly elaborate routines available for calculating bond values and yields; many of these routines involve details that we have not discussed. However, setting up a simple spreadsheet to calculate prices or yields is straightforward, as our next two spreadsheets show:

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate bond yields**							
3								
4	Suppose we have a bond with 22 years to maturity, a coupon rate of 8 percent, and a price of							
5	$960.17. If the bond makes semiannual payments, what is its yield to maturity?							
6								
7	Settlement date:	1/1/00						
8	Maturity date:	1/1/22						
9	Annual coupon rate:	.08						
10	Bond price (% of par):	96.017						
11	Face value (% of par):	100						
12	Coupons per year:	2						
13	Yield to maturity:	.084						
14								
15	The formula entered in cell B13 is = YIELD(B7, B8, B9, B10, B11, B12); notice that face value and bond							
16	price are entered as a percentage of face value.							
17								

(continued)

	A	B	C	D	E	F	G	H
1								
2	**Using a spreadsheet to calculate bond values**							
3								
4	**Suppose we have a bond with 22 years to maturity, a coupon rate of 8 percent, and a yield to**							
5	**maturity of 9 percent. If the bond makes semiannual payments, what is its price today?**							
6								
7	Settlement date:	1/1/00						
8	Maturity date:	1/1/22						
9	Annual coupon rate:	.08						
10	Yield to maturity:	.09						
11	Face value (% of par):	100						
12	Coupons per year:	2						
13	Bond price (% of par):	90.49						
14								
15	The formula entered in cell B13 is = PRICE(B7, B8, B9, B10, B11, B12); notice that face value and bond							
16	price are entered as a percentage of face value.							
17								

In our spreadsheets, notice that we had to enter two dates, a settlement date and a maturity date. The settlement date is just the date you actually pay for the bond, and the maturity date is the day the bond actually matures. In most of our problems, we don't explicitly have these dates, so we have to make them up. For example, because our bond has 22 years to maturity, we just picked 1/1/2000 (January 1, 2000) as the settlement date and 1/1/2022 (January 1, 2022) as the maturity date. Any two dates would do as long as they were exactly 22 years apart, but these are particularly easy to work with. Finally, notice that we had to enter the coupon rate and yield to maturity in annual terms and then explicitly provide the number of coupon payments per year.

2000) as the settlement date and 1/1/2022 (January 1, 2022) as the maturity date. Any two dates would do as long as they were exactly 22 years apart, but these are particularly easy to work with. Finally, notice that we had to enter the coupon rate and yield to maturity in annual terms and then explicitly provide the number of coupon payments per year.

CONCEPT QUESTIONS

6.1a What are the cash flows associated with a bond?

6.1b What is the general expression for the value of a bond?

6.1c Is it true that the only risk associated with owning a bond is that the issuer will not make all the payments? Explain.

6.2 MORE ON BOND FEATURES

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In this section, we continue our discussion of corporate debt by describing in some detail the basic terms and features that make up a typical long-term corporate bond. We discuss additional issues associated with long-term debt in subsequent sections.

Securities issued by corporations may be classified roughly as *equity securities* and *debt securities*. At the crudest level, a debt represents something that must be repaid; it is the result of borrowing money. When corporations borrow, they generally promise to make regularly scheduled interest payments and to repay the original amount borrowed (i.e., the principal). The person or firm making the loan is called the *creditor*, or *lender*. The corporation borrowing the money is called the *debtor*, or *borrower*.

Information for bond investors can be found at www.investinginbonds.com.

From a financial point of view, the main differences between debt and equity are the following:

1. Debt is not an ownership interest in the firm. Creditors generally do not have voting power.
2. The corporation's payment of interest on debt is considered a cost of doing business and is fully tax deductible. Dividends paid to stockholders are *not* tax deductible.
3. Unpaid debt is a liability of the firm. If it is not paid, the creditors can legally claim the assets of the firm. This action can result in liquidation or reorganization, two of the possible consequences of bankruptcy. Thus, one of the costs of issuing debt is the possibility of financial failure. This possibility does not arise when equity is issued.

Is It Debt or Equity?

Sometimes it is not clear if a particular security is debt or equity. Suppose a corporation issues a perpetual bond with interest payable solely from corporate income if, and only if, earned. Whether or not this is really a debt is hard to say and is primarily a legal and semantic issue. Courts and taxing authorities would have the final say.

Corporations are very adept at creating exotic, hybrid securities that have many features of equity but are treated as debt. Obviously, the distinction between debt and equity is very important for tax purposes. So, one reason that corporations try to create a debt security that is really equity is to obtain the tax benefits of debt and the bankruptcy benefits of equity.

As a general rule, equity represents an ownership interest, and it is a residual claim. This means that equity holders are paid after debt holders. As a result, the risks and benefits associated with owning debt and equity are different. To give one example, note that the

As a general rule, equity represents an ownership interest, and it is a residual claim. This means that equity holders are paid after debt holders. As a result, the risks and benefits associated with owning debt and equity are different. To give one example, note that the maximum reward for owning a debt security is ultimately fixed by the amount of the loan, whereas there is no upper limit to the potential reward from owning an equity interest.

Long-Term Debt: The Basics

Ultimately, all long-term debt securities are promises made by the issuing firm to pay principal when due and to make timely interest payments on the unpaid balance. Beyond this, there are a number of features that distinguish these securities from one another. We discuss some of these features next.

The maturity of a long-term debt instrument is the length of time the debt remains outstanding with some unpaid balance. Debt securities can be short term (with maturities of one year or less) or long term (with maturities of more than one year).[1] Short-term debt is sometimes referred to as *unfunded debt*.[2]

Debt securities are typically called *notes*, *debentures*, or *bonds*. Strictly speaking, a bond is a secured debt. However, in common usage, the word *bond* refers to all kinds of secured and unsecured debt. We will, therefore, continue to use the term generically to refer to long-term debt.

The two major forms of long-term debt are public issue and private issue. We concentrate on public-issue bonds. Most of what we say about them holds true for private-issue, long-term debt as well. The main difference between public-issue and private-issue debt is

[1]There is no universally agreed-upon distinction between short-term and long-term debt. In addition, people often
refer to intermediate-term debt, which has a maturity of more than 1 year and less than 3 to 5, or even 10, years.
[2]The word *funding* is part of the jargon of finance. It generally refers to the long term. Thus, a firm planning to
"fund" its debt requirements may be replacing short-term debt with long-term debt.